# A METHOD FOR ESTIMATING THE INSTANTANEOUS FREQUENCY OF NON-STATIONARY HEART SOUND SIGNALS

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#### ABSTRACT

Practical signals such as speech, biomedical measurement and communications turn out to be extremely non-stationary and nonlinear time series. Traditional FFT-based power spectral analysis fails to deal with these transient signals. To provide more efficient way for investigating non-stationary and nonlinear signals with high time-frequency resolution and extract more information regarding the transient frequency features involved in the signals, a novel method based on the instantaneous frequency distribution is developed in this paper to provide the time-frequency distribution of the practical signals. The aim of this contribution is to explore the role that both empirical mode decomposition and Hilbert transform can be used to play in such practical signals. Both simulation and experimental results were presented and analyzed to demonstrate the power and effectiveness of the proposed new time-frequency distribution.

## 1. INTRODUCTION

The assumption of stationarity in signal analysis has long been dominated by FFT with its end products in the form of power spectra [1]. However, the Fourier analysis fails to deal with the non-stationary signals whose frequency contents change with time. For non-stationary process, different time-frequency representations, such as STFT, spectragram, WVD and wavelet transform, were proposed [1-5]. To provide the technique with higher time-frequency resolution, a new method for analyzing the non-stationary nonlinear random sequences is developed to study the time-varying frequency of the practical signals. We explore the role that both Empirical Mode Decomposition (EMD) and Hilbert transform can be employed to play in such non-stationary signals. The EMD analysis for a non-stationary nonlinear signal includes decomposing a signal into a series of intrinsic oscillatory mode, known as intrinsic mode functions. Hilbert transform is applied to each intrinsic mode function for

the purpose of providing the global time-frequency distribution of the underlying signal with a point of view of instantaneous frequency (IF) [6-9]. Two kinds of real heart sound signals with normal and abnormal cardiac functions were analyzed by using the proposed method. The instantaneous frequency distributions of the signals were also compared with the results by using the wavelet transform. Both simulation and the experimental results indicate that the novel method provides us a new way to effectively deal with the time-varying spectral characteristics of the non-stationary nonlinear signals in a wide range of practical application.

# 2. ANALYTIC SIGNAL

Let the Hilbert transform of a real signal x(t) is defined as

$$H[x(t)] = X_H(t) = \frac{P}{\pi} \int \frac{x(t-\tau)}{t-\tau} d\tau \tag{1}$$

where P indicates the Cauchy principal value. This transform exists for all functions of class  $L^P$  [7]. The analytic signal of x(t) can be defined as

$$z(t) = x(t) + iX_H(t) = e(t)e^{i\theta(t)}$$
 (2)

Another important concept for analytic signal is about instantaneous phase which is defined as

$$\theta(t) = \arctan \frac{X_H(t)}{x(t)}.$$
 (3)

## 3. INSTANTANEOUS FREQUENCY

Hilbert transform and the analytic signal enable to detect the instantaneous information regarding both amplitude and phase of the given non-stationary signals. The instantaneous frequency is defined as

$$\omega(t) = \frac{d\theta(t)}{dt} = \operatorname{Im} \left[ \frac{d \ln X_H(t)}{dt} \right]$$
 (4)

Thus, the IF of the signal can be obtained by using the above definition, forming a series of instantaneous frequency values changing from point to point in time domain. Only one frequency value at any given time can be obtained since the instantaneous frequency defined in equation (4) is a single value function of time. As a result, it can only precisely provide the IF of one component at any time when using (4).

## 4. THE EMD SCHEME

The IF cannot be directly computed from the equation (4) for multi-components signals. Method is therefore needed to separate the multi-components signal into a series of simple signals with only one frequency component at any time so that the definition of instantaneous frequency can be applied to obtain the global TFD of the multi-component signal. A complicated signal decomposition for well-behaved Hilbert transform was discussed [10-11]. A so called empirical mode decomposition (EMD) is introduced to solve this problem. A non-stationary nonlinear signal can be decomposed into a finite set of functions that have meaningful instantaneous frequencies defined by equation (4). These functions are called intrinsic mode functions (IMFs) in which each mode should be independent of the others and only one frequency component exists at a given time. Then the instantaneous frequencies can be computed by taking Hilbert transform of each IMF. The decomposition of a signal s(t) into a series of IMFs is implemented with a so called sifting process. First, all the maxima and minima of the signal are located. In addition, the maxima and minima are connected to construct the up and low envelopes with a cubic spline fitting so that all the data set are between the upper and lower envelopes. Finally, the mean value of each point on the maxima and minima spline envelopes can be obtained, designated as m(t), and then the m(t) is subtracted from the original signal to produce the first IMF candidate, i.e.

$$h_1(t) = s(t) - m(t) \tag{5}$$

If  $h_1(t)$  is taken as an IMF, it is extracted from the original signal. However, if  $h_1(t)$  do not satisfy the conditions above, the step like extracting  $h_1(t)$  should be repeated for k siftings until the condition is satisfied such that  $h_1$ ,  $h_{12}$ , ..., $h_{1k}$  are computed and lead to the first IMF component:

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t) \tag{6}$$

After the  $h_{1k}$  is subtracted from the original signal, it can be designated as  $c_1 = h_{1k}$ , producing the residue

$$r_1(t) = s(t) - c_1$$
 (7)

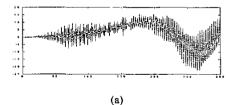
Replace the original signal with r<sub>1</sub> and repeat the process as mentioned above to decide the next IMF component. The IMFs decomposition will be stopped once a preset value for the residue is met or when the residue becomes monotonic function from which no more IMF can be extracted. Thus, the original signal are separated into n IMFs plusing an expected residue value such as

$$s(t) = \sum_{j=1}^{n} c_j + r_n$$
 (8)

### 5. RESULTS AND DISCUSSIONS:

To test the performance of the presented method, some simulation and real heart sound signals were investigated in this section. As an example, a record of non-stationary nonlinear signal was shown in Fig. 1 (a). Its corresponding signal decompositions were presented in Fig. 1 (b). Obviously, the signal is decomposed into 4 basic components pulsing a zero residue. The IF estimation of the signal before and after smoothing were shown in Fig. 2 (a) and (b).

As a practical application, the IF of two kinds of real heart sound signals with normal and abnormal cardiac functions recorded from the surface of the chest were studied. The IF of the heart sounds were also compared with the results by using Morlet wavelet transform [12-13]. Fig. 3 shows an example of the TFD of the normal and abnormal heart sounds by using both Hilbert TFD and wavelet transform. Fig. 3 (a) and (d) show two records of normal and abnormal heart sound signals. With the new time-frequency technique discussed in this paper, the time-varying frequency components of the heart sound signals can be clearly identified, as shown in Fig. 3 (b) and (e). The Morlet wavelet transforms for two heartsound signals were also demonstrated in Fig. 3(c) and (f). It can be seen that the time-frequency structures of heart sound series are too vague to identify the individual frequency component when using the wavelet analysis since the wavelet transform is a mathematical decomposition and the energy in high frequency was spread.



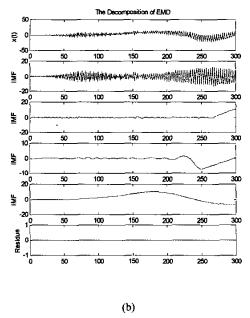


Fig. 1 A non-stationary process (a) and its corresponding empirical mode decomposition. The original signal was decompounded into 4 basic components (b).

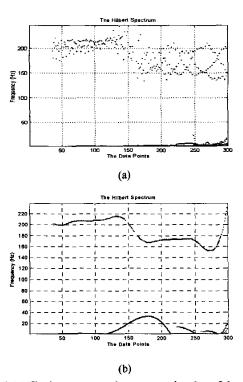
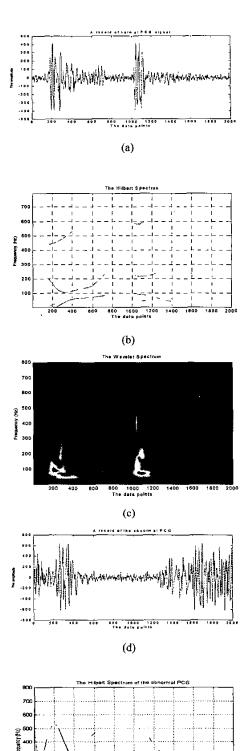
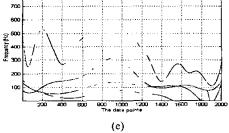


Fig. 2 (a) The instantaneous frequency estimation of the signal before smoothing. (b) The smoothed instantaneous frequency distribution of the signal via a specified smoothing method.





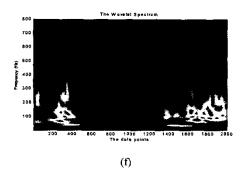


Fig.3 (a) and (d): Two records of heart sound signals with normal and abnormal cardiac function. (b) and (e): The corresponding Hilbert spectral estimations of the signals in (a) and (d). (c) and (f): The corresponding wavelet spectra of the signals in (a) and (d), as the results compared with the Hilbert spectra

#### 6. CONCLUSIONS

It can be concluded that the significant results of Hilbert spectral analysis exhibited the concept of IF for multi-component non-stationary signals. The proposed novel approach is adequate for investigating the non-stationary nonlinear signals. The IF analysis gives a more distinct definition of the particular events in time-frequency space than wavelet analysis or WVD. The presented new method enables to resolve changes in the frequency content of the given non-stationary multi-component signals. Both EMD method and the associated Hilbert spectral analysis offer a new technique for non-stationary nonlinear signal analysis. Sifting process to produce the necessary IMFs based on the EMD is a procedure of signal pre-processing, which enables a complicated signal to be reduced into a finite set of amplitudeand frequency-modulated form. The instantaneous frequency can be properly estimated with the IMFs from the given signal. The IF spectrum can be regarded as a local and adaptive method for time-frequency analysis. This novel method has provided us a new physical insight in understanding the non-stationary and nonlinear phenomena in many applications.

## 7. ACKNOWLEDGEMENT

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