Analytical studies of groundwater-head fluctuation in a coastal confined aquifer overlain by a semi-permeable layer with storage

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Abstract

Analytical studies are carried out to investigate groundwater head changes in a coastal aquifer system in response to tidal fluctuations. The system consists of an unconfined aquifer, a semi-confined aquifer and a semi-permeable confining unit between them. An exact analytical solution is derived to investigate the influences of both leakage and storage of the semi-permeable layer on the tide-induced groundwater head fluctuation in the semi-confined aquifer. This solution is a generalization of the solution obtained by (Jiao JJ and Tang Z. Wat Resour Res 1999;35:747-751) which ignored the storage of the semi-confining unit. The analytical solution indicates that both storage and leakage of the semi-permeable layer play an important role in the groundwater head fluctuation in the confined aquifer. While leakage is generally more important than storage, the impact of storage on groundwater head fluctuations changes with leakage. With the increase of leakage the fluctuation of groundwater head in the confined aquifer will be controlled mainly by leakage. The study also demonstrates that the influence of storativity of the semi-permeable layer on groundwater head fluctuation is negligible only when the storativity of the semi-permeable layer is comparable to or smaller than that of the confined aquifer. However, for aquifer systems with semi-permeable layer composed of thick, soft sedimentary materials, the storativity of the semi-permeable layer is usually much greater than that of the aquifer and its influence should be considered.

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1. Introduction

Since the 1950's, research on the dynamic interaction between groundwater and seawater has attracted much attention from hydrogeologists. Jacob [1950] and Ferris [1951] derived an equation to describe the tide-induced groundwater fluctuation in a confined aquifer. This equation has been widely used to study groundwater head and to estimate aquifer parameters in coastal areas [e.g., Carr and van der Kamp, 1969; Drogue et al., 1984, Erskine, 1991; Pandit, 1991; Serfes, 1991]. Meanwhile, different analytical solutions related to sea tidal fluctuations have been derived for coastal confined aquifers. For example: Li and Chen [1991] used an analytical solution to determine the length of a confined aquifer roof extending under the sea; Sun [1997] developed an analytical solution to understand the groundwater response to tidal fluctuations in an estuary using a two-dimensional tidal loading boundary condition. These studies focused on a single confined aquifer only. In many coastal aquifer systems, there is an unconfined aquifer above one or more confined aquifers and these aquifers are separated by semi-permeable layer(s) [e.g., Li and Chen, 1991; Jiao and Tang, 1999; Sheahan, 1977; Serfes, 1991]. Jiao and Tang [1999] derived an analytical solution to study the groundwater head fluctuations in the confined aquifer of a coastal multi-layered groundwater system consisting of a confined aquifer, an unconfined aquifer, and a semi-permeable layer between them. The following assumptions were used in their solution: 1) The aquifer system satisfies the assumptions introduced by Hantush and Jacob [1955]. That is, there is leakage through the semi-permeable layer but the storage of the layer is ignored. 2) Based on numerous field studies [White and Roberts, 1994; Chen and Jiao, 1998], they assumed that the water table fluctuation in the shallow unconfined aquifer can be neglected compared to the fluctuation of the water head in the confined aquifer since the specific yield of an unconfined aquifer usually is several orders of magnitude greater than the storage coefficient of the confined aquifer. This assumption may not be valid when the leakance of the leaky aquifer system is great [Volker and Zhang, 2000]. However, Jiao and Tang [2000] examined the leaky aquifer systems reported in literature and found that the leakance is usually very small for a real leaky aquifer system and there is no problem to use the assumption. On the basis of their analytical solution, Jiao and Tang [1999] found that the leakage has a significant damping effect on the groundwater fluctuation amplitude in the confined aquifer. When there is significant leakage from the unconfined aquifer, the lack of groundwater fluctuations in the confined aquifer cannot be considered to be indicative of poor hydraulic connection between the confined aquifer and the seawater.

According to various pumping test data available in literature on leaky confined aquifer systems (see Table 1), if the semi-permeable layer is composed of thick, soft sedimentary materials, its storage is much greater than that of the main aquifer. In this case, the assumption which ignores the storage may be questionable and the influence of the storage of the semi-permeable layer on the groundwater head fluctuation in the coastal confined aquifer should be investigated. It is also worthwhile to understand when the storage of the semi-permeable layer can be neglected without losing much accuracy. Based on this motivation, an attempt is made in this paper to derive an exact analytical solution that includes both the storage and leakage of the semi-permeable layer in a coastal multi-layered aquifer system. The assumptions underlying the solution are: (a) no head variation in the upper unconfined aquifer, (b) no horizontal flow in the leaking layer and (c) all formations have a clear-cut vertical boundary with seawater. After the analytical solution is derived, the influence of both storage and leakage of the semi-permeable layer on the groundwater head fluctuation behavior in the confined aquifer is discussed. A hypothetical example is used to demonstrate how the storativity of the semi-permeable layer will influence the estimated aquifer parameters when it is ignored.

2. Conceptual model and analytical solution

Consider a subsurface system consisting of a leaky confined aquifer, an unconfined aquifer and a semi-permeable layer between them (Figure 1). All the layers extend landward infinitely. The unconfined aquifer and the semi-permeable layer are assumed to terminate at the coastline, which is defined as the intersection of the mean sea level and the beach face. Both the aquifer and the semi-permeable layer are homogeneous, horizontal, and with constant thickness. Following Hantush [1960] and Neuman and Witherspoon [1969], the flows in the confined aquifer and in the semi-permeable layer are assumed to be horizontal and vertical, respectively. The coastline is straight. Let the x axis be positive landward and coincide with the middle line of the semi-permeable layer with its origin at the coastline. Let the z axis be vertical, positive upward with its origin coinciding with that of the x axis (see Figure 1). Following Jiao and Tang [1999] and White and Roberts [1994], assume that the shallow unconfined aquifer has a large specific yield which can damp effectively the tidal effect so that the tidal fluctuation in the unconfined aquifer is negligible compared to that in the confined aquifer. That is to say, the water table of the unconfined aquifer is a constant. According to these assumptions and the theory of Hantush [1960], the mathematical model for the subsurface system can be written as the following boundary-value problem including two unknown functions of groundwater heads in the semi-permeable layer and the confined aquifer.

1. Groundwater flow in the semi-permeable layer:

$$S_S' \frac{\partial h'}{\partial t} = K' \frac{\partial^2 h'}{\partial z^2}, \quad -\infty < t < \infty, \quad -\frac{b'}{2} < z < \frac{b'}{2}, \tag{1}$$

$$h'(b'/2, t; x) = h_z = 0, \tag{2}$$

$$h'(-b'/2, t; x) = h(t, x), \tag{3}$$

where $h'(z,t;x)$ denotes the hydraulic head [L] of the groundwater in the semi-permeable layer at the location (x,z) and time t; S_S' and K' are the specific storativity [L^{-1}] and vertical hydraulic conductivity [LT^{-1}] of the semi-permeable layer, respectively, b' is the thickness of the semi-permeable layer [L], h_z is the constant water table of the unconfined aquifer overlying the semi-permeable layer [L]. The water table surface is chosen to be the datum of hydraulic head so that $h_z = 0$, and $h(x,t)$ is the hydraulic head [L] of the main aquifer at the instant t and location x.

2. Groundwater flow in the main aquifer:

$$S \frac{\partial h}{\partial t} = T \frac{\partial^2 h}{\partial x^2} + K' \frac{\partial h'}{\partial z}\left(-\frac{b'}{2}, t; x\right), \qquad -\infty < t < \infty, \quad x > 0, \tag{4}$$

$$h(0,t) = A\cos(\omega t + c), \tag{5}$$

$$h(\infty, t) = 0, \tag{6}$$

where $h(0,t)$ is the head at the coastline $x = 0$; S and T are the storativity (dimensionless) and the transmissivity [L^2T^{-1}] of the main aquifer, respectively [Hantush, 1960]; A is the amplitude of the sea tide [L]; c is the phase shift [dimensionless]; and ω is the angular velocity [T^{-1}] of tide and equals $2\pi / t_0$, where t_0 is the tidal period [T] [Fetter, 1994]. Equation (6) gives the boundary condition of $h(x,t)$ on the inland side, which states that the tide has no effect far inland as x approaches infinity.

The derivation of the solutions $h'(z,t;x)$ and $h(x,t)$ to the boundary value system (1)-(6) is presented in detail in the appendix. The analysis will focus on the groundwater head $h(x,t)$ in the main aquifer because it is much more useful than the groundwater head $h'(z,t;x)$ in the semi-permeable layer in practice. Hence, only the expression of $h(x,t)$ will be given here. Details about $h'(z,t;x)$ are presented in the appendix. For convenience of discussion, three new parameters are introduced. They are the main aquifer's tidal propagation parameter a [L^{-1}], the storativity ratio s (dimensionless) and the dimensionless leakage u

$$a = \sqrt{\frac{\omega S}{2T}} = \sqrt{\frac{\pi S}{T t_0}}, \tag{7}$$

$$s = \frac{S'}{S} = \frac{S'_S b'}{S}, \tag{8}$$

$$u = \frac{L}{\omega S} = \frac{K'}{\omega S b'}, \tag{9}$$

where $L = K'/b'$ [T^{-1}] is the specific leakage of the semi-permeable layer [Hantush and Jacob, 1955; Hantush, 1960]. Then the solution $h(x,t)$ can be written as

$$h(x,t) = Ae^{-pax} \cos(\omega t - qax + c), \tag{10}$$

where p and q are dimensionless constants defined as

$$p(s,u) = \sqrt{\sqrt{\xi^2(s,u) + \eta^2(s,u)} + \eta(s,u)}, \tag{11.a}$$

$$q(s,u) = \frac{\xi}{p} = \sqrt{\sqrt{\xi^2(s,u)+\eta^2(s,u)} - \eta(s,u)}\,, \tag{11.b}$$

in which the functions $\xi(s,u)$ and $\eta(s,u)$ are given by

$$\xi(s,u) = 1 + u\theta \frac{1-2e^{-2\theta}\sin 2\theta - e^{-4\theta}}{1-2e^{-2\theta}\cos(2\theta)+e^{-4\theta}}\,, \tag{12}$$

$$\eta(s,u) = u\theta \frac{1+2e^{-2\theta}\sin 2\theta - e^{-4\theta}}{1-2e^{-2\theta}\cos(2\theta)+e^{-4\theta}}\,, \tag{13}$$

where $\theta(s,u)$ is a dimensionless parameter defined as

$$\theta = \sqrt{\frac{s}{2u}}\,. \tag{14}$$

3. Discussion

3.1 Comparison with existing analytical solutions

The solution by Jacob [1950] assumed that there is only a confined aquifer or $K' = 0$. The solution by Jiao and Tang [1999] assumed that the storage of the semi-permeable layer is negligible, or $S' = 0$. In this section it will be demonstrated that these solutions are special cases of the solution (10).

When $K' = 0$ or equivalently u =0, from (12) and (13), one finds that

$$\lim_{u\to+0} \xi(s,u) = 1, \qquad \lim_{u\to+0} \eta(s,u) = 0\,. \tag{15}$$

Substituting (15) back into (11.a) and (11.b), yields

$$p\,|_{u=0} \equiv 1\,, \qquad q\,|_{u=0} \equiv 1\,. \tag{16}$$

Combination of (7), (10) and (16) directly leads to

$$h(x,t) = Ae^{-ax}\cos(\omega t - ax + c)\,. \tag{17}$$

This is the traditional solution by Jacob [1950] and Ferris [1951].

When $S' = 0$ or equivalently $s = 0$, from (12) and (13), one has

$$\lim_{s\to+0} \xi(s,u) = \lim_{\theta\to+0} \xi(s,u) = 1, \qquad \lim_{s\to+0} \eta(s,u) = \lim_{\theta\to+0} \eta(s,u) = u. \tag{18}$$

Substituting (18) back into (11.a) yields

$$p(0,u) = \sqrt{\sqrt{1+u^2} + u}, \qquad q(0,u) = \sqrt{\sqrt{1+u^2} - u}. \tag{19}$$

Using (18), (19) and (10), it follows that

$$h(x,t) = Ae^{-ap(0,u)x} \cos(\omega t - q(0,u)ax + c), \tag{20}$$

which is the same as the solution expressed by equation (4) in Jiao and Tang [1999].

3.2 Influence of various parameters on groundwater head fluctuation

From equations (7)-(14), it can be seen that three important aquifer parameters, the main aquifer’s tidal propagation parameter a, the storativity ratio s and the dimensionless leakage u, are involved in the model. It is important to know the rough ranges of these three parameters in real aquifer systems. To this end, the values of a, s and u corresponding to some case studies are calculated by using equations (7), (8) and (9) for the semidiurnal sea tide whose angular velocity $\omega = 0.506\,\mathrm{h}^{-1}$. The results are listed in Table 1. One can see that a ranges from $9\times10^{-4}\ \mathrm{m}^{-1}$ to $9\times10^{-3}\ \mathrm{m}^{-1}$, s from 5.22 to 117 and u from 0.147 to 8.921. Based on these data, the discussion ranges of the parameters will be chosen 0 to 100 for s and 0 to 10 for u.

Solution (10) shows that the groundwater head fluctuation with time at a fixed inland location x is also sinusoidal if the sea tide is a sinusoidal wave.

Assume that at a fixed inland location *x*, the ratio of the groundwater head fluctuation amplitude to the sea tide amplitude is A_x, and the time lag [T] of groundwater response to sea tidal fluctuation is t_{lag}, then in view of (10), one has

$$h(x,t)/A = A_x \cos(\omega(t - t_{lag}) + c) . \qquad (21)$$

Comparison of (21) and (10) yields

$$A_x = \exp(-p(s,u)ax) , \qquad (22)$$

$$\omega t_{lag} = \frac{a\xi(s,u)x}{p(s,u)} = q(s,u)ax . \qquad (23)$$

The parameters *s* and *u* in the parentheses after *p* and *q* highlight their functional relationship. Theoretically, if one has the observed data such as A_x, *x*, t_{lag} and ω, and knows one of the three parameters *a*, *s* and *u*, then the other two unknowns of the three can be estimated by solving equations (22) and (23).

There are three independent parameters (*a*, *u*, s) but only two equations (22 and 23) to characterize them. This phenomenon of over-parameterization cannot be improved even if one has observed data of A_x and t_{lag} at different observation wells (i.e., for different values of *x*) because the equations derived from (22) for different values of *x* are not independent and so are the equations derived from (23). This fact implies that the three parameters (*a*, *u*, *s*) cannot be estimated uniquely by using (22) and (23). It also indicates that different aquifer systems with different values of (*a*, *u*, *s*) may produce exactly the same responses to a given sea tide as long as the values of $p(s, u)a$ and $q(s, u)a$ remain the same for different (*a*, *u*, *s*).

Equations (22) and (23) suggest that the groundwater head fluctuation amplitude decreases with the landward distance from the coastline exponentially, while the time lag increases with it linearly. When the tidal propagation parameter *a* is fixed, the groundwater head fluctuation in the confined aquifer is determined by the leakage and storage of the overlying semi-permeable layer via the two dimensionless parameters $p(s, u)$ and $q(s, u)$. The greater the parameter *p* is, the more quickly will the fluctuation amplitude of the groundwater head damp landward. The greater the parameter *q*, the longer will the time lag be. Hence, *p* is called dimensionless damping coefficient and *q* dimensionless time lag coefficient.

Figure 2 shows how the dimensionless damping coefficient *p* changes with dimensionless leakage *u* for different storativity ratio *s*. One can see that *p* increases with both *s* and *u*. If the storativity of the semi-permeable layer is not much greater than that of the main aquifer, its influence on the groundwater head fluctuation is insignificant, as can be seen from the small discrepancy of *p* between the two curves corresponding to $s = 0$ and $s = 1$ in Figure 2.

Figure 3 shows how the time lag coefficient *q* changes with dimensionless leakage *u* for different storativity ratio *s*. One can see that, for any fixed nonzero *u*, *q* increases with *s*. But for a fixed nonzero value of *s*, *q* increases with *u* first to a peak and then decreases to zero monotonically as *u* tends to infinite. The value of *u* at which the peak of *q* occurs increases as the storativity ratio *s* increases.

The storage of the leaky layer will damp the groundwater head fluctuation in the main aquifer and increase the time lag by taking water into the leaky unit when the head in the main aquifer tends to increase and releasing water from the leaky layer when the head tends to decrease. This is why both the damping coefficient *p* and time lag coefficient *q* increase with the storativity ratio *s* in Figures 2 and 3.

Due to the constant water table in the unconfined aquifer, the leakage of the leaky layer will tend to damp the groundwater head fluctuation in the main aquifer. This explains why the damping coefficient *p* increases with the dimensionless leakage *s* in Figure 2. When $s = 0$, which is the case of Jiao and Tang [1999], an increase in leakage *u* will have the same effect on the time lag as an increase in the transmissivity of the main aquifer because both increases enhance the water transfer speed in the aquifer and therefore will lead to a decrease in time lag [see equations (5) and (7) in Jiao and Tang, 1999]. For a fixed nonzero *s*, its influence on the head behavior in the confined aquifer is subject to the leakage when leakage is small. When there is no leakage ($u = 0$), there will be no water transferred between the leaky layer and the confined aquifer, hence, the storage of the leaky layer has no influence on the head behavior in the confined aquifer at all, no matter how great the storativity ratio *s* is. This explains why $p|_{u=0} \equiv 1$ and $q|_{u=0} \equiv 1$, or why all the curves for different values of *s* in Figure 2 and 3 coincide with each other at the point (0, 1). Similarly, very small *u* will hinder the groundwater from transferring between the confined aquifer and the leaky layer so that the storage capacity of the leaky layer cannot be fully used. In this case an increase in *u* will speed up the rate of water transferring between the leaky layer and the main aquifer and raise the utilization ratio of the storage capacity of the leaky layer. When *u* increases and becomes greater than a certain value, say, u_0, the storage capacity of the leaky layer becomes to be fully used. These explain the non-monotonic behavior of *q* with respect to *u* in Figure 3: For a fixed nonzero *s*, an increase of *u* from zero will, on the one hand, have a decreasing effect on *q* due to its own influence pattern when $s = 0$, on the other hand, have an increasing effect on *q* because it raises the utilization ratio of the storage capacity of the leaky layer before u increases to u_0 and the whole storage capacity is used. This increasing effect is stronger than the decreasing one when u is small enough. As a result, q increases with u at the beginning. When u is greater than u_0 the whole storage capacity is used, the increasing effect on q ceases to work so q decreases with u. Consequently, somewhere between zero and u_0, say, u^*, the two opposite effects balance and q reaches its maximum, and naturally, u^* increases as the storativity ratio s increases because greater s will lead to greater increasing effect on q when u increases from zero."

Figure 4 demonstrates the different influences of the storage and leakage of the semi-permeable layer on the amplitude of groundwater head fluctuation. Figure 4(a) shows the variation of the dimensionless groundwater head amplitude A_x defined by (22) with dimensionless distance for different storativity ratio s when dimensionless leakage u is fixed at 5. Figure 4(b) shows the variation of the dimensionless groundwater head amplitude A_x with dimensionless distance ax for different dimensionless leakage u when storativity ratio s is fixed at 50. Comparison of Figures 4(a) and 4(b) demonstrates that the landward attenuation of the amplitude of groundwater head fluctuation appears to be more sensitive to the dimensionless leakage than to the storativity ratio. When s increases from 0 to 100, the dimensionless landward distance from the coastline disturbed by the sea tide decreases from 1.5 to 0.8 or so, but when u increases from 0 to 10, the dimensionless landward distance disturbed by the sea tide decreases from more than 4.0 to only 0.8 or so.

Figure 5 shows how the dimensionless groundwater head $h(x,t)/A$ defined by (21) changes with the dimensionless time ωt at the fixed inland location $ax = 0.3$ from the coastline. Figure 5(a) is for different values of the storativity ratio s when the dimensionless leakage u is fixed at 5.0. Figure 5(b) is for different values of the dimensionless leakage u when the storativity ratio s is fixed at 50. The fluctuation amplitude A_x decreases with both s and u, but is more sensitive to the latter.

The fixed value u=5 in Figures 4(a) and 5(a) is chosen to be approximately the middle point of its real range listed in Table 1, and so is the fixed value s=50 in Figures 4(b) and 5(b). The trend of A_x defined by (22) is the same as that implied in Figures 4 and 5 for other fixed values of u (>0) and s.

Figure 5 shows that the fluctuation amplitude A_x of the groundwater head in a coastal aquifer is decreased when leakage or storativity ratio is increased. This suggests that, if the confined aquifer is overlain by a semi-permeable layer and an unconfined aquifer, the traditional explanation and understanding of the relationship between the confined aquifer and its connected coastal tide may be too simple and misleading. It was believed that a quicker damping of tide-induced groundwater head fluctuation in the confined aquifer was caused by a greater tidal propagation parameter a of the aquifer [e.g. Ferris 1951, Gregg 1966, White and Roberts, 1994]. This may not be necessarily true if there is a semi-permeable layer with significant leakage and storage. A similar finding was also obtained by Jiao and Tang [1999] when they considered only the leakage from the semi-permeable layer.

The above discussion shows that the impact of the storativity ratio on the fluctuation of groundwater head in the aquifer system is not negligible unless the storativity ratio is small (≤ 1), as shown in Figures 2 and 3. When the storativity ratio is small the groundwater head fluctuation can be described approximately by the solution of Jiao and Tang [1999]. It is of interest to know the rough range of storativity ratios in real aquifer systems. Table 1 lists the storativity ratio in some aquifer systems discussed in literature [Hantush, 1956 and 1960; Neuman and Witherspoon, 1972; Sheahan, 1977; Dawson and Istok, 1991; Batu 1998; Chen and Jiao, 1999]. It can been seen that the ratio in most of the aquifer systems listed in Table 1 is much greater than 1. This suggests that for most coastal leaky aquifer systems the equation derived in this paper which includes the storativity of the semi-permeable layer should be employed.

3.3. Hypothetical example

A hypothetical example is designed to understand how much error can be introduced in estimating aquifer parameters if the storage of the semi-permeable layer is ignored. The approach used is as follows: assume first that all the "true values" of the three parameters *a*, *u*, *s* are known and equation (10) is used to generate groundwater head fluctuation data, and the data are then treated as "observed" data and an inverse problem is solved to estimate only the two parameters *a* and *u* based on the equation derived by Jiao and Tang [1999] which does not include the storativity of the semi-permeable layer. By comparing the "estimated" and the "true" values of the parameters *a* and *u*, one can see the parameter estimation errors caused by the neglect of the storativity of the semi-permeable layer.

Assume that the sea tide is semidiurnal with period $t_0 = 12.42$ h [Jacob, 1950; Carr and van der Kamp, 1969], amplitude $A = 1.0$ m and phase shift $c = 0$. The angular velocity of the sea tide $\omega = \dfrac{2\pi}{12.42} \approx 0.506$ h^{-1}. An observation well is screened in the confined aquifer at an inland point which is $x_0 = 50$ m far from the coastline. The "true" values of the aquifer parameters are $a = 0.001$ m^{-1}, $u =$ 5 and $s = 10$. Based on equation (10), the "observed" groundwater head fluctuation in the observation well should be:

$$h(x_0, t) = 0.84\cos(0.506t - 0.061)\,, \tag{27}$$

where the units of $h(x_0, t)$ and t are meter and hour, respectively. Then the solution of Jiao and Tang [1999] (i.e., equation (20) of this paper), which assumed $s = 0$, is used to estimate the parameters a and u by solving the two equations

$$0.84/1.0 = \exp(-p(0,u)ax_0)\,, \tag{28}$$

$$0.061 = q(0,u)ax_0\,. \tag{29}$$

The "true" and estimated parameters as well as their relative errors are presented in the row of Table 2 beginning with Aquifer system 1. As can be seen, a is overestimated by 106% and u underestimated by 74%. The errors for the two estimated parameters are very significant when the storage of the semi-permeable layer is ignored.

Then the above procedures are repeated for other 6 different aquifer systems corresponding to smaller values of s. The estimated parameters and their errors are shown in the rows of Table 2 beginning with Aquifer system 2 to 7. As shown in Table 2, for these hypothetical aquifer systems, the parameter estimation errors decrease as s reduces and become less than 15% only when $s < 0.5$.

4. Summary

This paper investigates the groundwater head fluctuation in a coastal aquifer with a confined aquifer, an unconfined aquifer, and a semi-permeable layer between them. An exact analytical solution is derived to examine the influence of both the storage and leakage of the semi-permeable layer on tidal response in the confined aquifer. This solution is based on the assumption that water table fluctuations in the shallow unconfined aquifer can be neglected compared to that in the confined aquifer. The analytical solution is a generalization of the solution obtained by Jiao and Tang [1999] which ignores the storage of the overlying semi-permeable layer.

The discussion on the analytical solution indicates that both storage and leakage of the semi-permeable layer play an important role in the groundwater head fluctuation in the confined aquifer. While leakage is generally more important than storage, the impact of storage on groundwater head fluctuations changes with leakage. With the increase of leakage, the fluctuation of groundwater head in the confined aquifer will be controlled mainly by leakage.

The study also demonstrates that the influence of storativity of the semi-permeable layer on groundwater head fluctuation is negligible only when the storativity of the semi-permeable layer is comparable to or smaller than that of the confined aquifer ($s \leq 1$). The data of some actual leaky confined aquifer systems compiled from publications available to the authors show, however, that the storativity ratio s is far more than 1. This indicates that, for most aquifer systems, the storativity of the semi-permeable layer should be considered. This is further demonstrated by a hypothetical example, which shows that the errors in estimating the aquifer parameters can be very significant if the storativity of the semi-permeable layer is ignored.

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Appendix

Assume that

$$h(x,t) = A\mathrm{Re}[X(x)\exp(i(\omega t + c))], \tag{A1}$$

$$h'(z,t;x) = A\mathrm{Re}[Z(z)X(x)\exp(i(\omega t + c))], \tag{A2}$$

where $X(x)$ and $Z(z)$ are complex functions, Re denotes the real part of the followed complex expression, $i = \sqrt{-1}$. Substituting (A2) back into (1)-(3), and extending the three resultant real equations into complex ones with respect to the unknown function $Z(z)$, yields

$$i\omega S_S' Z = K'Z'', \qquad -\frac{b'}{2} < z < \frac{b'}{2}, \tag{A3}$$

$$Z(b'/2) = h_z = 0\,, \tag{A4}$$

$$Z(-b'/2) = 1\,. \tag{A5}$$

The solution of (A3)-(A5) is

$$Z = \frac{\exp(-(1+i)\sigma(z-b'/2)) - \exp((1+i)\sigma(z-b'/2))}{\exp((1+i)\sigma\, b') - \exp(-(1+i)\sigma b')}, \tag{A6}$$

where

$$\sigma = \sqrt{\frac{\omega S_S'}{2K'}}\,. \tag{A7}$$

Using (A6), one obtains

$$Z'(-b'/2) = -\sigma(f_r(\theta) + if_i(\theta))\,, \tag{A8}$$

where

$$f_r(\theta) = (1 + 2e^{-2\theta}\sin(2\theta) - e^{-4\theta})/\rho(\theta)\,, \tag{A9}$$

$$f_i(\theta) = (1 - 2e^{-2\theta}\sin(2\theta) - e^{-4\theta}) / \rho(\theta)\,, \tag{A10}$$

$$\rho(\theta) = 1 - 2e^{-2\theta}\cos(2\theta) + e^{-4\theta}, \tag{A11}$$

$$\theta = \sigma b' = b'\sqrt{\frac{\omega S'_S}{2K'}} = \sqrt{\frac{s}{2u}}\ . \tag{A12}$$

Now substituting eqns (A1), (A2) and (A8) back into eq. (4), and eq. (A1) back into eqns (5) and (6), and extending the three resultant real equations into complex ones with respect to the unknown function *X*(*x*), yields

$$[i(\omega S + K'\sigma\, f_i(\theta)) + K'\sigma\, f_r(\theta)]X = TX'', \qquad 0 < x < +\infty, \tag{A13}$$

$$X(0) = 1\,, \tag{A14}$$

$$X(+\infty) = 0\,. \tag{A15}$$

The solution of (A13)-(A15) is $X = \exp(-a(p + iq)x)$,

(A16)

where a, p and q are given by eqns (7), (11.a) and (11.b), respectively. Substituting (A16) back into (A1) leads to solution (10).

Substituting (A6), (A16) back into (A2) leads to the solution of (1)-(3)

$$\begin{aligned} h'(z,t;x) = Ae^{-pax}\{&e^{-\sigma(z+b'/2)}[r_c\cos(\omega t - \sigma(z+\tfrac{b'}{2}) - qax + c) + r_s\sin(\omega t - \sigma(z+\tfrac{b'}{2}) - qax + c)] \\ &- e^{-\theta+\sigma(z-b'/2)}[r_c\cos(\omega t + \sigma(z-\tfrac{b'}{2}) - \theta - qax + c) + \\ &+ r_s\sin[\omega t + \sigma(z-\tfrac{b'}{2}) - \theta - qax + c]\}, \end{aligned} \tag{A17}$$

where

$$r_c(\theta) = [1 - e^{-2\theta}\cos(2\theta)] / \rho(\theta), \tag{A18}$$

$$r_s(\theta) = e^{-2\theta}\sin(2\theta) / \rho(\theta)\,. \tag{A19}$$

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List of Captions

Table 1. Ranges of parameters *a, s* and *u* in leaky confined aquifer systems reported in literature #

Case No.	Leaky aquifers			Semi-permeable layers			Model parameters			References
	T, m^2/d	S	b, m	K', m/d	S'	b', m	a, 1/m	s	u	
1	108	0.000036	14	0.013	0.001	4	0.0014	27.8	7.43	Sheahan, 1977
	158	0.000045	14	0.013	NA	4	0.0013	NA	5.95	Batu, 1998
	114	0.00003	14	0.013	0.0035	4	0.0013	116.7	8.92	Batu, 1998
2	1624	0.000112	28	0.0016	0.003	9.1	0.0006	26.8	0.13	Neuman & Witherspoon, 1972
	1624	0.000112	28	0.0016	0.012	15.2	0.0006	107.1	0.08	Neuman & Witherspoon, 1972
3	7.31	0.000098	6.1	0.0082	0.004	3	0.009	40.8	2.29	Dawson & Istok,1991
4	733	0.00009	16	0.0021	0.0005	13.1	0.0009	5.2	0.15	Chen & Jiao,1999
5	753	0.00004	NA	NA	NA	NA	NA	NA	0.02	Batu 1998 & Hantush 1956
6	1097	0.000092	NA	NA	NA	NA	NA	NA	0.004	Batu 1998 & Hantush 1956

Note: “NA” means that data are not available; The three sets of data of in Case 1 were obtained using different aquifer parameter estimation method based on the same set of field pumping test data; The units in Cases 2 and 3 were in English system in the original paper and are converted into metric system; The data of Case 4 are arithmetic average values of 4 different zones in the original paper.

Table 2 Impact of storativity ratio on parameter estimation when it is ignored

	True Parameters			Estimated parameters a_e and u_e when s is ignored			
Aquifer Systems	a (m^{-1})	u	s	a_e (m^{-1})	$(a_e-a)/a$	u_e	$(u_e-u)/u$
1	0.001	5	10	0.00206	106%	1.28	-74%
2	0.001	5	5	0.00163	63%	1.924	-62%
3	0.001	5	1	0.00115	15%	3.75	-25%
4	0.001	5	0.8	0.00113	13%	3.95	-21%
5	0.001	5	0.5	0.00108	8%	4.29	-14%
6	0.001	5	0.2	0.00103	3%	4.69	-6%
7	0.001	5	0.1	0.00102	2%	4.84	-3%

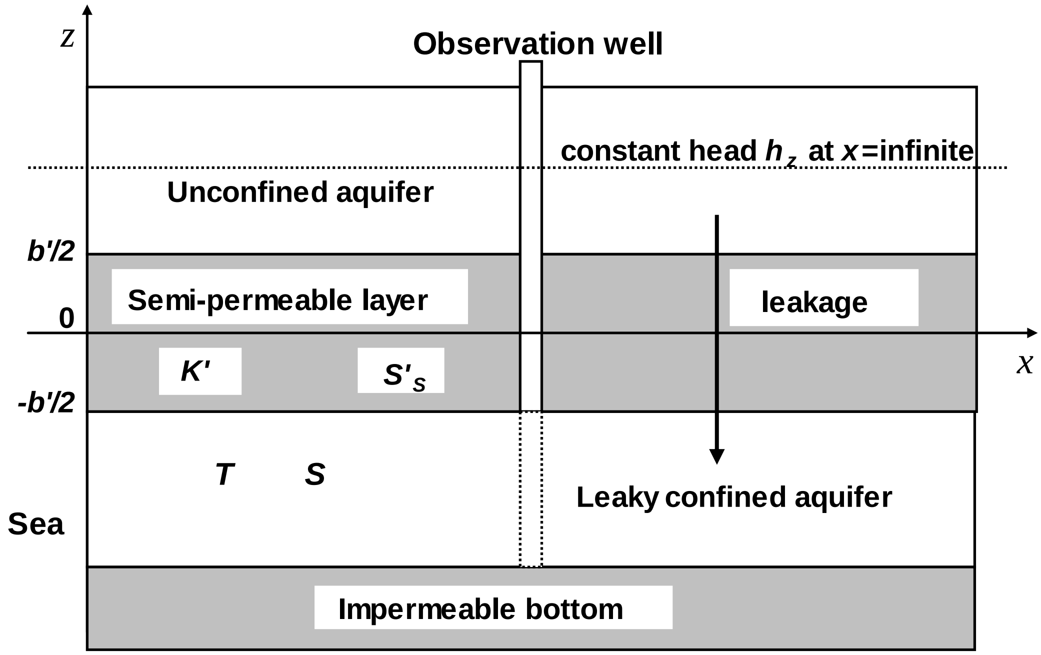


Figure1. Schematic representation of a leaky confined aquifer system near open tidal water.

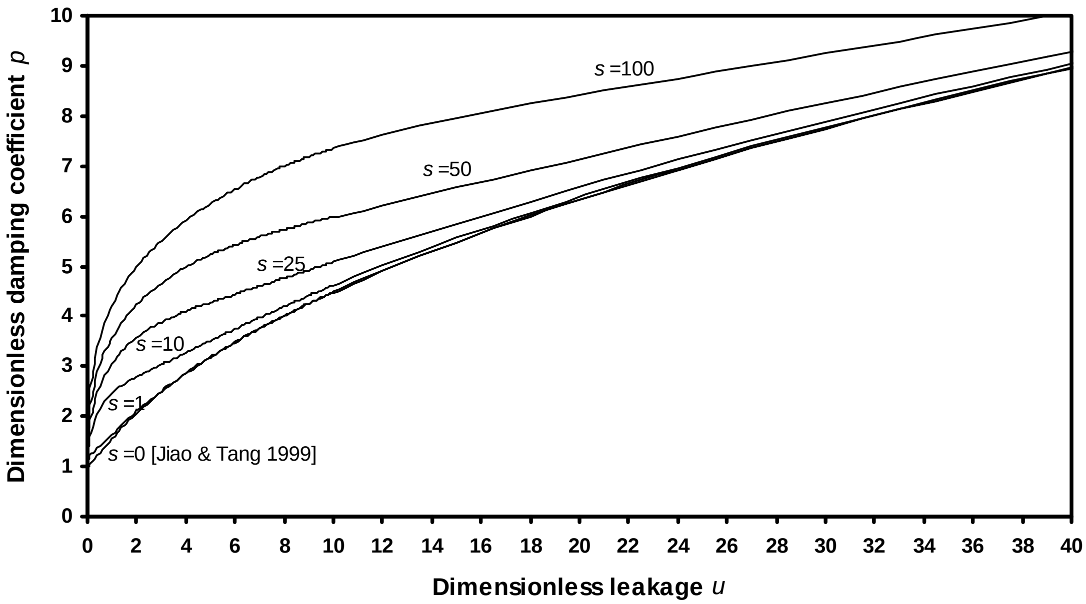


Figure 2. Dimensionless leakage u versus dimensionless damping coefficient p curves for different values of storativity ratio s

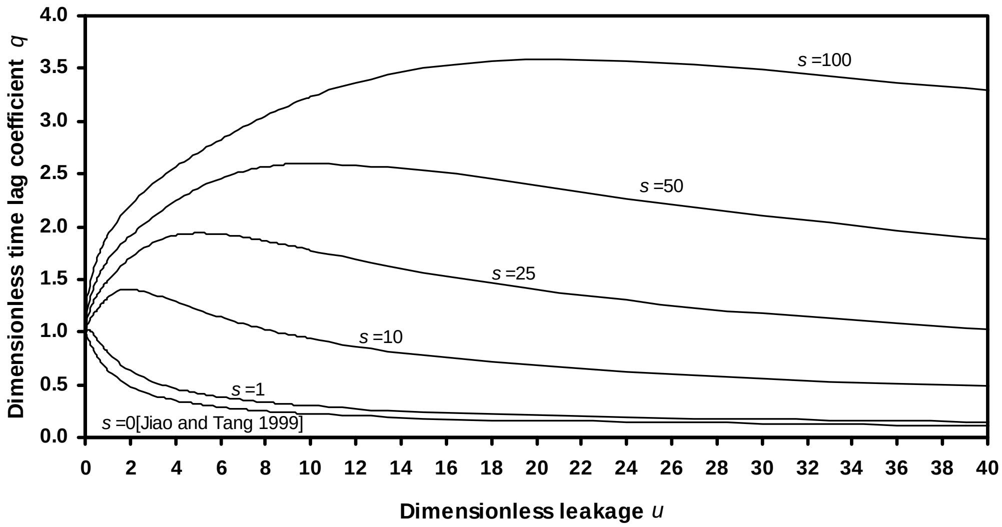


Figure 3. Dimensionless leakage u versus time lag coefficient q curves for different values of storativity ratio s

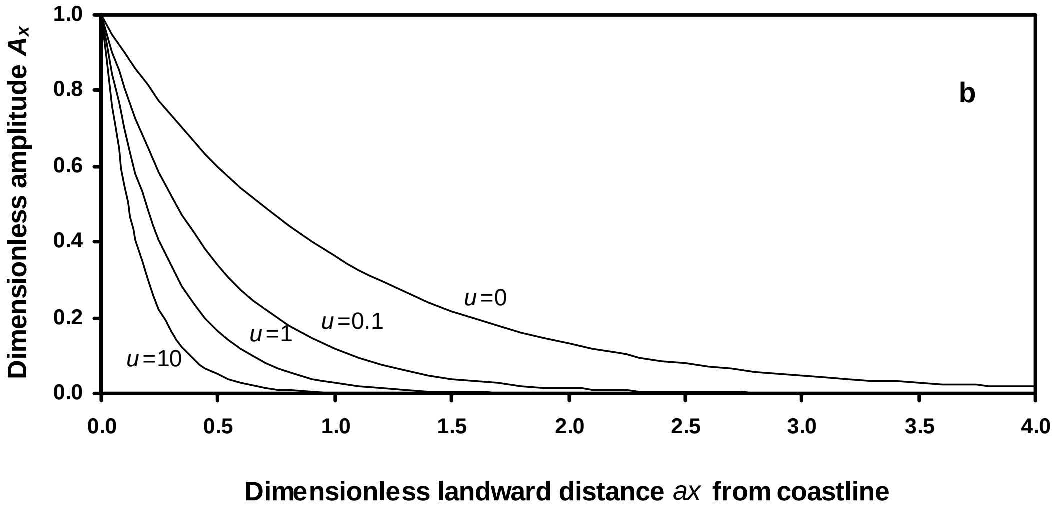
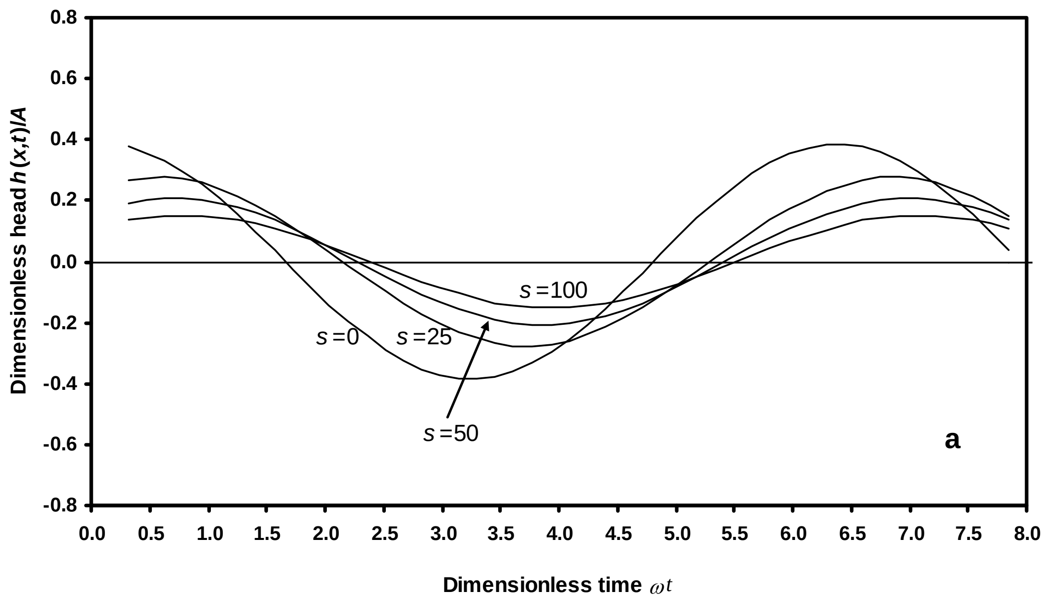


Figure 4. Change of dimensionless groundwater head fluctuation amplitude A_x with dimensionless landward distance ax from the coastline (a) for different values of storativity ratio s as dimensionless leakage is fixed to be u=5.0 and (b) for different values of dimensionless leakage u as storativity ratio s=50.0



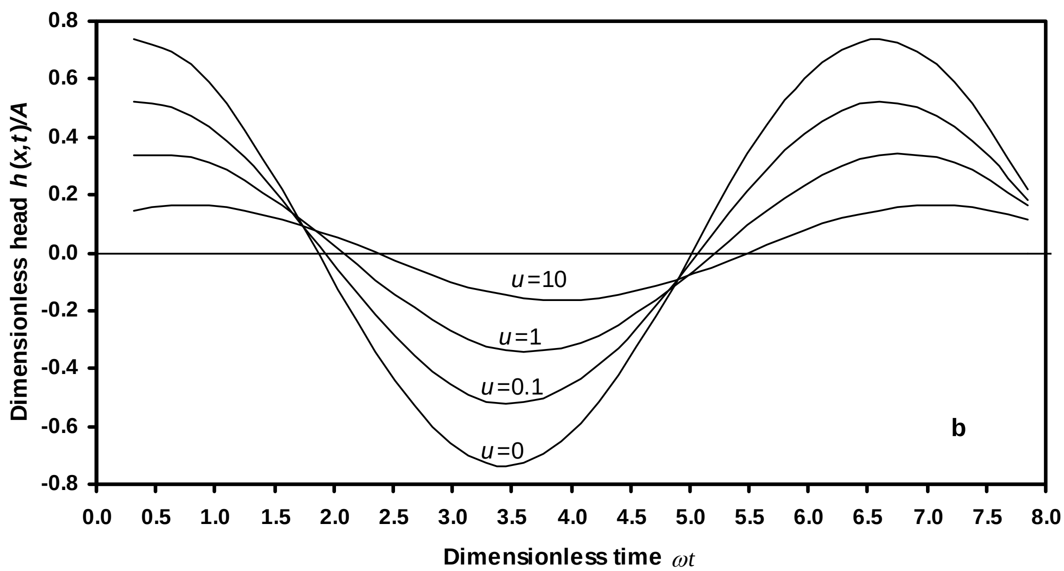


Figure 5. Change of dimensionless groundwater head $h(x, t)/A$ with dimensionless time ωt at inland location ax=0.3 (a) for different values of storativity ratio s when dimensionless leakage is fixed to be u=5.0 and (b) for different values of dimensionless leakage u when storativity ratio is fixed to be s=50.0