

# Effective Throughput for Coded OFDM/SDMA Systems with Pilot-Assisted Channel Estimation

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*Abstract*— This paper investigates the performance of coded orthogonal frequency division multiplexing (OFDM) systems with receiving adaptive array, through which multiple users with single-element transmitting antenna are supported simultaneously by spatial division multiple access (SDMA). We characterize the performance of an OFDM/SDMA systems by effective throughput, which is essentially the average number of data bits in an OFDM symbol after considering the erroneous packet transmissions and modulation scheme by excluding the overhead from coding and pilots for channel estimation. Optimization of system operating parameters can be achieved through the maximization of effective throughput. The focus of this paper is to study the impact of pilot density and the number of users on the performance of coded OFDM/SDMA systems. Through extensive computer simulation, we show that using more pilots always improves bit error rate (BER) performance, but may reduce effective throughput. The optimal number of pilots together with the modulation scheme can be determined by maximizing the effective throughput for given operating signal-to-noise ratio (SNR). It is also shown that the system performance degrades gradually with the increase of users. For a system with a six-element adaptive array, the effective throughput with 5 users is lower than that with 4 users for a certain range of SNR. This indicates that the maximal number of users supportable by the system should consider the effective throughput.

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1] is a promising technology for high speed data communications, and has been adopted in wireless local area networks (LAN) standards such as IEEE 802.11a [2] and HIPERLAN II [3]. While OFDM is capable of combating inter-symbol interference (ISI), its performance can still be severely degraded by channel fading. An effective method to mitigate the influence of fading is to employ adaptive array [4], [6], through which multiple simultaneous users can further be supported by spatial division multiple access (SDMA).

In this paper, we study the uplink performance of an coded OFDM/SDMA wireless communication system. The base station (BS) is equipped with an adaptive antenna array for signal reception, and each mobile user transmits through a single-element antenna.

The performance of coded OFDM/SDMA systems can be characterized by effective throughput proposed in [5], which essentially represents the average number of data bits carried in an OFDM symbol after considering the modulation scheme and packet error while excluding the overhead from channel estimation and coding. Therefore, the optimization of system performance is equivalent to the maximization of effective throughput. Through the characterization by effective throughput, the influence of various factors on system performance can be quantified.

In this paper, the focus is on studying how the system performance is influenced by the pilot-aided channel estimation [8],

modulation, coding schemes, and the number of simultaneously transmitting users. Through simulation, we show that it is not always beneficial to have more pilots for channel estimation, because the use of pilots reduces the throughput for data. The optimal number of pilots should be determined together with the modulation scheme in order to maximize the effective throughput given the operating signal-to-noise ratio (SNR). Similarly, supporting more users may result in the reduction of effective throughput. In particular, simulation results demonstrate for a system with a six-element adaptive array, the effective throughput with 5 users can be lower than that with 4 users for a certain range of SNR. This indicates that the maximal number of users supportable by the system is not only limited by the freedom of the system (number of receiving antenna elements), but is also conditioned upon effective throughput and operating SNR.

This paper is organized as follows. In Section II, we describe the system. The definition of effective throughput is presented in Section III. In Section IV, the performance of an OFDM/SDMA system is investigated. We conclude this paper in Section V.

## II. SYSTEM DESCRIPTION

The baseband processing of a mobile user  $u$ ,  $u = 1, \dots, U$  is depicted in Fig. 1. We assume the number of simultaneous users  $U$  is no greater than the number of antenna elements  $A$  at the BS. Data bits are first encoded by a convolutional encoder, and interleaved by a random interleaver. The conventional block interleaver is inappropriate because convolutional code performs unsatisfactorily with bursty errors [10] due to the correlated fading on adjacent subcarriers over both time and frequency. The interleaved binary bits are then mapped to a modulation symbol using quadrature amplitude modulation (QAM). After serial-to-parallel conversion, inverse fast Fourier transform (IFFT) is performed on the symbols  $S_u(i)$ ,  $i = 1, \dots, N_c$  where  $i$  denotes the subcarrier index and  $N_c$  is the number of subcarriers in an OFDM symbol. The outputs from the IFFT transformer correspond to the time samples of the transmitting signal and are parallel-to-serial converted. After cyclic prefix is added, the resulting signal is finally transmitted from a single-element antenna.

At the BS (see Fig. 2), signal is received by  $A$  antenna elements. We assume the channels between a user and every antenna element are uncorrelated, and signals from different users are independent, which leads to a full diversity system. After sampling and removing cyclic prefix, the samples are serial-to-parallel converted, and demodulated by the fast Fourier

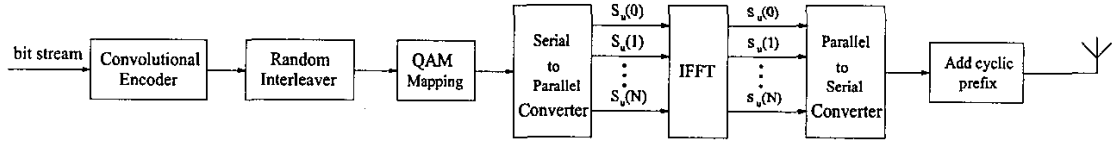


Fig. 1. Baseband processing of a mobile user.

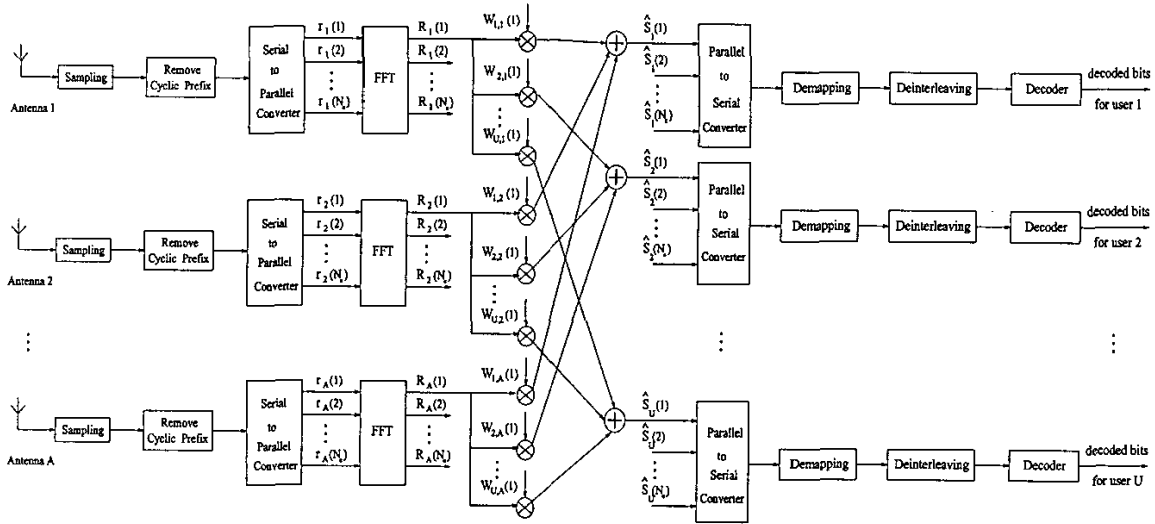


Fig. 2. Baseband processing at the base station. We only show the antenna processing on subcarrier 1, while the processing on other subcarriers is identical and is omitted.

transform (FFT) processor to produce  $R_a(i)$ ,  $a = 1, \dots, A$ ,  $i = 1, \dots, N_c$  at antenna  $a$ . For subcarrier  $i$ , the signals from different antenna elements  $R_a(i)$ ,  $a = 1, \dots, A$  are weighted and combined to form an estimate  $\hat{S}_u(i)$ ,  $u = 1, \dots, U$  for the transmitted signal  $S_u(i)$ ,  $u = 1, \dots, U$ . The estimated signal is finally mapped back to binary bits, which are then deinterleaved and decoded.

To illustrate how the antenna processing works, let  $\mathbf{S}(i) = [S_1(i), \dots, S_U(i)]^T$  be the vector of transmitted signal on subcarrier  $i$ , where  $T$  denotes the transpose operation. Also, let  $\mathbf{R}(i) = [R_1(i), \dots, R_A(i)]^T$  be the received signal vector, and  $\mathbf{n}(i) = [n_1(i), \dots, n_A(i)]^T$  be the noise vector, where  $n_a(i)$  is the noise on subcarrier  $i$  at antenna  $a$ . Since users are assumed to be perfectly synchronized in both time and frequency, the received signal for subcarrier  $i$  is expressed as

$$\mathbf{R}(i) = \mathbf{H}(i)\mathbf{S}(i) + \mathbf{n}(i), \quad i = 1, \dots, N_c \quad (1)$$

where  $\mathbf{H}(i)$  is the channel transfer matrix on subcarrier  $i$

$$\mathbf{H}(i) = \begin{bmatrix} H_{1,1}(i) & \cdots & H_{U,1}(i) \\ \vdots & \ddots & \vdots \\ H_{1,A}(i) & \cdots & H_{U,A}(i) \end{bmatrix} \quad (2)$$

with  $H_{u,a}(i)$  being the fading coefficient between user  $u$  and antenna element  $a$  at the BS. Denoting the antenna weight matrix

$$\mathbf{W}(i) = \begin{bmatrix} W_{1,1}(i) & \cdots & W_{U,1}(i) \\ \vdots & \ddots & \vdots \\ W_{1,A}(i) & \cdots & W_{U,A}(i) \end{bmatrix}, \quad (3)$$

where  $W_{u,a}(i)$  is the antenna weight for the  $i$ th subcarrier of user  $u$  at antenna  $a$ , the transmitted signal vector  $\mathbf{S}(i)$  can be estimated by

$$\hat{\mathbf{S}}(i) = \mathbf{W}^H(i)\mathbf{R}(i) \quad (4)$$

where  $H$  represents the Hermitian transpose.

With channel state information, the optimal weight matrix  $\mathbf{W}(i)$  can be computed by [9]

$$\mathbf{W}(i) = [\mathbf{H}(i)\mathbf{H}^H(i) + \sigma^2\mathbf{I}]^{-1}\mathbf{H}(i), \quad i = 1, \dots, N_c, \quad (5)$$

where  $\mathbf{I}$  is an  $A \times A$  identity matrix. In this paper, the channel state information is obtained through the use of pilots [8].

### III. EFFECTIVE THROUGHPUT

We define the effective throughput in this section. This concept is devised to characterize the performance of an OFDM/SDMA system by considering the packet error rate (PER), overheads from pilots, modulation scheme and coding rate.

**Definition 1—User Effective Throughput:** Let  $P_e$  be the PER,  $N_d$  the number of data subcarriers in an OFDM symbol,  $M_j$  the

modulation index on data subcarriers, and  $r_c$  the coding rate. The effective throughput for a user is defined as

$$T_u \triangleq (1 - P_e)N_d M_I r_c. \quad (6)$$

The normalized effective throughput is defined as the ratio between the user effective throughput and the number of subcarriers in an OFDM symbol. That is

$$\begin{aligned} \bar{T}_u &\triangleq \frac{T_u}{N_s} \\ &= \frac{(1 - P_e)N_d M_I r_c}{N_s}. \end{aligned} \quad (7)$$

**Definition 2—System Effective Throughput:** Let  $T_u(i)$  be the effective throughput for user  $i$ . When there are  $U$  users transmitting simultaneously, the system effective throughput is defined as

$$T_s \triangleq \sum_{i=1}^U T_u(i). \quad (8)$$

The normalized system effective throughput is similarly defined as

$$\begin{aligned} \bar{T}_s &\triangleq \sum_{i=1}^U \bar{T}_u(i) \\ &= \frac{T_s}{N_s}. \end{aligned} \quad (9)$$

#### IV. DISCUSSIONS AND SIMULATION RESULTS

In this section, we discuss the impact of various factors on the performance of coded OFDM/SDMA through simulation, with the emphasis on the pilot density and number of users. The performance criterion we use is the normalized effective throughput for a user, but sometimes BER is used as a supplement. Throughout this section, we shall use the term effective throughput interchangeably with normalized effective throughput for convenience.

##### A. Simulation Setup

We assume all users apply the same operating parameters, and the channels between every user and the BS are statistically independent and identical. Therefore, all users would have statistically the same performance.

The multipath fading channel is assumed to be wide sense stationary with uncorrelated scattering (WSSUS). In the time domain, channel impulse response is modeled as a tapped delay line at tap spacing  $T_s$ , with each path following an exponential power delay profile. The amplitude on each path is Rayleigh distributed, and the phase is uniformly distributed between  $[0, 2\pi]$ . The method in [11] is used for the simulation of channel fading. The simulation parameters are summarized in Table I.

In our simulation, pilots are inserted for channel estimation. To confine the overheads from pilots, only one of every  $M_t$  OFDM symbols is inserted with pilots on subcarriers that are  $M_f$  subcarriers apart, and the arrangement is illustrated in Fig. 3. When a subcarrier is used as a pilot subcarrier for a user, other users place null symbols on the subcarrier to avoid interference, i.e., no signal (neither pilot nor data) is carried by the subcarrier. After estimating the channel on pilot subcarriers in a pilot-bearing OFDM symbol, the channels on all

TABLE I  
PARAMETERS FOR AN OFDM/SDMA SYSTEM.

Number of antennas	$A = 6$
Number of users	$U = 3, 4$ or $5$
Carrier frequency	$f_c = 5.4$ GHz
Bandwidth	20 MHz
Number of subcarriers	$N_c = 128$
Carrier spacing	$\Delta f = 20/128 = 162.5$ kHz
Sampling interval	$T_s = 1/N\Delta f = 0.05$ $\mu$ s
FFT period	$T_F = 1/\Delta f = 6.4$ $\mu$ s
Guard interval	$T_G = 0.8$ $\mu$ s
Symbol interval	$T_S = T_F + T_G = 7.2$ $\mu$ s
Packet length	500 bytes
Modulation schemes	QPSK, 16-QAM, 64-QAM
Coding scheme	Convolutional code
Coding rate	$r_c = 1, 1/2, 1/3, 1/4$
Generator functions (in octals)	(753, 561), (557, 663, 771), (765, 671, 513, 473)
Interleaver	Random interleaver
Channel delay spread	0.8 $\mu$ s
Doppler frequency	$F_d = 1389$ Hz
Pilot density	$M_f = 4$ or $8, M_t = 5$
Pilot SNR	20 dB

other subcarriers of that OFDM symbol are estimated by a maximal likelihood (ML) estimator [12]. Then linear interpolation is performed based on the estimation from two consecutive pilot-bearing OFDM symbols for estimating the channels of the  $M_t - 1$  OFDM symbols in between [13]. To ensure the quality of channel estimation, we require the SNR on the pilot subcarriers to be always maintained at 20 dB regardless of the SNR on data subcarriers.

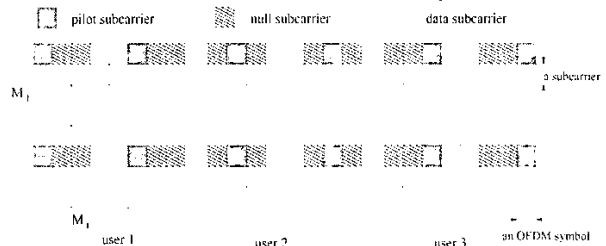


Fig. 3.. An example for pilot arrangement with three users.

##### B. Pilot Density

We first compare the performance under different number of pilots. In the simulation,  $M_t$  is fixed at 5,  $M_f$  is 4 or 8 and  $U = 3$ .

The BER for the uncoded QPSK, 16-QAM, and 64-QAM with  $M_f = 4$  or  $8, M_t = 5, U = 3$  is plotted in Fig. 4. From Fig. 4, we notice that by doubling the pilot density in the frequency domain (from  $M_f = 8$  to  $M_f = 4$ ), there is consistently about 2 dB gain in SNR for all modulation schemes under investigation. At the same  $M_f$  and  $M_t$ , there is 6 to 8 dB difference

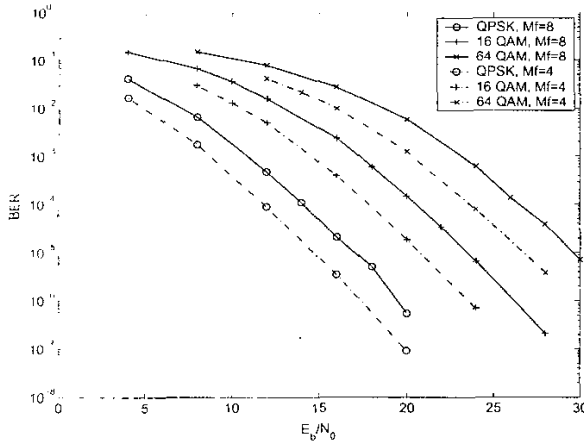


Fig. 4. BER of uncoded QPSK, 16-QAM, and 64-QAM with  $M_f = 4$  or  $8$ ,  $M_t = 5$ ,  $U = 3$ .

in SNR between QPSK and 16-QAM for the same BER, and 4 to 6 dB difference between 16-QAM and 64-QAM.

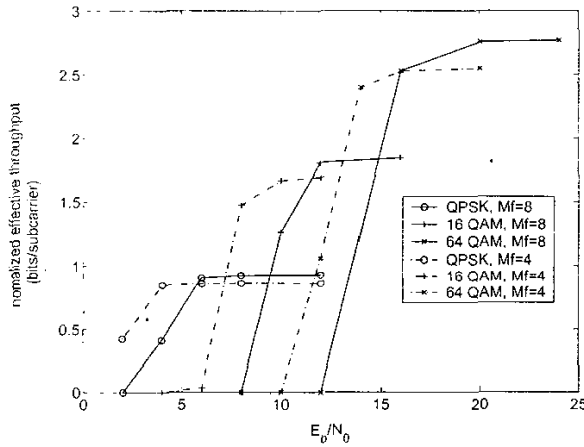


Fig. 5. Effective throughput for rate- $\frac{1}{2}$  coded QPSK, 16-QAM, and 64 QAM with  $M_f = 4$  or  $8$ ,  $M_t = 5$ ,  $U = 3$ .

The effective throughput for a rate- $\frac{1}{2}$  coded system with  $M_f = 4$  or  $8$ ,  $M_t = 5$ ,  $U = 3$  is plotted in Fig. 5. We first observe from Fig. 5 that the curve of effective throughput can be characterized by three segments depending on the SNR. When SNR is lower than a certain threshold, effective throughput is zero. When SNR is higher than a certain cutoff value, effective throughput becomes saturated. Between the two values, effective throughput curve is almost linear. Further, the effective throughput with  $M_f = 4$  is higher than that with  $M_f = 8$  before it becomes saturated, and is lower than that with  $M_f = 8$  in the saturation segment.

The above phenomena can be explained as follows. The normalized effective throughput is expressed in (7) as  $\bar{T}_u = (1 - P_e)N_d M_f r_c / N_c$ , where  $N_d$  is the average number of data subcarriers in an OFDM symbol. From our arrangement of pi-

lots,  $N_d$  can be calculated as

$$N_d = N_c - N_c / M_f / M_t. \quad (10)$$

Thus when SNR is low,  $P_e \approx 1$ , so that  $\bar{T}_u \approx 0$ ; when SNR is high,  $P_e \rightarrow 0$ , so that  $\bar{T}_u$  is close to the upper limited

$$N_d M_f r_c / N_c. \quad (11)$$

Obviously the limit with  $M_f = 8$  is higher than that with  $M_f = 4$ . Although more pilots always improves the BER performance, it increases overheads and thus reduces effective throughput, as shown in Fig. 5.

To maximize effective throughput, the optimal number of pilots can be selected together with the modulation scheme from the effective throughput curves in Fig. 5 when given the operating SNR. For example, when SNR is 10 dB, 16-QAM with  $M_f = 4$  is used; when SNR is 20 dB, 64-QAM with  $M_f = 8$  is chosen.

### C. Number of Users

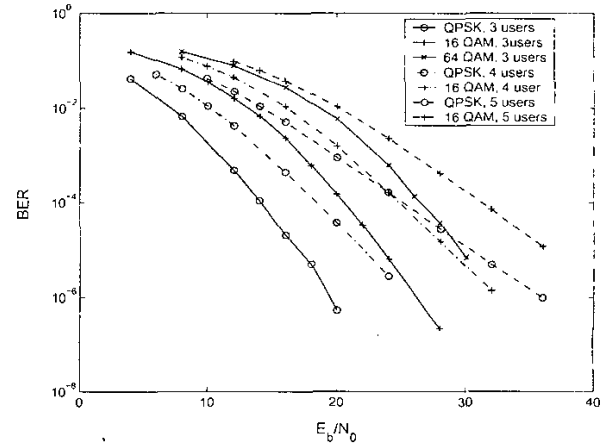


Fig. 6. BER for uncoded QPSK, 16-QAM and 64-QAM with 3, 4 and 5 users,  $M_f = 8$ ,  $M_t = 5$ .

We investigate the performance impact by the number of users. According to [9], an  $A$ -element antenna array has  $A - 1$  degrees of freedom. To cancel the co-channel interference (CCI) from a user, one degree of antenna freedom is needed. As a result, with  $U$  users ( $U \leq A$ ),  $U$  degrees of freedom are needed for CCI cancellation, so that the degree of freedom for diversity is  $A - U$ . Thus the degree of diversity diminishes with more users.

We plot the BER performance of uncoded systems in Fig. 6, and the corresponding uncoded effective throughput in Fig. 7. It is clear that both the BER and effective throughput performance degrades with more users.

An interesting observation from Fig. 7 is that the effective throughput for uncoded 16-QAM with 4 users is always higher than that of 5 users using either QPSK or 16-QAM. We can also calculate from the definition in (9) that the system effective throughput with 5 users is lower than that with 4 users when

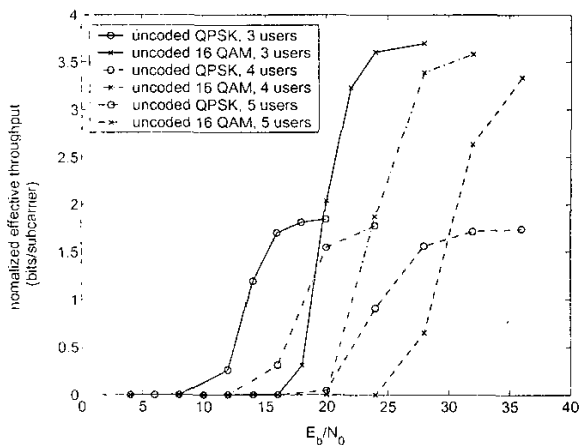


Fig. 7. Effective throughput for uncoded QPSK, 16-QAM and 64-QAM with 3, 4 and 5 users,  $M_f = 8$ ,  $M_t = 5$ .

SNR is lower than 35 dB. For example, when SNR is 32 dB, the effective throughput for a user is about 3.6 bits/subcarrier for 16-QAM when there are 4 users, and is about 2.7 bits/subcarrier for 16-QAM when there are 5 users. The corresponding system effective throughput is  $3.6 \times 4 = 14.4$  bits/subcarrier and  $2.7 \times 5 = 13.5$  bits/subcarrier respectively. This reveals that the addition of a user can degrade the effective throughput performance for a user and the system at the same time, which is obviously undesirable.

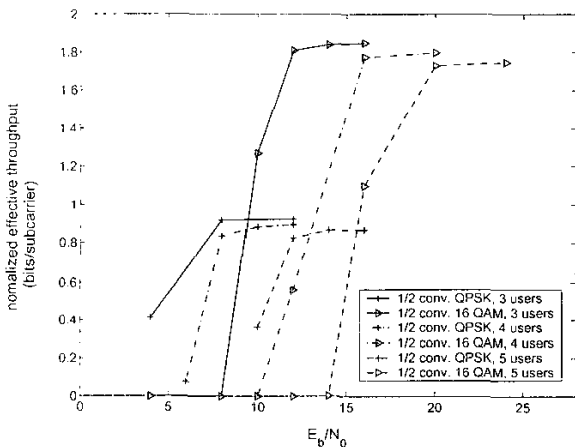


Fig. 8. Effective throughput for rate-1/2 coded QPSK and 16-QAM with 3, 4, and 5 users,  $M_f = 8$ ,  $M_t = 5$ .

The coded performance for effective throughput is plotted in Fig. 8. The effective throughput with 5 users employing QPSK is better than that of 4 users employing 16-QAM when SNR is lower than 16 dB, which is much improved as compared with the uncoded case. However, when SNR is above 16 dB, the effective throughput for 5 user with either QPSK or 16-QAM is still lower than that of 4 users with 16-QAM.

## V. CONCLUSION

In this paper, we study the performance of coded OFDM/SDMA systems through effective throughput. Specifically, the focus is on investigating the influence of pilot density and number of users. We show that the BER performance improves consistently with more pilots, but the effective throughput can be reduced due to the overhead from pilots. The optimal number of pilots can be determined together with the modulation scheme by maximizing effective throughput at the operating SNR. Simulation results also show that system performance degrades with more users. It is possible for the effective throughput for a user and the system be reduced at the same time by the addition of a user. Thus the maximal number of users that can be supported by an OFDM/SDMA system should be carefully chosen by considering effective throughput.

## ACKNOWLEDGMENT

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