Macro-approach of Cell Formation Problem with Consideration of Machining Sequence

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Abstract—**Cellular Manufacturing System (CMS) which is based on the concept of Group Technology (GT) has been recognized as an efficient and effective way to improve the productivity in the factory. In recent years, there has been much effort done for continuing to improve CMS. Most researches concentrated on distinguishing the part families and machine cells either simultaneously or individually by considering of minimizing intercellular and intracellular part movements. However, fewer researches have studied the impact of the sequencing of machine cells. In light of this, the main aim of this present work is to study the impact of the sequencing of allocating the machine cells in minimizing intercellular part movement. The problem scope, which is also called as machine-part grouping problem (MPGP) together with the background of cell layout problem (CLP), has been identified. A mathematical model is formulated and part incidence matrix with operational sequence is often used. Since MPGP has been proved as an NP complete, genetic algorithm (GA) is employed as cell formation algorithms in solving this problem.**

Keywords—**Cell layout problem, genetic algorithm, machine-part grouping problem, operational sequence, sequencing,**

I. Introduction

In the era of vigorous competition, people in manufacturing industry have been striving to increase their competence so as to make them outstanding among their competitors. Specifically, for the sake of improving the productivity and efficiency in the production floor, Cellular Manufacturing Systems (CMS) with the superior concept of Group Technology (GT), which is the one of most popular methodology, have been employed. GT is a manufacturing philosophy which analyses, determines and assigns the parts, which are to be manufactured, into a number of part families and assigns the machines into a number of cells as well. This is based on taking the advantage of part similarity in processing and design functions. The essential problem of CMS is to group the parts into part families and group the machines into machine cells correspondingly.

According to [1] and [2], by implementing CMS, one can achieve the merits of minimizing the total production costs, namely material handling cost, processing cost, machine duplication cost, fixed terms costs, and part families setup cost. The total cost is thus highly dependant on the cell configuration.

Particularly, machine-part grouping problem (MPGP), which is also called cell formation problem (CMP), is a critical and crucial element in CMS [3, 4, 5]. MPGP is solved by applying the concept of GT which is mentioned in the above. Generally, in MPGP, the main aim is to minimize total intercellular part movement and intracellular part movement as a result of minimizing the total cost [6]. To find an optimal or near-optimal solution for MPGP, numerous research efforts have been done for it. Different research may different on different topics on MPGP. Some may focus on developing efficient algorithms or heuristics to solve MPGP in which parts families and machine cells were formed either sequentially or simultaneously while considering system constraints and various direct or indirect factors [7, 8, 9]; some may focus on solving multi-criterion objectives MPGP with the consideration of routing flexibility [10, 11]. Moreover, people have also take some specific constraints into account in MPGP, these constraints or preferences are machine aggregation and machine disaggregation [12].

Even many researches have done on MPGP, however, a very few efforts have made for cell layout problem (CLP) [13, 14]. In reality, some components may not be finished within only one cells, they have to travel to another cell(s) for further operation(s). Under this circumstance, intercellular part movement will occur. Different order/sequence of machine cells allocation may result in different total intercellular movement unit distance. Thus, the sequence of machine cells is important in this aspect.

In this present study, a macro-approach on cell formation problem will be investigated. A step will be made further from cell formation problem. The problem will incorporate operational sequence of parts as well as production volume of each part. Based on the counting of total intercellular part movement distance unit, an optimal sequence of cells arrangement will be obtained. A mathematical model will be developed first and solved by using concept of genetic algorithm (GA). The proposed method will only consider linear arrangement of cell layout. Other form of cell layout such as circular arrangement will not be considered in this study as it would make the problem complicated.

The paper is presented in the following sequence. Literature review is made in Section II. Then a mathematical model is presented in Section III. Afterwards, solution algorithm is shown in Section IV. Section V, which is dedicated for illustration and result. Lastly, conclusion is discussed in section six.

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II. MATHEMATICAL MODELING

For describing this problem, a mathematical model is formulated.

Notations

i = the part number;
q = the operation number;
cn = the cell number;
P = the total umber of parts to be manufactured;
Pm = $\{pm_1, pm_2, pm_3, \ldots\ldots pm_P\}$ be the set of P parts to be manufactured;
M = the total number of machines;
Mh = $\{mh_1, mh_2, mh_3, \ldots\ldots, mh_M\}$ be the set of M machines;
V_i = the volume of part i to be manufactured;
$m_i(k)$ = machine being used to manufacture part i under kth operation correspondingly;
no_i = the total number of operations of part i;
o_i = $\{m_i(1), m_i(2), m_i(3), \ldots, m_i(no_i)\}$ be the set of operations of part i;
NC = the total number of existing cells;
C = $\{c_1, c_2, c_3, \ldots, c_{NC}\}$ be the set of NC cells;
PF = $\{pf_1, pf_2, pf_3, \ldots, pf_{NC}\}$ be the set of part families;
MC = $\{mc_1, mc_2, mc_3, \ldots, mc_{NC}\}$ be the set of machine cells;
SOC = $\{sc_1, sc_2, sc_3, \ldots, sc_{NC}\}$ be the set of sequence of the cells;
$SCm_i(k)$ = the order/sequence number of Cell in which machine, $m_i(k)$, is in that cell;
$popsize$ = the size of population;
Z = the objective function value;

The ultimate objective function is shown as follows,

Minimize

$$Z = \sum_{i=1}^{P} \sum_{q=1}^{no_i - 1} V_i m_i(q)(q+1) \left| SC_{m_i(q)} - SC_{m_i(q+1)} \right| \quad (1)$$

and the function is subjected to the following constraints:

$$m_i(q)(q+1) \begin{cases} 0 & \text{if machine } m_i(q) \text{ and machine } m_i(q+1) \text{ are in the same cell;} \\ 1 & \text{otherwise;} \end{cases} \quad (2)$$

$\forall i$ and (q+1) $\in \{2,3,\ldots,no_i\}$

$$sc_1 \neq sc_2 \neq sc_3 \neq \ldots \neq sc_{NC} \quad (3)$$

$$\text{Max } SC_{m_i(k)} \leq NC; \quad (4)$$

Equation (1) indicates the ultimate objective function value which is to calculate the total intercellular part distance movement (2)

Equation (3) restricts the sequence value assigned to each cell should not be equal. The maximum value assigned to the cell should be less than or equal to the total number of available cells which is indicated by (4).

III. SOLUTION ALGORITHM

A. Genetic Algorithm

In this study, GA will be employed in solving CLP. The general outline of GA is summarized below:

1) Generate random population with n chromosomes by using symbolic representation scheme (suitable size of solutions for the problem).
2) Evaluate the fitness f(x) of each chromosome x in the population by using the proposed objective function.
3) Create a new population by iterating loop highlighted in the following steps until the new population is complete
   i) Select two parent chromosomes from a population according to their fitness from step 2. Those chromosomes with the better fitness will have higher chance to be chosen.
   ii) With a preset crossover probability, crossover operation will perform on the selected parents and to form new offsprings (children). If no crossover was performed, offsprings are the exact copy of parents. Here partially matched crossover (PMX) is employed.
   iii) With a preset mutation probability, mutation will perform on new offspring at each gene. Chosen genes are swapped to perform mutation process.
   iv) Place new offsprings in the new population.
4) Use new generated population for a further run of the algorithm.
5) If the end condition is satisfied, stop, and deliver the best solution in current population.
6) Go to step 2.

B. GA in Cell Layout Problem

With reference to the description of GAs in the above, the proposed method should be specially designed in accordance with the problem nature. In light of this, the aspects including chromosome representation, fitness evaluation, parent selection, crossover (reproduction), and mutation will be tailor-made to the problem in this study.

Each chromosome in population with a given size represents a solution of CLP. When the iterative loop comes to the end in which the condition is satisfied, the ultimate solution will be found and it will also be the ultimate solution of the problem. So, in the very beginning, suitable representation of the chromosome (solution) should be carefully made. In order to apply the proposed approach, a path representation for each chromosome is used to encode the sequence of the existing cells. If the CLP involves NC cells, a string with the defining length is NC is needed to encode any

candidate solutions. The path representation is perhaps the most natural representation for this problem. For instance, the cell sequence is presented pictorially as follows:

Cell	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7
Sequence No.	2	3	5	6	4	7	1

Fig. 1. Chromosome representation of CLP

Therefore, the solution (chromosome) is simply represented as (2, 3, 5, 6, 4, 7, 1). In this chromosome representation, Cell 7 should be arranged all ahead of other cells. Cell 1 is then located just after Cell 7. So the entire linear arrangement of the cells is like Cell 7, Cell 1, Cell 2, Cell5, Cell 3, Cell 4, and lastly is Cell 6.

1) Fitness evaluation

Before performing the crossover function to produce offsprings, each solution in the population pool should be calculated to determine its fitness value. So that according to their fitness values, a probability will be assigned to each of them. The higher probability, the bigger chance to be chosen for crossover in such way. In this study, the objective is to minimize the total intercellular traveling unit distance. Equation (1) is used to determine the objective function value of each chromosome. One with the most minimum objective value will be the best chromosome in that population, in which the objective value is denoted as Z_{best} in each generation. Other chromosomes are denoted as Z_a in generation g. Therefore, the fitness value of chromosome a with the objective function value Z_a will be calculated in (5).

$$\Gamma_a = \frac{Z_{best}}{Z_a} \tag{5}$$

From (5), the fitness value of chromosome a will be found. It shows that chromosome with lower objective function value will result in having higher fitness value. The chromosome with higher fitness value will then have better combination of genes. This facilitates the selection process, which will be mentioned in the following section, and this also allows selection of the individuals on which the operators (crossover and mutation) are going to be performed and this also conforms with the purpose of minimizing the total intercellular traveling unit distance in this study.

2) Parent selection

Selection is the process of picking individuals for reproduction depending on their calculated fitness value from (5). This is the vital process in the algorithm since it selects the parents to produce offsprings and optimal solution will be obtained among the new solutions. In principle, individual with higher fitness value will be selected more often. In contrast, individual with lower fitness value will receive a lower or even no chance to be as a parent for reproduction. In light of this, a chromosome with higher fitness value may be chosen more than once to be a parent in current generation. According to this evolutionary principle, the chromosomes in the next generation generally must be better than or equal to those in previous generation. Hence, the generation will evolve gradually for producing better chromosomes after each iteration of parent selection, and finally the optimal solution may be obtained.

3) Reproduction

In this stage, crossover operation, which is the most reproduction operator in GA, will be performed on the selected parents. Two parents will exchange segments, which consist of genes, between them, and two new offsprings will be created as result for next generation. For optimization problem, like QAP, two crossover operators are found to be appropriate. They are partially matched crossover (PMX) [15] and order crossover (OX) operators [16].

In solving the CLP here, PMX will be used as crossover operator as it is especially suitable for TSP and QAP. Partially matched crossover (PMX) may be viewed as a crossover of permutations. It tends to preserve the positions of symbols from parents to offspring. PMX generates one offspring from two parents. It chooses a random segment within one parent and swaps each symbol in this segment with the symbol appearing at the same position in the second parent. The offspring permutation inherits the selected segment exactly from the second parent. It inherits the remaining positions primarily from the first parent.

Pictorial description of PMX is shown as follows:

P1: 2-4-5-|3-8-9|-6-1-7 P2: 3-9-8-|6-5-4|-2-7-1
P1': 2-9-8-|6-5-4|-3-1-7 P2': 6-4-5-|3-8-9|-2-7-1

Here, gene 3 and gene 6 are the crossover points for PMX.

PMX proposes a scheme to replace the repeated genes with the missing genes, which should have been there in the chromosome.

4) Mutation

Generally, normal mutation is not possible for TSP and QAP. It is because if one digit is mutated with one another, it will produce an illegal cell sequence. For example if a sequence is {1, 4, 6, 3, 7, 2, 5} and mutate at digit 5 with 4, the new sequence would be {1, 4, 6, 3, 7, 2, 4} which is absolutely illegal. In light of this, swap mutation is employed in this problem. In swap mutation two random points are found, swap the two digits at these positions. For example suppose the random points is p1=3, p2=6; the digits at positions 3 and 6 are 6 and 2.Swapping them produce {1, 4, 2, 3, 7, 6, 5}. By the way, for the sake of improving the efficiency and speed of convergence, the value of

probability of whether mutation is performed on the genes of the chromosome is usually very low.

IV. Illustration and Discussion

In order to demonstrate the effectiveness of the proposed model and methodology in solving CLP, a published problem is studied. In the following, we use an example taken from [1]. A preliminary data of machining sequence of the parts without production volume is shown as Table I and Table II.

The results are obtained with the parameters as follows: population size = 200; crossover rate = 0.8; and mutation rate = 0.001. And the results are shown in Table III.

TABLE I
MACHINING SEQUENCE OF EACH PART

Parts	Mac 1	Mac 2	Mac 3	Mac 4	Mac 5
1	1	13	21	22	
2	3	20	24		
3	7	14	23	24	
4	6	8	12	15	18
5	6	8	12	15	18
6	9	10	17		
7	9	10	17		
8	4	16			
9	1	13	21	22	
10	2	5	11	19	21
11	3	20			
12	3	20			
13	2	11	19		
14	2	5	11	19	21
15	3	20			
16	1	13	21	22	
17	1	13	21	22	
18	6	8	12	15	18
19	4	16			
20	2	5	11	19	21
21	4	16			
22	2	5	11	19	21
23	3	20			
24	3	20	12		
25	7	14	23		
26	6	8	15	18	10
27	6	8	12	15	18
28	4				
29	9	17			
30	6	8	12	18	
31	3	20	17		
32	7	14	23	24	16
33	1	13	21	22	2
34	3	20			
35	5	11	19	21	
36	5	11	19	21	
37	16	15			
38	4	16			
39	4	16			
40	9	10	17		

TABLE II
EXISTING CELL FORMATION

Cell Number	Machines
1	1, 13, 21, 22
2	3, 20
3	4, 16
4	2, 5, 11, 19
5	6, 8, 12, 15, 18
6	7, 14, 23, 24
7	9, 10, 17

TABLE III
SIMULATION RESULT

No.	Cell Sequence
1	1-5-6-2-4-7-3
2	1-5-6-2-3-7-4
3	1-5-7-2-4-6-3
4	1-5-7-2-3-6-4
5	7-3-1-6-5-2-4
6	7-3-1-6-4-2-5
7	7-3-2-6-5-1-4
8	7-3-2-6-4-1-5

According to the above results, there are seven different combinations of cell sequence which also obtain the same objective function value, 18. In the other way, there are seven options to the decision maker to decide which sequence can be used. However, data of production volume of each part have not been input. If it is considered, there maybe a significant impact of the cell sequencing. This is a future recommendation of this present study.

V. Conclusion

In this study, a mathematical model has been proposed to describe the characteristics of the CLP with the consideration of machining sequence of each part. Obviously, the objective is to minimize intercellular part traveling distance unit. The CLP in this study was solved by the approach of GA. Basically, the mathematical model with the method used possesses a number of merits.

- The ability to enhance the efficiency of individual cells and the entire cellular manufacturing systems by reducing the total intercellular traveling distance unit;
- Machining sequence and production volume of each part has been considered. This would make the problem more practical and realistic;
- From Table 3, it shows that the proposed method is able to generate more than one optimal solution in which the objective function values are the same.

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