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Reverse Nearest Neighbors in Large Graphs

Man Lung Yiu, Dimitris Papadias, Nikos Mamoulis, and Yufei Tao

Abstract—A reverse nearest neighbor (RNN) query returns the data objects that have a query point as their nearest neighbor (NN). Although such queries have been studied quite extensively in Euclidean spaces, there is no previous work in the context of large graphs. In this paper, we provide a fundamental lemma, which can be used to prune the search space while traversing the graph in search for RNN. Based on it, we develop two RNN methods; an eager algorithm that attempts to prune network nodes as soon as they are visited and a lazy technique that prunes the search space when a data point is discovered. We study retrieval of an arbitrary number $k$ of reverse nearest neighbors, investigate the benefits of materialization, cover several query types, and deal with cases where the queries and the data objects reside on nodes or edges of the graph. The proposed techniques are evaluated in various practical scenarios involving spatial maps, computer networks, and the DBLP coauthorship graph.

Index Terms—Query processing, spatial databases, graphs and networks.

1 INTRODUCTION

Given a multidimensional data set $P$ and a point $q$, a (monochromatic) reverse nearest neighbor (RNN) query retrieves all the points $p \in P$ that have $q$ as their nearest neighbor, i.e., $\text{RNN}(q) = \{p \in P | d(p, q) \leq d(p', q), \forall p' \in P\}$, where $d$ is a distance metric. Given two data sets $P$ and $Q$ and a point $q$, a bichromatic (b RNN) query retrieves all points $p \in P$ that are closer to $q$ than to any point of $Q$, i.e., $\text{bRNN}(q) = \{p \in P | d(p, q) < d(p', q), \forall p' \in Q\}$ such that $d(p, q') < d(p, q)$. The problem has received considerable attention in the last few years [9], [17], [1], [10], [14], [18] due to its relevance in several applications involving decision support, resource allocation, profile-based marketing, etc. However, all the existing work focuses exclusively on Euclidean spaces, whereas in several domains, the data are modeled as large disk-based graphs.

Let $G = (V, E, W)$ be an undirected weighted graph (we use the terms graph and network interchangeably), where $V$ is the set of nodes and $E$ is the set of edges. $W$ associates each edge $n_i, n_j$ with a positive real number $w(n_i, n_j)$ (i.e., the weight). The interpretation of the weights is determined by the application domain, e.g., in road networks $w(n_i, n_j)$ may denote the travel time or the driving distance of the road segment connecting $n_i$ and $n_j$, whereas in peer-to-peer (P2P) systems, it may denote the network latency of the corresponding link, or simply be set to 1 (if the cost of a path is based on the number of hops).

The network distance $d(n_i, n_j)$ between two nodes $n_i, n_j \in V$ is defined as the minimum sum of weights of any path between them. We assume that the cost of traversing an edge is the same for both directions, i.e., $w(n_i, n_j) = w(n_j, n_i)$. Therefore, the network distance is symmetric and satisfies the inequality $d(n_i, n_j) \leq d(n_i, n_k) + d(n_k, n_j)$ (because $d(n_i, n_j)$ is the shortest distance between $n_i$ and $n_j$), i.e., it is a metric. The network distance definition necessitates dedicated techniques for RNN processing, since the existing (i.e., Euclidean) methods are inapplicable.

Depending on the domain, the characteristics of the RNN problem differ. In the P2P context, each point (i.e., peer) $p \in P$ lies on a node $n \in V$, but some nodes may not contain relevant peers to a specific query. We call such networks restricted. For instance, in Fig. 1a, assume that a new user $q$ interested in music enters the system. A (monochromatic) RNN query retrieves among the existing users also interested in music ($p_1$ to $p_5$), the ones for which $q$ will become their new NN. On the other hand, nodes such as $n_1$ and $n_2$ are irrelevant to the user (e.g., they may represent peers with other types of content) and they are considered empty. Given the edge costs of Fig. 1a, $\text{RNN}(q) = \{p_3\}$, e.g., $q$ is beneficial to $p_3$ since it is its closest NN in terms of network cost and shares the same interests. In a collaborative environment, $q$ would inform $p_3$ about its arrival, so that $p_3$ could address future requests directly to $q$, minimizing the network latency. Furthermore, the set $\text{RNN}(q)$ reflects the potential workload of $q$; thus, by knowing this set, each peer could manage/control its available resources. Note that the NN of $q$ (point $p_3$) is not an RNN (the NN of $p_3$ is $p_5$).

In unrestricted networks [12], [21], the query and the data points can reside anywhere on the edges of the graph. Fig. 1b shows an example of a bichromatic query in a road network, where points $p_1$ to $p_5$ stand for residential blocks and $q_1$, $q_2$ indicate restaurants. Nodes $n_1$ and $n_2$ are empty road junctions (i.e., they do not contain residential blocks or restaurants). Given several choices for the location of a new restaurant, the query $\text{bRNN}(q)$ may be used to evaluate the benefit of $q$ in terms of the customers that it may attract from rival restaurants based on proximity. Specifically, the set $\text{bRNN}(q) = \{p_1, p_2, p_3\}$ represents the blocks that are closer to $q$ than to any other competitor. Similarly,
bRNN(q₁) = \{p₄, p₅\} and bRNN(q₂) = \emptyset. Note that unrestricted networks can be transformed to restricted ones by adding a node for each data point that lies on a graph edge. We do not adopt this transformation since, as discussed in [12], the separation of the network component from the data points provides flexibility and facilitates updates. In particular, although the network can be considered static, the data points may change with high frequencies. Furthermore, new data sets (e.g., hotels) can be incorporated in the system as more information or services become available, without affecting the network storage scheme.

The above examples refer to single RNN retrieval. On the other hand, a (monochromatic) reverse k nearest neighbor (RkNN) query retrieves all the points \( p \in P \) that have \( q \) as one of their \( k \) nearest neighbors, i.e., \( \text{RkNN}(q) = \{ p \in P | d(p, q) \leq d(p, q_k(p)) \}, \) where \( p_k(p) \in P \) is the \( k \)th NN of \( p \). For instance, assuming that in the P2P scenario each node propagates a query to four other peers (e.g., this value is used in Gnutella), the arrival of a new peer could trigger a R4NN query. In case of bichromatic queries:

\[
\text{bRkNN}(q) = \{ p \in P | d(p, q) \leq d(p, q_k(p)) \},
\]

where \( q_k(p) \in Q \) is the \( k \)th NN of \( p \) (only considering objects of \( Q \)), e.g., in Fig. 1b, bR2NN(q) = \{p₁, p₂, p₃, p₁\}, bR2NN(q₁) = \{p₃, p₁, p₅\}, and bR2NN(q₂) = \{p₁, p₂, p₃\}.

In the sequel, we propose algorithms for processing both monochromatic and bichromatic RNN queries involving arbitrary values of \( k \) in restricted or unrestricted graphs. For our discussion, the term distance refers to the network distance defined using the edge weights. The rest of the paper is organized as follows: Section 2 overviews related work on RNN and graph algorithms. Section 3 introduces the architecture for graph storage and presents our basic methods. Section 4 focuses on optimizations and Section 5 discusses variations of RNN queries. Section 6 evaluates the proposed techniques in several practical scenarios involving spatial maps, computer networks, and the DBLP coauthorship graph. Section 7 concludes the paper.

2 RELATED WORK

Section 2.1 surveys methods for RNN search in the Euclidean space. Section 2.2 presents related query processing techniques for large graphs.

2.1 RNN Search in the Euclidean Space

We start with methods for monochromatic RNN queries. The first algorithm [9] precomputes for each data point \( p \) its nearest neighbor NN(p). Then, it represents \( p \) as a vicinity circle \( (p, \text{dist}(p, \text{NN}(p))) \) centered at \( p \) with radius equal to the Euclidean distance between \( p \) and its NN. The MBRs of all circles are indexed by an R-tree, called the RNN-tree. Using the RNN-tree, the reverse nearest neighbors of \( q \) can be efficiently retrieved by a point location query, which returns all circles that contain \( q \). Fig. 2a illustrates the concept using four data points, each associated with a vicinity circle. Since \( q \) falls in the circles of \( p₃ \) and \( p₄ \), the result of the query is RNN(q) = \{p₃, p₄\}.

Because the RNN-tree is optimized for RNN, but not NN search, Korn and Muthukrishnan [9] use an additional (conventional) R-tree on the data points for computing nearest neighbors and the NN radii of the RNN-tree. In order to avoid the maintenance of two separate structures, Yang and Lin [20] combine the two indexes in the RdNN-tree. Similar to the RNN-tree, a leaf node of the RdNN-tree contains vicinity circles of data points. On the other hand, an intermediate node contains the MBR of the underlying points (not their vicinity circles), together with the maximum distance from every point in the subtree to its nearest neighbor.

Techniques (e.g., [9], [20]) that rely on preprocessing cannot deal efficiently with updates. Stanoi et al. [13] eliminate the need for precomputing all NNs by utilizing some interesting properties of RNN retrieval. Consider Fig. 2b, which divides the space around a query \( q \) into six equal regions \( S₁ \) to \( S₆ \). Let \( p \) be the NN of \( q \) in some region \( Sᵢ \); it can be proven that: 1) either \( p \in \text{RNN}(q) \) or 2) there is no RNN of \( q \) in \( Sᵢ \). For instance, in Fig. 2b, the NN of \( q \) in \( S₁ \) is point \( p₂ \). However, the NN of \( p₂ \) is \( p₁ \). Consequently, there is no RNN of \( q \) in \( S₁ \) and we do not need to search further in this region. The same is true for \( S₂ \) (no data points), \( S₃ \), \( S₄ \) (\( p₁, p₃ \) are NNs of each other), and \( S₅ \) (the NN of \( p₃ \) is \( p₁ \)).

The actual result is RNN(q) = \{p₃\}. Based on the above property [13] adopts a two-step processing method. First, six constrained NN queries retrieve the nearest neighbors of \( q \) in regions \( S₁ \) to \( S₆ \). These points constitute the candidate result. Then, at a second step, a NN query is applied to find the NN \( p' \) of each candidate \( p \). If \( \text{dist}(p, q) < \text{dist}(p, p') \), \( p \) belongs to the actual result; otherwise, it is a false hit.

Assuming that the data points are indexed by an R-tree, Tao et al. [18] also adopt a two-step framework that retrieves a set of candidate RkNNs and then removes the...
false hits. The two steps are combined into a single traversal of the tree. In particular, the algorithm first retrieves a set of potential candidates in ascending order of their distance to the query point \( q \). Nodes that cannot contain candidates are pruned by a half-plane strategy and are inserted (without being visited) in a refinement set \( S_{j/f} \). At the second step, the entries of \( S_{j/f} \) are used to eliminate false hits. Singh et al. [14] propose another multistep method for monochromatic RNN retrieval in high dimensionality. The algorithm may incur false misses, i.e., it may fail to discover some RNN that is far from the query point. Benetis et al. [1] study continuous RNN, where \( P \) contains linearly moving objects with fixed velocities, and the goal is to retrieve all RNNs of \( q \) for a future interval.

Regarding bichromatic queries, the methods of [9], [20] can be applied directly by setting the vicinity circle of each point \( p_i \in P \) using its distance from its NN \( q_i \in Q \). However, they still have the problem of expensive updates. Stanoi et al. [17] propose an improved technique, which first computes the Voronoi cell of the query point (based on the objects of \( Q \)) and then retrieves the data points that fall in the cell (using a range query on the R-tree of data points). Assuming that the data arrive in the form of streams, Korn et al. [10] report aggregate results over the RNNs of a set of query points.

All the above techniques are inapplicable to graphs for several reasons. First, most techniques rely on indexes (e.g., R-trees) which can only handle Cartesian spaces. Furthermore, properties such as the one exploited by [13] do not hold in networks, i.e., the maximum number of RNNs is not constrained by the dimensionality (i.e., six in two dimensions). Similarly, the half-plane pruning technique of [18] or the Voronoi cells of [17] are specific to Euclidean distance. Finally, the above approaches target special instances of the problem (e.g., monochromatic versus bichromatic queries) and are usually limited to retrieval of the single RNN.

Since the network distance is a metric, an alternative solution could rely on indexes for general metric spaces (e.g., [19], [3]). However, such indexes do not capture the connectivity of nodes, which can be utilized to speed-up search compared to simply using the triangular inequality. In this paper, we use the network connectivity to solve RNN queries on large graphs.

### 2.2 Graph Algorithms

Our problem is related to shortest path computation in graphs. Given a source \( n_i \) and a destination node \( n_f \), Dijkstra’s algorithm [4] expands the network from \( n_i \) until \( n_f \) is reached. A priority queue \( H \) organizes the neighbors of the nodes found so far, so that intermediate nodes are visited in increasing order of their distances from \( n_i \). A* search (e.g., see [15]) speeds up shortest path computation by using the Euclidean distance bounds to guide the search. Materialization techniques that precompute and store distances between nodes can be applied to retrieve the shortest path distance in constant time. However, the high storage cost renders full materialization of (distances for all node pairs) infeasible for most practical networks. For instance, given a graph of \( V = 100K \) nodes, we need to store \( |V|(|V| - 1)/2 \approx 5 \times 10^9 \) distances. HiTi [7] and HEPV [6] avoid the extreme space requirements by partial materialization of a subset of the distances.


Most related to our techniques is the concept of network expansion as performed by Dijkstra’s algorithm and some of the methods for spatial networks (e.g., [12], [8], [21]). However, there is also an important difference. Whereas in spatial networks, the Euclidean distance constitutes a bound of the network distance, used to prune the search space (e.g., in the A* algorithm or the Euclidean restriction framework of [12]), in our case the Euclidean distance may be undefined (e.g., in P2P applications), or (if it is defined) it may not provide a bound for the network distance (e.g., in road networks where edge costs correspond to travel time). For generality, we do not exploit the Euclidean distance in the proposed algorithms.

### 3 Preliminaries and Basic Algorithms

Section 3.1 briefly describes the storage scheme for the network and a lemma for early search termination. Sections 3.2 and 3.3 present two algorithms for RNN search, based on eager and lazy evaluation, respectively. For simplicity, we focus on monochromatic queries in restricted networks. The extensions to all other cases are elaborated in Section 5.

#### 3.1 Architecture and Pruning Method

We use a file of adjacency lists to represent the graph. The adjacency list of node \( n \) keeps the neighboring nodes of \( n \) together with the weights of the corresponding edges. In order to minimize the I/O cost in the presence of a buffer, a disk page stores lists of neighboring nodes, grouped together using the method of [2]. Furthermore, we build an index on node id; for each node id in the index, there is a pointer to the file of lists and the data point that it contains (if any). Fig. 3b shows the storage scheme for the network of Fig. 3a assuming that the adjacency lists of \( \{n_1, n_2, n_3\}, \{n_2, n_3\} \), and \( \{n_3, n_1\} \) are stored in the same disk page.

Given a source node \( n \) and an integer \( k \), a nearest-neighbor query retrieves the \( k \) nearest data points of \( n \) in terms of network distance. NN search can be efficiently processed by the above storage scheme and an algorithm similar to Dijkstra that utilizes a heap \( H \) to expand the network around \( n \). In particular, the entry of \( n \) is retrieved using the index on node id. If \( n \) contains a point \( p \) and \( k = 1 \), \( p \) is added to the result and the search terminates. Otherwise, the nodes in the adjacency list of \( n \) are inserted to \( H \). Subsequently, nodes are dequeued, their potential points are added to the result, and their lists are fetched. The process is repeated until \( k \) neighbors are found.

A simple method for retrieving the RNN set is to traverse the network from \( q \), and for each point \( p \in P \) encountered
issue a nearest-neighbor query; \( p \in \text{RNN}(q) \), if \( d(p, q) \) is no greater than the distance between \( p \) and its NN. The problem is that the set \( \text{RNN}(q) \) does not have a fixed size and may contain points which are far from \( q \). Thus, the above approach would have to visit all data points. The proposed algorithms minimize the extent of network traversal by utilizing the following lemma.

**Lemma 1.** Let \( q \) be a query point, \( n \) a graph node, and \( p \) a data point satisfying \( d(q, n) > d(p, n) \). For any point \( p' \neq p \) whose shortest path to \( q \) passes through \( n \), \( d(q, p') > d(p, p') \), i.e., \( p' \notin \text{RNN}(q) \).

**Proof.**

\[
\begin{align*}
\quad d(q, p') &= d(q, n) + d(n, p') > d(p, n) + d(n, p') \\
&\geq d(p, p').
\end{align*}
\]

For instance, in Fig. 3a, \( d(q, n_3) = 4 > d(p_1, n_3) = 3 \). Thus, any data point (other than \( p_1 \)) whose shortest path to \( q \) passes from \( n_3 \) cannot be a RNN of \( q \) because it is closer to \( p_1 \) than \( q \). On the other hand, \( p_1 \in \text{RNN}(q) \) if there is no other data points within distance \( d(p_1, q) \) from \( p_1 \) (which is true in this example).

Our algorithms exploit two alternatives of NN search, which we call range-NN and verification queries. Specifically, a range-NN(\( n, k, e \)) retrieves the \( k \) nearest data points with (network) distance smaller than \( e \) from \( n \), if such \( k \) points exist. Otherwise, it returns a smaller number (possibly 0) of NNs. For instance, in Fig. 3a the range-NN query with parameters \( n = n_4, k = 1 \), and \( e = 7 \) has no results because the NN \( p_1 \) of \( n_4 \) has distance \( d(p_1, n_4) = 7 \geq e \).

Given two points \( p \) and \( q \), a verification query \( \text{verify}(p, k, q) \) checks whether \( q \) is among the \( k \)-NNs of a data point \( p \) by applying a range-NN query around the node that contains \( p \). \( \text{verify}(p, k, q) \) is in fact equivalent to range-NN(\( p, k, d(p, q) \)), i.e., search terminates as soon as \( q \) is encountered (the maximum range \( e \) in this case is implied by the distance \( d(p, q) \)).

### 3.2 Eager Algorithm

Eager traverses the network around \( q \) (in a way similar to Dijkstra’s algorithm), using Lemma 1 to eliminate nodes that may not lead to RNN results. For simplicity, we first assume \( k = 1 \) and then deal with arbitrary values of \( k \). The algorithm initializes a heap \( H \) by inserting the (source) node containing the query point \( q \). When a node \( n \) is deheaped, eager applies Lemma 1 in order to determine whether the expansion should proceed. In particular, it first retrieves the NN of \( n \) by performing a range-NN query \((n, 1, d(n, q))\). If no data point is discovered within distance \( d(n, q) \) from \( n \), the algorithm en-heaps the adjacent nodes of \( n \). If there is a point \( p \), such that \( d(n, q) > d(n, p) \), the expansion does not proceed further because (according to Lemma 1) \( n \) cannot lead to a RNN of \( q \). In this case, however, we need to verify if \( p \in \text{RNN}(q) \) because Lemma 1 is only true for points \( p' \neq p \). Thus, eager issues a verify\((p, 1, q)\) query. If \( q = \text{NN}(p) \), \( p \) is added to the result. Furthermore, \( p \) is marked as verified in order not to be expanded, if it is found again in the future through another node.

As an example consider the RNN query of Fig. 3 initiated at node \( n_4 \), which is inserted into \( H = \langle n_4, 0 \rangle \). Then, \( n_4 \) is deheaped and its adjacent nodes are added to \( H = \langle n_4, 4 \rangle, < n_1, 5 \rangle \). The subsequent removal (from \( H \)) of \( n_3 \) triggers a range-NN(\( n_3, 1, 4 \)) around \( n_3 \) for finding potential data points closer than the query. In this case, \( d(n_3, p_1) = 3 < d(n_3, q) \), meaning (by Lemma 1) that we do not search farther, i.e., the adjacent nodes of \( n_3 \) are not inserted to \( H \). Nevertheless, we have to check if \( p_1 \in \text{RNN}(q) \) by verify\((p_1, 1, q)\). Since \( q = \text{NN}(p_1) \), \( p_1 \) is an actual result. Next, \( n_1 \) is deheaped, its NN \( p_2 \) is discovered (by range-NN(\( n_1, 1, 5 \))) and search terminates because \( d(n_1, q) = 5 > d(n_1, p_2) = 3 \). The final step simply verifies \( p_2 \) as an actual result because \( q = \text{NN}(p_2) \).

Fig. 4 illustrates a pseudocode of eager for an arbitrary order-\( k \)-of RNNs. The main difference with respect to single RNN retrieval is that we can stop the expansion of the deheaped node \( n \), only if \( k \) data points are found within distance \( d(q, n) \) from \( n \). Eager verifies all data points encountered by the range-NN query. Specifically, a point \( p \in \text{RNN}(q) \), iff \( q \) is discovered by verify\((p, k, q)\); otherwise, \( p \notin \text{RNN}(q) \). Note that for each point \( p \), we only know \( d(p, n) + d(n, q) \), which is an upper bound for \( d(p, q) \) (i.e., if the shortest path from \( p \) to \( q \) does not pass through \( n \), then \( d(p, q) < d(p, n) + d(n, q) \)). This does not affect the correctness of the verification phase since the search will anyway terminate as soon as \( q \) is discovered.
3.3 Lazy Algorithm

Although eager minimizes the number of nodes inserted into the heap, it may perform numerous local network expansions for 1) retrieving the nearest point \( p \) of a deheaped node and 2) for verifying whether \( p \in \text{RNN}(q) \). The lazy algorithm delays pruning until a point is visited.

In particular, assuming single RNN retrieval, when the deheaped node \( n \) does not contain a point, lazy simply inserts its adjacent nodes into \( H \). If \( n \) contains a point \( p \), the expansion stops since every subsequent node is closer to \( p \) than to \( q \). Lazy issues a verification query to check whether \( p \) is a result. Verification queries are exploited to prune the search space. Specifically, let \( n' \) be a node visited by the verification phase of a data point \( p \). If \( n' \) has not been visited by the expansion around the query, we know that \( d(n', p) < d(n', q) \) since nodes are visited in ascending order of their distances and the verification query has maximum range \( d(p, q) \) (i.e., \( d(n', p) < d(p, q) < d(n', q) \)). Therefore, (by Lemma 1) \( n' \) cannot lead to any RNNs of \( q \). On the other hand, if \( n' \) has already been visited by the expansion around \( q \), we compare the distances \( d(n', p) \) and \( d(n', q) \). If \( d(n', p) < d(n', q) \), nodes inserted into \( H \) by the expansion of \( n' \) are eliminated.

An in-memory hash table is maintained in order to facilitate pruning for visited nodes. When a node \( n \) is deheaped its id is stored in the table, together with its distance from \( q \) and pointers to all the heap entries that were inserted during its processing. Thus, if \( n \) is invalidated in the future by a verification query, its adjacent nodes are removed from the heap by following the pointers. Furthermore, a table entry is deleted if it does not have any pointers to heap entries. Thus, the total number of pointers in the hash table equals the heap cardinality.

Fig. 5 illustrates an example where the first data point \( (p_1) \) is discovered when node \( n_4 \) is deheaped (nodes are numbered according to their distance from \( q \)). Lazy checks whether \( p_1 \in \text{RNN}(q) \) by a query verify\( (p_1, 1, q) \) that visits nodes \( n_5, n_2, \) and \( n_3 \) before determining that \( q = \text{NN}(p_1) \). The verification of \( p_1 \) will encounter \( n_5 \); thus, when \( n_5 \) is deheaped later it will be immediately discarded. Fig. 6 illustrates the contents of the heap and the hash table for the example of Fig. 5. \( n_7 \) (inserted during the processing of \( n_2 \)) cannot lead to a RNN since \( d(n_2, p_1) < d(n_2, q) \). Similarly, \( n_6 \) (inserted during the processing of \( n_4 \)) does not need to be expanded because \( d(n_3, p_1) < d(n_3, q) \) and lazy terminates with \( \text{RNN}(q) = \{ p_1 \} \).

The retrieval of RNNs is more complex because lazy may have to expand nodes that contain data points. In this case, we keep a count of the number of times that a node \( n \) is encountered during the verification phase. As shown in Fig. 7, if \( \text{count}(n) \geq k \); \( n \) is not visited by the RNN query (line 4) because it is closer to at least \( k \) data points than \( q \). Furthermore, if during a verification query the count of a visited node \( n_i \) becomes \( k \), the entries that were inserted into \( H \) during the processing of \( n_i \) are removed (using the hash table). This process is not repeated if \( \text{count}(n_i) \) subsequently exceeds \( k \) since it would be redundant. Unvisited nodes, whose counter reaches \( k \), are not removed from the heap since they will be captured when visited.

4 Optimizations

In this section, we discuss some optimizations to speed up performance. Section 4.1 proposes a materialization-based

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<th>Heap Content</th>
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<td>( n_4 ) ( 0 )</td>
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<td>de-heap ( n_3 )</td>
<td>( n_0 ) ( 10 )</td>
<td>( &lt;n_{p,15}&gt; &lt;n_{p,16}&gt; &lt;n_{n,18}&gt; &lt;n_{n,19}&gt; &lt;n_{n,20}&gt; )</td>
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<tr>
<td>de-heap ( n_4 )</td>
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<td>( &lt;n_{n,16}&gt; &lt;n_{n,18}&gt; &lt;n_{n,19}&gt; &lt;n_{n,20}&gt; )</td>
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</tbody>
</table>
method for eager, and Section 4.2 an extended pruning technique for lazy.

4.1 Materialization for eager

Eager can take advantage of materialized information to avoid local expansions incurred by range-NN and verification queries. As discussed in Section 2.2, full materialization of the graph requires the storage of $|V| (|V| - 1)/2$ distances. In order to avoid the quadratic space overhead, we follow an alternative approach that materializes the KNNs of each node $n$, where $K$ is the maximum number of RNNs to be requested by any query. Practical values of $K$ are fairly small (e.g., 20 for 2D space in [18] and 50 for high-dimensional spaces in [14]).

A naïve solution for computing the $K$ closest data points of all nodes, would simply apply a KNN query for each node. Instead, we use the algorithm of Fig. 8 that expands the network only once. All-NN initializes a single heap $H$ that stores the nodes containing the data points together with their distance (initially 0) from the corresponding point. When a node $n$ is deheaped, if all its KNNs have been found or $n$ has been visited by the same point, we ignore it. Otherwise, we update the K NN set of $n$ and en-heap the entries for its unvisited neighbor nodes (by the same point). Each completed KNN list is flushed to the disk. The process terminates when $H$ becomes empty. The worst-case complexity is $O(K \cdot |E| \cdot \log(K \cdot |E|))$, where $|E|$ is the number of edges in the graph, because an edge may be inserted in the heap at most $K$ times and each heap operation has logarithmic cost. The space overhead for materialization is $O(K \cdot |V|)$.

Fig. 9 illustrates an example for materializing the single NN ($K = 1$) of each network node in Fig. 3, where $P = \{p_1, p_2, p_3\}$. All-NN(1) initializes the heap to $H = <n_0, p_1, 0>, <n_0, p_2, 0>, <n_0, p_3, 0>$. Next, it deheaps $<n_0, p_1, 0>$, updates NN$(n_0) = p_1$, and en-heaps the entries $<n_3, p_1, d(n_6, p_1) + w(n_6)n_3 >$ and $<n_2, p_1, d(n_6, p_1) + w(n_6)n_2 >$.

Algorithm all-NN($K$)
1. for each node $n$ containing a point $p \in P$
2. insert $<n, d(n,p), 0>$ into $H$
3. while (not-empty($H$))
4. if $H$ has not been visited before and count($n$)<$K$
5. mark $n$ as visited
6. if $n$ contains a data point $p$
7. KNN($p$) := verify($p, k, q$)
8. if $q$ discovered by verification, add $p$ to RkNN($q$)
9. for each node $n$, encountered by the verification
10. count($n$)++
11. if count($n$)=k, is visited and $d(n,p)<d(n,q)$
12. remove from $H$ all entries inserted by $n$.
13. if count($n$)<$k$, insert $<n, d(n,q)>$ to $H$
14. return RkNN($q$)

Fig. 8. All-NN algorithm.

Similarly, after the deheaping of $<n_5, p_2, 0>$ and $<n_5, p_3, 0>$, the nearest-neighbor lists are updated to $NN(n_5) = p_2$, $NN(n_1) = p_3$, and

$$H = <n_3, p_1, d(n_6, p_1) + w(n_6)n_3 >,$$
$$<n_1, p_2, d(n_5, p_2) + w(n_5)n_1 >,$$
$$<n_2, p_2, d(n_5, p_2) + w(n_5)n_2 >,$$
$$<n_2, p_1, d(n_6, p_1) + w(n_6)n_2 >,$$
$$<n_3, p_2, d(n_5, p_2) + w(n_5)n_3 >.$$

The subsequent processing: 1) of $<n_3, p_1, d(n_6, p_1) + w(n_6)n_3>$ will lead to $NN(n_3) = p_1$ and 2) of $<n_1, p_2, d(n_5, p_2) + w(n_5)n_1>$ to $NN(n_1) = p_1$ and 3) of $<n_2, p_2, d(n_5, p_2) + w(n_5)n_2>$ to $NN(n_2) = p_2$. The algorithm terminates after processing $n_4$ since all the NN lists are completed.

Eager-M (M stands for materialization) utilizes the stored information to avoid network expansions as follows: When a node $n$ is deheaped, instead of applying range-NN($n, k, d(n,q)$), it retrieves its KNNs directly. Furthermore, the verification of a data point $p \in KNN(n)$ must compare $d(p, q)$ with $d(p, p_k(n'))$, where $p_k(n')$ is the $k$th NN of the node $n'$ that contains $p$. However, when $p$ is discovered, we only have $d(q, n) + d(n, p)$, which is an upper bound for $d(p, q)$. Thus, if $d(q, n) + d(n, p) \leq d(p, p_k(n'))$, then $p$ is in RkNN($q$). Otherwise, eager-M performs a query verify($p, k, q$).

Now, it remains to clarify the maintenance of materialized information in the presence of object updates. An insertion of a new data point $p$ is handled by a variation of all-NN that initializes $H$ to $<n, p, 0>$, where $n$ is the node containing $p$. Suppose that we insert the point $p_1$ at node $n_4$ in Fig. 9. First, we en-heap the entry $<n_3, p_1, 0>$. The materialized NN of $n_4$ is updated to $NN(n_4) = p_1$ and the entries $<n_3, p_1, 4>, <n_1, p_1, 5>$ are inserted into $H$. Then, $<n_3, p_4, 4>$ is deheaped; since $d(n_3, p_4) \geq 3$ (i.e., the distance between $n_3$ and its existing NN $p_1$), the materialized NN of $n_4$ is not updated and the neighbor nodes are en-heaped. A similar situation happens when $<n_1, p_1, 5>$ is deheaped (because $d(n_1, p_1) \geq d(n_1, p_3)$) and the algorithm terminates.

On the other hand, deletion is more complex because we have to modify all influenced nodes, i.e., the nodes for which the deleted point is a NN. This is essentially a RKN query; the difference with respect to the algorithms of Section 3, is that 1) now we are looking for the RNN nodes, as opposed to the RNN data points (i.e., we also consider nodes that do not contain data points), and 2) we

1. Each entry in the materialized list KNN($n$) for node $n$ is a pair $<p, d(p, n)>$, where $p_i, 1 \leq i \leq K$, is the $i$th NN of $n$. 

Algorithm lazy($q,k$)
1. insert $<n(q), 0>$ into $H$ // $n(q)$ is the node containing $q$
2. while (not-empty($H$))
3. $<n, d(n,q)>$ := de-heap($H$)
4. if $n$ has not been visited before and count($n$)<$k$
5. mark $n$ as visited
6. if $n$ contains a data point $p$
7. KNN($p$) := verify($p, k, q$)
8. if $q$ discovered by verification, add $p$ to RkNN($q$)
9. for each node $n$, encountered by the verification
10. count($n$)++
11. if count($n$)=k, is visited and $d(n,p)<d(n,q)$
12. remove from $H$ all entries inserted by $n$.
13. if count($n$)<$k$, insert $<n, d(n,q)>$ to $H$
14. return RkNN($q$)
need to update the affected nodes by retrieving their new NNs. The deletion algorithm (shown in Fig. 10) applies two steps. The first step expands the network around \( p \), removing \( p \) from the materialized lists of all nodes \( n \) for which \( p \in \text{KNN}(n) \). The expansion stops at some border nodes, whose \( K \)-NNs do not change. Let \( n \) be a border node; then, \( n \) has at least one affected neighbor \( n_0 \), such that \( |\text{KNN}(n_0)| < K \) (since \( p \) was removed from \( \text{KNN}(n_0) \)). For each \( n_0 \), the algorithm will insert \( K \) entries of the form \(<n_0, p_i; d(n_0, p_i) + w(n_0 n_i)>\) to a second heap \( H_0 \), where \( p_i \), \( 1 \leq i \leq K \), is the \( i \)-th NN of \( n_0 \). Then, the second step will complete the \( K \)-NN list of all affected nodes by using the contents of \( H_0 \) to perform another expansion. The intuition is that the new NN (to replace \( p_0 \)) of \( n_0 \) is one of the NNs of the adjacent border nodes.

Fig. 11 illustrates the deletion of \( p_1 \) in the example of Fig. 3, assuming that \( K = 1 \). Point \( p_1 \) is the NN of \( n_6 \), \( n_3 \), and \( n_4 \). The border nodes are \( n_2 \), \( n_5 \), and \( n_1 \) since they are neighbors of the affected nodes (and their NNs do not change). During the first step of deletion, the third node deheaped from \( H \) is \( n_2 \) (\( \text{NN}(n_2) = p_2 \)), which causes the insertion of entry \(<n_6, p_2; d(n_6, p_2) + w(n_6 n_2)\>\) to \( H' \). Similarly, the processing of border nodes \( n_5 \) and \( n_1 \) will add entries \(<n_3, p_2; d(n_3, p_2) + w(n_3 n_5)\>\) and \(<n_4, p_2; d(n_4, p_2) + w(n_4 n_1)\>\), respectively, and the first step terminates. Note that the expansion does not proceed after a border node is visited; e.g., in this case, the processing of \( n_5 \) will not cause the insertion of \(<n_1, 3\>\) in \( H \) (observe that there is only one entry \(<n_1, 12\>\) in \( H \)). The second step computes the NN sets of the affected nodes in the order that they are deheaped from \( H' \), i.e., the processing of \( n_3 \) will lead to \( \text{NN}(n_3) = p_2 \) and the insertion of

\[
< n_1, p_2; d(n_1, p_2) + w(n_1 n_3) >, \\
< n_4, p_2; d(n_4, p_2) + w(n_4 n_3) >
\]

to \( H' \). The final result will update: \( \text{NN}(n_4) = p_2 \) and \( \text{NN}(n_6) = p_2 \).

Each insertion and deletion is expected to affect a small percentage of the nodes. Thus, the memory requirements of the update algorithms are relatively small. On the other hand, for some large problems, the heap size for all-NN may exceed the available main memory. In this case, we can divide \( P \) into partitions and apply an incremental batch insertion (a variation of all-NN) for each partition. Finally, note that materialization is not beneficial for \textit{lazy} because 1) it does not apply range-NN queries (therefore, it cannot benefit as much as \textit{eager}) and 2) the expansion during the verification queries constitutes an essential pruning component of the algorithm. In the next section, we discuss an alternative optimization technique for \textit{lazy}.

**Algorithm deletion-M\((p, K)\)**

1. insert \(<n(p), 0>\) into \( H /\!\!/ n(p) \) is the node containing \( p \)
2. while (not-empty(\( H \))) // first step
3. \(<n, d(n,q)\) := de-heap(\( H \))
4. if \( n \) has not been visited before
5. mark \( n \) as visited
6. if \( p \in \text{KNN}(n) \)
7. remove \( p \) from \( \text{KNN}(n) \)
8. for each adjacent non-visited node \( n_i \) of \( n \)
9. insert \(<n_i, d(n_i, p) + w(n_i n)\>\) to \( H \)
10. else // \( p \notin \text{KNN}(n) \); i.e., \( n \) is a border node
11. for each \( p_i \in \text{KNN}(n) \)
12. for each adjacent node \( n' \) of \( n \) with \(|\text{KNN}(n')| < K \)
13. insert \(<n', p_i; d(n_i, n') + w(n' n)\>\) to \( H' \)
14. end while //empty(\( H \))
15. while (not-empty(\( H' \))) // second step
16. \(<n', p', d'\) := de-heap(\( H' \))
17. if \(|\text{KNN}(n')| < K \)
18. add \(<p', d'\) to \( \text{KNN}(n') \)
19. for each adjacent node \( n_i \) of \( n' \) with \(|\text{KNN}(n_i)| < K \)
20. insert \(<n_i, p', d' + w(n' n_i)\>\) to \( H' \)
21. end while //empty(\( H' \))

Fig. 10. Object deletion for materialized KNN.
distance is less than the last deheaped distance of \( n \).
Continuing the example, \( n \) is closer to \( p_1 \) than \( q \) and, therefore, according to Lemma 1, it cannot lead to any results.

The lazy-EP algorithm (EP stands for extended pruning) avoids such cases by expanding the network in parallel using two heaps \( H \) and \( H' \). The heap \( H' \) applies the pruning effect of the discovered points. In the above example, when the entry \( <n_2, 1> \) is processed in the conventional heap \( H \), a new entry \( <n_2, p_1, 0> \) is inserted in \( H' \). Then, expansion proceeds with the heap \( H' \) whenever its top distance is less than the last deheaped distance of \( H \). Continuing the example, \( <n_2, p_1, 0> \) is deheaped and \( <n_1, p_1, 1> \), \( <n_4, p_1, 2> \) are inserted into \( H' \). Since the last deheaped distance from \( H \) is 1, we now switch back to \( H \). The entry \( <n_3, 3> \) is deheaped and \( <n_3, 4> \) is inserted into \( H \). The expansion proceeds with \( H \) and the entry \( <n_1, p_1, 1> \) and \( <n_4, p_1, 2> \) are deheaped. The processing of \( <n_4, p_1, 2> \) marks node \( n_4 \) using the distance \( d(p_1, n_4) \). After that, \( <n_4, 4> \) is deheaped from \( H' \); because \( n_4 \) is closer to \( p_1 \) than \( q \), the adjacent unvisited nodes of \( n_4 \) are not inserted to \( H \) and the algorithm terminates.

Fig. 13 illustrates lazy-EP for arbitrary values of \( k \). Whereas lazy uses verification to update counters and prune the search space, lazy-EP eliminates nodes during the expansion of \( H' \). In addition, lazy-EP maintains the kNN of each node found so far in order to avoid visiting the same point again during the expansion of \( H' \). The expansion in \( H \) is pruned (by Lemma 1) if \( d(n, q) > d(n, p_k(n)) \), where \( p_k(n) \) is the \( k \)-th neighbor of \( n \) (if \( |\text{kNN}(n)| < k \), we consider that \( d(n, p_k(n)) = \infty \)).

5 **Variants of RNN Queries**

Section 5.1 studies bichromatic and continuous RNN retrieval. Section 5.2 extends our techniques to unrestricted networks with data points lying on the edges.

### 5.1 Bichromatic and Continuous RNN Queries

Given two data sets \( P \) and \( Q \) and a point \( q \), the output of a bichromatic RNN query is

\[
\text{bRNN}(q) = \{ p \in P \mid d(p, q) \leq d(p, q_k(p)) \}
\]

where \( q_k(p) \in Q \) is the \( k \)-th NN of \( p \) only considering objects of \( Q \). The processing of a bichromatic query can be reduced to the monochromatic case, where the data set is \( Q \) (instead of \( P \)) and the result contains all nodes \( n \) (instead of data points) for which \( q \in \text{kNN}(n) \). A final step simply retrieves the points \( p \in P \) contained in these nodes. Specifically, Lemma 1 now requires that for a node \( n \) to be pruned it should be closer to \( k \) points of \( Q \) than \( q \). In both eager and lazy expansion stops at nodes pruned by Lemma 1: 1) for eager, it is not important if the node \( n \) under consideration contains a point and 2) for lazy, \( n \) is examined if it contains a point \( q \in Q \). Furthermore, the
optimizations of Section 4 can also be applied. For eager-M, we simply materialize the set \( \text{KNN}(n) \subset Q \) for each node \( n \), whereas lazy-EP remains the same but only considers \( Q \) during the parallel expansions.

The concept of continuous RNN queries, as defined for objects moving in the Euclidean space [1], is not applicable to graphs, because objects do not follow linear movement. Instead, in our case, a continuous query retrieves the RNNs for each node \( n_i \in V \) on a predefined route \( r = \langle n_1, n_2, \ldots, n_r \rangle \), where \( n_i n_{i+1} \in E \), i.e., \( c\text{RNN}(r) = \bigcup_{n_i \in r} \text{RNN}(n_i) \). The distance between \( r \) and a network node \( n \) is defined as:

\[
d(r, n) = \min \{d(n_i, n) | n_i \in r \}.
\]

**Eager** and **lazy** (as well as their optimizations) can be applied directly using the new distance definition \( d(r, n) \). For example, \( \text{eager}(r, k) \) initially inserts \( <n_1, 0>, \ldots, <n_r, 0> \) into \( H \). When a node \( n \) is dequeued for the first time, its distance corresponds to \( d(n, n) \) and \( n \) is marked as visited.

The range-NN(\( n, k, d(r, n) \)) retrieves the NNs of \( n \) within the range \( d(r, n) \). If the query returns \( k \) data points, \( n \) is pruned. Moreover, the maximum verification range of a point \( p \) is \( d(r, p) \), i.e., \( p \in c\text{RNN}(r) \), if the first node of the route is met before \( k \) other points are found. The same modification applies to lazy.

### 5.2 RNN Queries in Unrestricted Networks

In an unrestricted network, the position of a point \( p \) lying on the edge \( n_i n_j \) can be expressed by the triplet \( \langle n_i, n_j, \text{pos} \rangle \), where \( \text{pos} \in [0, w(n_i n_j)] \) is the distance of \( p \) from node \( n_i \) and \( w(n_i n_j) \) is the weight (cost) of the edge. In order to avoid ambiguity, we assume a lexicographic ordering of nodes so that \( p \) is assigned to the edge \( n_i n_j \), \( i < j \). The direct distance between a point \( p \) at \( \langle n_i, n_j, \text{pos} \rangle \) and node \( n_i \) is \( d_L(p, n_i) = \text{pos} \). The direct distance between \( p \) and \( n_i \) is \( d_L(p, n_i) = w(n_i n_j) - \text{pos} \). Let \( p \) and \( p' \) be two points at \( \langle n_i, n_j, \text{pos} \rangle \) and \( \langle n_i', n_j', \text{pos}' \rangle \), respectively. If \( n_i = n_i' \) and \( n_j = n_j' \) (i.e., \( p \) and \( p' \) lie on the same edge), the direct distance \( d_L(p, p') \) between \( p \) and \( p' \) is defined as \( |\text{pos} - \text{pos}'| \); otherwise, it is \( \infty \). The network distance \( d(p, p') \) is \( \min_{\text{pos}, \text{pos}'} (d_L(p, n_i) + d(n_i, n_j) + d_L(n_j, p')) \), if \( p \) and \( p' \) lie on different edges; otherwise, \( d(p, p') \) is the minimum of the previous quantity and \( d_L(p, p') \). The distance \( d(p, p') \) is symmetric and satisfies the triangular inequality.

For the following example, we use the network of Fig. 14a. Since the data points do not necessarily lie on nodes, the storage scheme of Fig. 3b does not apply. Instead, we store the data points in a separate file, pointed by the edges on which they reside (see Fig. 14b), so that when an edge (e.g., \( n_2 n_6 \)) is processed we can efficiently retrieve all the points (e.g., \( p_1 \)) on the edge. Although only \( d_L(p_1, n_2) = 4 \) is stored, \( d_L(p_1, n_6) = 1 \) can be computed as \( w(n_2 n_6) - d_L(p_1, n_2) \).

Lemma 1 does not presume that the query or data points lie on nodes; therefore, it is still applicable. The general framework of **eager** also remains the same as presented in Section 3. The difference is in the basic algorithms for retrieval and verification of the NNs of a node. We focus on range-NN, since verification is similar. In case that the...
query or the data points lie on the edges, the range-NN algorithm discussed in Section 3.1 is inapplicable because 1) data points are not necessarily visited according to their distance from the source node, and 2) the same data point may be found twice (with two different distances). Consider, for instance, the query $q$ in Fig. 14a and point $p_3$ on the edge $n_3n_4$. When $n_3$ is processed, we can compute an upper bound for the distance between $q$ and $p_3$ as: $d(q, n_3) + d_L(n_3, p_3) = 10$. The subsequent processing of $n_5$ will provide another upper bound $d(q, n_5) + d_L(n_2, p_3) = 8$. The actual distance $d(q, p_3) = 8$ is the minimum of the two bounds.

The unrestricted-range-NN algorithm takes the above observations into account for finding the $k$NNs of node $n$ within a range $e$ (if $k$ such points exist). When a node $n_i$ is deheaped, each point $p$ on its adjacent edges is inserted back to $H$, provided that $d(q, n_i) + d_L(n_i, p) < e$. Thus, points are deheaped (and inserted to $kNN(n)$) in ascending order of their distance from $n$. Multiple insertions of the same point visited via different paths are avoided. The search terminates when the number of discovered points equals $k$, the range $e$ is exceeded, or the heap becomes empty.

Verification queries follow the same approach. By substituting the original with the unrestricted functions, we can obtain a version of eager for unrestricted networks. Similarly, Eager-M requires minor modifications of the all-NN and the update algorithms. Note that although the $k$NNs of a point $p$ (lying on the edge $n_i n_j$) are not stored, they can be computed efficiently using $kNN(n_i), kNN(n_j), d_L(n_i, p)$, and $d_L(n_j, p)$.

On the other hand, recall that lazy initiates the pruning process during the deheaping of nodes that contain data points. For unrestricted networks, the pruning occurs during the processing of edges. Assuming, for example, R1NN retrieval, when a node $n$ is deheaped, we check the adjacent edges. If the edge $nn'$ (where $n'$ is an unvisited node) contains a data point $p$, then $n'$ is not en-heaped because it is closer to $p$ than $q$. The algorithms can also be applied for bichromatic and continuous queries in unrestricted networks with straightforward modifications. In the next sections, we evaluate the proposed techniques under various settings.

### 6 Experimental Evaluation

Let $|V|$ be the node cardinality and $|P|$ be the data cardinality; the ratio $|P|/|V|$ corresponds to the data density $D$. In general, high density leads to low processing cost since it limits the extent of expansions. For instance, assuming a restricted network, in the extreme case that $D = 1$ (every node contains a point), the output of a R1NN query contains only the point that resides on the query node. In order to provide meaningful results, we set the maximum value of density to 0.1 in all cases. The diagrams display the average cost of workloads containing 50 queries. Each query is randomly chosen from the set of data points, so that the queries follow the data distribution. For all experiments, we use a Pentium 4 CPU (2.3GHz) with page size set to 4Kb and an LRU buffer of 1Mb (256 pages). Section 6.1 focuses on restricted networks and Section 6.2 on unrestricted ones.

#### 6.1 Experiments with Restricted Networks

The first set of experiments uses the coauthorship graph of DBLP (database.cs.ualberta.ca/coauthorship/) [11]. Each node corresponds to an author that has published in SIGMOD, VLDB, ICDE, or ACM PODS. Two authors are linked by an edge if their names appear together in some paper. All the edge weights are set to 1 such that the length of the shortest path between two authors models their degree of separation. The graph is “cleaned” to form a connected network with 4,260 nodes and 13,199 edges.

Given an author $q$, an ad hoc query retrieves the RNN nodes of $q$, where 1) the distance corresponds to the degree of separation and 2) the result contains only authors satisfying an input condition (e.g., they should have exactly two SIGMOD papers). Table 1 shows the cost of eager and lazy for three query conditions (the CPU-cost is measured in CPU-sec).

<table>
<thead>
<tr>
<th>Density</th>
<th>eager-I/O</th>
<th>lazy-I/O</th>
<th>eager-CPU</th>
<th>lazy-CPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>87</td>
<td>88</td>
<td>2.176</td>
<td>0.124</td>
</tr>
<tr>
<td>0.02</td>
<td>85</td>
<td>87</td>
<td>1.316</td>
<td>0.110</td>
</tr>
<tr>
<td>0.05</td>
<td>82</td>
<td>85</td>
<td>0.338</td>
<td>0.099</td>
</tr>
<tr>
<td>0.1</td>
<td>78</td>
<td>82</td>
<td>0.104</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Table 2 shows the cost of ad hoc queries with respect to the number of nodes (DBLP, $k = 1$).

<table>
<thead>
<tr>
<th>Condition</th>
<th>eager-I/O</th>
<th>lazy-I/O</th>
<th>eager-CPU</th>
<th>lazy-CPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 SIGMOD</td>
<td>64</td>
<td>74</td>
<td>0.053</td>
<td>0.060</td>
</tr>
<tr>
<td>1 SIGMOD</td>
<td>73</td>
<td>77</td>
<td>0.077</td>
<td>0.057</td>
</tr>
<tr>
<td>2 SIGMOD</td>
<td>86</td>
<td>88</td>
<td>0.201</td>
<td>0.091</td>
</tr>
</tbody>
</table>

Fig. 15. Cost versus $|V|$ ($BRITE, D = 0.01, k = 1$).
seconds). We do not include eager-M because materialization is not possible for ad hoc queries (i.e., the set of NNs of a node depends on the condition) and lazy-EP because its cost is similar to lazy.

The number of SIGMOD papers determines the selectivity of the condition. In particular, most authors have 0 papers and the selectivity increases with the number of papers leading to larger expansion around the query point, which explains the rising cost of the algorithms. Eager is slightly better than lazy in terms of I/O cost but worse in CPU time (especially for the most selective condition) because of the range-NN queries, which may visit the same node multiple times. Although, due to locality, these queries are likely to access pages already in the buffer, the overhead is reflected in the CPU-time.

Next, we randomly select “interesting” nodes (i.e., that correspond to data points) and test the performance of the algorithms on the resulting network. Table 2 shows the cost of eager and lazy as a function of the data density. As expected, the cost decreases as \( D \) increases. Although the algorithms incur very similar I/O cost, eager is much more CPU-intensive and the difference exceeds an order of magnitude for the low density values. This agrees with Table 1 because low density corresponds to high selectivity.

The second set of experiments simulates a P2P scenario. We use BRITE (www.cs.bu.edu/brite/) to generate graph topologies with average degree (number of edges adjacent to each node) equal to 4. Data points (i.e., peers) are located at random network nodes. Graphs generated by BRITE contain arbitrary connections between nodes and, consequently, the number of nodes visited during expansions increases exponentially and converges fast to the node cardinality \( |V| \) since eventually all the nodes are reached with a relatively small number of hops. We call this phenomenon exponential expansion.

Fig. 15 illustrates the overall cost of eager, eager-M, lazy and lazy-EP as a function of \( |V| \) ranging from 90K to 360K nodes (\( D = 0.01, k = 1 \)). Abbreviations of the algorithms (E for eager, L for lazy, EM for eager-M, and LP for lazy-EP) are shown on the top of each column. The cost is measured in seconds, after charging 10ms for each random I/O (a common value used in the literature [12]). The variations of eager perform well because they prune the search space early, thus reducing the effect of exponential expansion. On the other hand, the pruning strategy of lazy fails completely because, as explained in the example of Fig. 12, lazy may expand some nodes that can be pruned and, subsequently, access most of the network. The extended pruning strategy of lazy-EP pays-off only in terms of CPU time because it leads to fewer verification queries (when compared to lazy).

Fig. 16 evaluates the algorithms as a function of the density in a network with \( |V| = 160K \) nodes. Similar to Fig. 15, lazy and lazy-EP visit most of the network (independently of the density value) because of the exponential expansion effect. The performance of eager and eager-M improves significantly for large values of \( D \), since each node is surrounded by more data points (and, therefore, it can be pruned by the range-NN queries). The irregularity of the network leads to high standard deviations of the I/O costs, which range from 25 percent to 67 percent of the mean values shown in the figure.

Fig. 17. Cost versus \( D(SF, k = 1) \).

Fig. 18. Cost versus \( k(SF, D = 0.01) \).

Fig. 19. Cost of continuous queries versus route size (\( SF, D = 0.01, k = 1 \)).
6.2 Experiments with Unrestricted Networks

For unrestricted networks we use spatial graphs. In such (planar) graphs, adjacent nodes are likely to be near in space (therefore, they do not incur exponential expansion). The first network is generated from the San Francisco (SF) map (www.maproom.psu.edu/dcw) after extracting its largest connected component. The number of nodes and edges of the cleaned network are 174,956 and 223,001, respectively. The coordinates of the nodes are normalized in the range \([0, \frac{10000}{\text{C138}}]\) and edge weights are set to the Euclidean distance of the connected nodes. Fig. 20 evaluates performance versus the data density. The density is again defined as \(D = \frac{|P|}{|V|}\), but the data points are distributed randomly on the edges. Similar to the DBLP graph, eager is better than lazy in terms of I/O, but worse in CPU-time. The extended pruning technique improves the performance of lazy at \(D < 0.01\), because the parallel expansion of lazy-EP eliminates many nodes which cannot be pruned lazy at low density values. Nevertheless, the performance of lazy improves significantly for high density (as opposed to the case of exponential expansion—see Fig. 16). Materialization is beneficial for eager, and eager-M has the lowest I/O and CPU cost. The standard deviation of the I/O costs ranges from 42 percent to 79 percent of the mean values (shown).

Fig. 18 illustrates the cost of the algorithms as a function of the number of retrieved RNNs. As expected, the performance deteriorates with \(k\). This is most obvious for lazy because of the diminishing pruning effect of the verification queries. On the other hand, lazy-EP scales better since all discovered points are utilized for pruning. Eager and Eager-M are again the best choices. The I/O cost of Eager-M for accessing the materialized NNs increases with \(k\), and exceeds that of eager for \(k = 8\).

Fig. 19 evaluates the algorithms on continuous \(^2\) RNN queries as a function of the route size (i.e., the number of nodes in the route). Each route is a random walk without repeated nodes. The cost of eager and eager-M grows linearly with the route size. Interestingly, the cost of lazy variants first decreases because a longer route leads to earlier discovery of points (close to the path), limiting the scope of verification ranges. After the length of the path exceeds 20 nodes, the cost increases because a longer route leads to more RNNs and, thus, more network expansions.

The next set of experiments applies synthetic grid networks [7], [5] to test the behavior of the algorithms for different node cardinalities and average degrees. The standard grid map has an average degree of 4. To generate maps with higher degree, new edges are randomly added between nearby nodes. As shown in Fig. 20a, \(|V|\) does not have a serious effect on the cost because the expansion of all algorithms terminates around the query point (i.e., it does not depend on the size of the network). Fig. 20b investigates the effect of the average degree. In general, the performance deteriorates with the degree since the processing of each node must consider a larger number of adjacent nodes. Lazy EP scales worse than the other algorithms, because of the additional expansions incurred by the second heap \(H'\).

As discussed in the beginning of the section, all previous experiments were performed with an LRU buffer of 1Mb (256 pages). Fig. 21 evaluates the effect of the buffer size on the performance. Due to the log scale, the CPU cost is not visible because it is dominated by the I/O cost. At buffer size = 0, every network node access incurs a page fault. The cost of eager in this case is much higher than that of lazy due to the range-NN queries. However, since these queries may visit the same node multiple times, even a small buffer improves the performance of eager considerably. After the

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2. We do not evaluate bichromatic search since, as discussed in Section 5.1, the cost of bichromatic query is similar to that of a monochromatic one (on the same network) assuming that the set of data points is \(Q\).
buffer size reaches 64, the cost of eager stabilizes, indicating that 64 pages are sufficient for keeping all the visited nodes in memory. On the other hand, the performance of lazy stabilizes after the buffer reaches 256 pages, implying a more extensive search in the network. This experiment verifies that eager visits a (much) smaller set of nodes than lazy, but possibly many times, which explains its higher CPU overhead in the majority of experiments not involving networks with exponential expansion. Nevertheless, since the I/O cost is the dominant factor, eager is, in general, preferable.

In summary, the problem characteristics have a significant effect on the behavior of the algorithms. Lazy has lower CPU overhead than eager (up to two orders of magnitude for selective queries in the DBLP coauthorship graph), but it has higher I/O cost and is unacceptable for networks that incur exponential expansion. Lazy-EP provides improvements over the basic algorithm, but in some cases, its parallel expansion may increase the I/O cost. Eager has a more balanced behavior and in most settings it is preferable to both lazy and lazy-EP. The best choice for all scenarios in which it is applicable (i.e., excluding ad hoc queries) is eager-M. However, it requires the materialization and maintenance of the NN points of all nodes.

The last set of experiments uses the San Francisco network to evaluate the cost of maintaining this information in the presence of object insertions and deletions. The inserted points follow the distribution of the nodes, whereas the deleted points are chosen randomly from the existing data points. Fig. 22a shows the cost of insertions and deletions as a function of the density. Deletions incur higher cost than insertions since, as discussed in Section 4.1, their expansion incurs two steps: identification of the affected nodes and then retrieval of the new NNs. Fig. 22b illustrates the cost as a function of the number K of stored NNs. As expected, the I/O overhead increases with K. Nevertheless, even for K = 8 each operation takes less than 0.8 seconds, and given that updates are not frequent in several practical applications, materialization is a feasible option.

7 CONCLUSION

This paper constitutes the first work that deals with RNN queries in large networks. We propose several techniques that can be used for a variety of queries in different types of graphs. Our evaluation covers applications including spatial maps, computer networks, and coauthorship graphs. The best and most robust algorithm (eager-M) relies on materialized information. In cases where the set of interesting data points is not known in advance (and, therefore, materialization is not possible), the choice of the processing method depends on the problem characteristics; the eager algorithm minimizes the I/O cost and is, in general, the method of choice, but it can be more CPU-intensive than lazy for certain networks (e.g., DBLP coauthorship graph).

The proposed techniques assume undirected graphs. An interesting direction for future work concerns their extensions to directed networks (e.g., spatial maps with one-way streets). In this case, the neighborhood relation is asymmetric, complicating query processing. We also intend to develop models for estimating 1) the cost of algorithms and 2) the selectivity of RNN queries in large graphs. Such models are useful both for selecting the best processing method given the problem characteristics, and optimizing complex spatial queries involving several operators. This is a particularly challenging problem, given that it has not been solved even for the apparently simpler (and more established) Euclidean version of RNN search.

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REFERENCES


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