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Super-resolution Time Delay Estimation in Multipath Environments†

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Abstract—The problem of super-resolution time delay estimation in multipath environments is addressed in this paper. Two cases, active and passive systems, are considered. The time delay estimation is first converted into a sinusoidal parameter estimation problem. Then the sinusoidal parameters are estimated by generalizing the Multiple Signal Classification (MUSIC) algorithm for single-experiment data. The proposed method, referred to as the MUSIC-type algorithm, approximates the Cramer-Rao bound (CRB) in terms of the mean square errors (MSEs) for different signal-to-noise ratios (SNRs) and separations of multipath components. Simulation results show that the MUSIC-type algorithm performs better than the classical correlation approach and the conventional MUSIC method for the closely spaced components in multipath environments.

Index Terms—time delay estimation, multipath environments, Eigenanalysis, Sequential Quadratic Programming

I. INTRODUCTION

The time delay estimation is a fundamental signal processing problem with application in many areas, such as radar [1], sonar [1], communications [2], etc. The classical time delay estimation techniques are based on correlations [1], and are only effective when multipath components are well separated in arrival time or when only one component is present in the received signal. Their performance will degrade significantly when the signal components are closely spaced in multipath environments.

A number of super-resolution techniques that can separate closely-spaced multipath components have been developed. Initially, the problem was studied in the time domain [3]. However, they offer marginal performance gain against the classical methods. Hou and Wu [4] first proposed a model-based sinusoidal estimation method, which converts the time delay estimation problem into a sinusoidal parameter estimation problem. Several other model-based sinusoidal estimation methods were also presented in [5], [6]. These methods involve a spectral-division operation, so they are only suitable for signals with flat spectra [7], [8]. The approaches in [9]-[11] can be applied to signals with non-flat spectra. However, these methods do not guarantee global convergence and require large numbers of data samples.

In this paper, a super-resolution time delay estimation method is proposed, based on a generalization of the Multiple Signal Classification (MUSIC) algorithm [17]. Two cases, active and passive systems, are considered. In particular, the transmitted signal is always unknown in passive systems. Our method is applicable to signals with non-flat spectra, as opposed to the model-based sinusoidal estimation methods [4]-[6]. In addition, our method requires much fewer samples than those in [9]-[11]. The performance of the proposed time delay estimator is evaluated by numerical simulations. The mean square errors (MSE) for different SNRs and the time delay separations in a multipath environment are shown to approximate the Cramer-Rao bound (CRB), and the proposed estimator performs better than the classical correlation approach and the conventional MUSIC method.

This paper is organized as follows. Section 2 presents the data models for estimating the time delay. In Section 3, we derive the super-resolution time delay estimators. Simulation results are shown in Section 4. Conclusions are provided in Section 5.

II. DATA MODEL

In this section, we give the data models for the time delay estimation. First, the notations in this paper are listed as follows:

\* , complex conjugate,
\( T \), transpose,
\( H \), Hermitian transpose,
\( \text{Re}(\cdot) \) and \( \text{Im}(\cdot) \), real part and imaginary part,
\( E(\cdot) \), expected value,
\( \|\cdot\| \), Euclidean norm,
\( \text{DFT} \), discrete Fourier transform (DFT),
\( \text{DFT}^{-1} \), inverse DFT (IDFT).

A. The Data Model in Active Systems

In active systems, the time delay estimation is always accomplished by matched filter or cross-correlation where the reference signal is the known transmitted signal. Suppose that the received signal is described as

\[ r(t) = \sum_{i=1}^{D} \lambda_i s(t - \tau_i) + w(t), \quad 0 \leq t \leq T_r, \quad (1) \]

where \( D \) is the number of multipath components, \( s(t) \) is the transmitted signal with duration \( T_s \), \( \lambda_i \) and \( \tau_i \) are the corresponding random amplitudes and time delays, \( w(t) \) is

\[ w(t) \]

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additive white Gaussian noise (AGWN), $T_r$ is the duration of $r(t)$. The resulting discrete-time sequence can be written as

$$r(n) = \sum_{i=1}^D \lambda_i s(n - \tau_i) + w(n), \quad n = 0, 1, \ldots, K_r - 1,$$

where $\tau_i, i = 1, 2, \ldots, D$, are the discrete time delays, and $K_r$ is the length of $r(t)$. We zero-pad $s(n)$ and $r(n)$ to length $K_{sA} = K_r + K_s - 1$, where $K_s$ is the length of $s(n)$. Then by computing the circular correlation between $s(n)$ and $r(n)$, and carrying out the square envelope detection, we have

$$|R_A(\tau)|^2 = \sum_{k=0}^{K_{sA}-1} S(k) \cdot W^*(k)$$

where $S(k)$ and $W(k)$ are the discrete Fourier transforms of $s(n)$ and $w(n)$, respectively. We denote

$$x_A(k) = \sum_{i=1}^D \lambda_i^* |S(k)|^2 e^{j2\pi k\tau_i/K_{sA}} + S(k) \cdot W^*(k),$$

and have the following relation,

$$|R_A(\tau)|^2 = (\text{DFT} [x_A(k)])^2.$$

In vector form, (4) is given by

$$x_A = x_A(0) \quad x_A(1) \quad \cdots \quad x_A(K_{sA} - 1)^T,$$

$$\lambda_A = [\lambda_1^* \quad \lambda_2^* \quad \cdots \quad \lambda_D^*]^T,$$

$$\Lambda(\tau_i) = \text{diag}\left(1, e^{j2\pi \tau_i/K_{sA}}, \ldots, e^{j2\pi(K_{sA} - 1)\tau_i}\right),$$

$$S = [\vert S(0) \vert^2 \quad \vert S(1) \vert^2 \quad \cdots \quad \vert S(K_{sA} - 1) \vert^2]^T,$$

$$\Phi_A = [\Lambda(\tau_1) S \quad \Lambda(\tau_2) S \quad \cdots \quad \Lambda(\tau_D) S]^T,$$

$$W_A = [S(0) \cdot W^*(0) \quad S(1) \cdot W^*(1) \quad \cdots \quad S(K_{sA} - 1) \cdot W^*(K_{sA} - 1)]^T.$$
C. About Models and Colored Noise

In the above two subsections, the time delay estimations in (2) and (8) are converted into sinusoidal parameter estimation problems as in (4) and (10), where \( |R_A(\tau)|^2 \) and \( |R_P(\tau)|^2 \) can be regarded as the power spectral densities of \( x_A(k) \) and \( x_P(k) \) as in (5) and (11), respectively. In later sections, we will propose a MUSIC-type algorithm to estimate \( \tau_i \) in (4) and \( \Delta \tau_i \) in (10). Following common practices as in [4]-[6], we assume the SNRs of the received signals in (2) and (8) are high, thus neglecting the colored noises \( S(k) \cdot W^* \) in (4) and \( W_P \) in (10) (see [5],[12] and references therein) in algorithm derivation.

III. TIME DELAY ESTIMATION

The time delay estimation problem in (4) and (10) is a parameter estimation of sinusoidal signals with lowpass envelopes [13]-[15]. The data models in (4) and (10) contain a multiplicative noise term \( S(k)^2 \). When the conventional MUSIC algorithm is applied to this case, Besson and Stoica [16] have shown that the estimator performance degrades slightly with very slowly varying envelopes, but the degradation is significant when the envelope fluctuation is not very slow. Thus the conventional methods should be modified to take into account the envelope variation.

A. The Time Delay Estimation in Active Systems

1) Multiple-experiment Data: Since \( |S(k)|^2 \) is the power spectral density of \( s(t) \) and deterministic, the covariance matrix of \( x_A \) in (6) can be written as

\[
R_{x_A} = \mathbf{E} \left( x_A x_A^H \right) = \Phi_A E \left( \lambda \lambda^H \right) \Phi_A^H + R_{W_A},
\]

where \( R_{W_A} \) is colored noise matrix and defined as

\[
R_{W_A} = \left\{ E \left[ \left( \sum_{i=1}^{D} \lambda_i \Lambda(\tau_i) S \right)^H \right] + E \left[ W_A \left( \sum_{i=1}^{D} \lambda_i \Lambda(\tau_i) S \right)^H \right] \right\}.
\]

Since \( |S(k)|^2 \) is always non-zero and the time of arrivals \( \tau_1, \tau_2, \cdots, \tau_D \) are different from each other, we have rank \( \left\{ \Phi \right\} = D \). Under the assumption that \( \lambda_1, \lambda_2, \cdots, \lambda_D \) are mutually uncorrelated, rank \( \left\{ E \left[ \lambda \lambda^H \right] \right\} = D \). Then we have the following criterion [15] for estimating \( \tau_i, i = 1, 2, \cdots, D, \) as in [17]

\[
\Phi_A^H G_A \simeq 0,
\]

where \( G_A \) is the noise subspace corresponding to \( R_{x_A} \). The approximation in (14) arises from \( R_{W_A} \).

2) Single-experiment Data: In many cases, multiple-experiment data may not be available. Thus, rank \( \left\{ x_A x_A^H \right\} = 1 \) if \( x_A \) is obtained from single-experiment data, which makes the above method incapable of resolving multipath components. Here we will present a method to estimate \( \tau_i, i = 1, 2, \cdots, D \) in the case of single-experiment data.

The estimate of covariance matrix \( \hat{R}_{x_A} \) in the case of single-experiment data can be expressed as [18]

\[
\hat{R}_{x_A} = \sum_{i=1}^{D} |\lambda_i|^2 \hat{R}_{\Phi_i} + \hat{R}_{W_A} = \hat{R}_{S_A} + \hat{R}_{W_A},
\]

where

\[
\hat{R}_{\Phi_i} = \hat{\Phi}_A (\tau_i) \hat{\Phi}_A^H (\tau_i) / (K_{x_A} - M_A + 1),
\]

\[
\hat{\Phi}_A (\tau_i) = \left[ \phi_A(0, \tau_i), \phi_A(1, \tau_i), \cdots, \phi_A(K_{x_A} - M_A, \tau_i) \right],
\]

\[
\phi_A(q, \tau_i) = [S(q)]^2 e^{j2\pi q \tau_i} [S(q + 1)]^2 e^{j2\pi q (\tau_i+1)} \cdots [S(q + M_A - 1)]^2 e^{j2\pi q (\tau_i + M_A - 1)}\]  

\[
q = 0, 1, \cdots, K_x - M_A,
\]

\( \hat{R}_{W_A} \) is the estimated covariance matrix of noise. We simply write \( \hat{R}_{\Phi_i} \) instead of \( \hat{\Phi}_A (\tau_i) \) whenever there is no confusion. According to the common assumptions of the MUSIC algorithm in [18], it is necessary to have \( K_{x_A} - M_A + 1 > M_A \) and \( M_A > D \), or equivalently \( (K_{x_A} + 1)/2 > M_A / D \). In general, the rank of \( \hat{R}_{\Phi_i} \) in (15) is larger than 1 and sometimes equals \( M_A \) [15],[19]. Even though the covariance matrix for the \( i \)th signal is full rank, the signal energy usually concentrates on the largest eigenvalue [15],[19]. The remaining dimensions of the matrix can be regarded as the quasi-noise subspace. Here, the concentration largely depends on \( |S(k)|^2 \) in (4) and (9) [13]-[15],[19]. Due to the orthogonality between signal and noise subspaces, there is

\[
\hat{R}_{S_A} \hat{G}_A \simeq 0,
\]

where \( \hat{G}_A \) is the noise subspace corresponding to \( \hat{R}_{x_A} \). Here the approximation also arises from the colored noise \( \hat{R}_{W_A} \). In practice, we can determine the approximate quasi-signal subspace dimension by examining the number of dominant eigenvalues. In this paper we assume that the number of signal components is known and only address the parameter estimation. Since \( \hat{R}_{S_A} \) is a linear combination of \( \hat{R}_{\Phi_i} \), we have

\[
\hat{R}_{\Phi_i} \hat{G}_A \simeq 0.
\]

By premultiplication of \( \hat{G}_A^H \) with (17), we have

\[
\hat{G}_A^H \hat{R}_{\Phi_i} \hat{G}_A = \hat{G}_A^H \hat{G}_A \hat{G}_A^H \simeq 0.
\]

Therefore, we have the following relation

\[
\hat{G}_A^H \hat{G}_A \simeq 0, \quad i = 1, 2, \cdots, D.
\]

Then we denote

\[
P_A(\tau) = \frac{1}{\hat{S}_A \hat{S}_A^H (\tau) \hat{G}_A},
\]

where \( \hat{A}_A(\tau) = \text{diag} \left( \frac{1}{e^{j2\pi \tau / K_x}} \cdots \frac{e^{j2\pi (M_A-1) \tau / K_x}}{e^{j2\pi \tau / K_x}} \right) \), \( \hat{S} \) is an \( M_A \times 1 \) sub-vector formed by \( |S(k)|^2 \). In our simulations, we always take the segment that contains the main spectral components of \( s(n) \). Since \( |S(k)|^2 \) is known, we can perform a 1-dimensional search with respect to \( \tau \) to find the maxima of (20), whose locations are the time delay estimates \( \hat{\tau}_i, i = 1, 2, \cdots, D \).
In this paper, since we have the power spectrum of \(x_A\) as in (5), we adopt the idea in [21] to estimate the covariance matrix from single-experiment data, instead of that in (15). We first obtain the correlation function \(R_{x_A}^{'}(l)\) from \(\hat{R}_A(\tau)\) according to the Wiener-Khinchine theorem [20]

\[
R_{x_A}^{'}(l) = \text{DFT}^{-1}\left[\hat{R}_A(\tau)\right]^2, \quad l = 0, 1, \ldots, 2K_{x_A} - 1.
\]

(21)

where \(\hat{R}_A(\tau)\) is a sequence formed by zero-padding \(\hat{R}_A(\tau)\) to length \(2K_{x_A} - 1\). Thus, \(R_{x_A}^{'}(l)\) can be viewed as the correlation function of an analytic signal. Then the covariance matrix can be formed based on \(R_{x_A}^{'}(l)\) [21]. The dimension of the covariance matrix is decreased to \(M_A\) to get the following truncated covariance matrix

\[
\tilde{R}_{x_A} = \left[R_{x_A}^{'}\right]_{M_A \times M_A},
\]

(22)

where \(R_{x_A}^{'} = R_{x_A}^{'}(i - j)\) for \(i \geq j\), and \(R_{x_A}^{'} = R_{x_A}^{'}(j - i)\) for \(i < j\). It has been shown that \(\tilde{R}_{x_A}\) in (15) and \(R_{x_A}^{'}\) in (22) are the consistent estimates of \(R_{x_A}\) [20]. Thus, we can substitute \(\tilde{R}_{x_A}\) for \(R_{x_A}^{'}\) in our algorithm.

\section{The Time Delay Estimation in Passive Systems}

In passive systems, if the transmitted signal is known, the time delay \(\Delta \tau_i\), \(i = 1, 2, \cdots, D\), in (12) can also be estimated by the above method. However, the transmitted signal in passive systems is always unknown. Therefore, the minimization requires the search not only on \(\Delta \tau\), but also on the unknown transmitted signal. Thus the above method can not be directly applied to obtain \(\Delta \tau_i\), \(i = 1, 2, \cdots, D\). However, in the case of single-experiment data, we still have

\[
\tilde{\Phi}_{\pi}^{\text{II}} \equiv 0,
\]

(23)

where

\[
\tilde{\Phi}_{\pi}^{\text{II}}(\Delta \tau_i) = \left[\Phi_P(0, \Delta \tau_i) \quad \Phi_P(1, \Delta \tau_i) \quad \cdots \quad \Phi_P(K_{x_P} - M_P, \Delta \tau_i)\right],
\]

\[
\Phi_P(q, \Delta \tau) = \left[|S(q)|^2 e^{j2\pi \frac{q}{N_P} \Delta \tau} |S(q + 1)|^2 e^{j2\pi \frac{(q+1)}{N_P} \Delta \tau} \quad \cdots \quad |S(q + M_P - 1)|^2 e^{j2\pi \frac{q+(M_P-1)}{N_P} \Delta \tau}\right]^T,
\]

\[q = 0, 1, \cdots, K_{x_P} - M_P,
\]

and \(\tilde{G}_P\) is the noise subspace corresponding to the estimated covariance matrix \(\tilde{R}_{x_P}\) of \(x_P\). Similar to that in active systems, \(\tilde{R}_{x_P}\) is substituted by \(\tilde{R}_{x_P}\), which can be obtained as in (24) and (25):

\[
R_{x_P}^{'}(l) = \text{DFT}^{-1}\left[\hat{R}_P(\tau)\right]^2, \quad l = 0, 1, \cdots, 2K_{x_P} - 1,
\]

(24)

where \(\hat{R}_P(\tau)\) is a sequence produced by zero-padding \(R_P(\tau)\) to length \(2K_{x_P} - 1\).

\[
\tilde{R}_{x_P} = \left[R_{x_P}^{'}\right]_{M_P \times M_P},
\]

(25)

where \(R_{x_P}^{'} = R_x^{'}(i - j)\) for \(i \geq j\), and \(R_{x_P}^{'} = R_x^{'}(j - i)\) for \(i < j\). It is necessary to have \((K_{x_P} + 1)/2 > M_P > 2D\) [15].

Here the delay estimation can be further expressed as

\[
\left(\Delta \tau_i, \tilde{S}\right) = \arg\min_{(\Delta \tau, \Psi)} \left\{\Psi^H \tilde{A}_G^{\text{II}}(\Delta \tau) \tilde{G}_P \tilde{A}_G^{\text{II}}(\Delta \tau) \Psi\right\},
\]

(26)

where \(\Psi\) is a vector of real-valued variables representing the power spectrum of the unknown transmitted signal \(s(t)\), and \(\tilde{A}_G(\Delta \tau) = \text{diag}\left[1, e^{j2\pi \frac{\xi_1}{M_P}}, \cdots, e^{j2\pi \frac{\xi_{M_P}}{M_P}}\right]\). In theory, a perfect estimate on \(\Psi\) should be equal to \(\tilde{S}\). Since \(\tilde{S}\) and \(\tilde{A}_G^{\text{II}}(\Delta \tau)\tilde{G}_P\tilde{A}_G^{\text{II}}(\Delta \tau)\Psi\) are real-valued, but \(\tilde{A}_G^{\text{II}}(\Delta \tau)\tilde{G}_P\tilde{A}_G^{\text{II}}(\Delta \tau)\) is complex-valued, there must be \(\Psi^H \text{Im} \left\{\tilde{A}_G^{\text{II}}(\Delta \tau)\tilde{G}_P\tilde{A}_G^{\text{II}}(\Delta \tau)\Psi\right\} = \Psi^H \text{Re} \left\{\tilde{A}_G^{\text{II}}(\Delta \tau)\tilde{G}_P\tilde{A}_G^{\text{II}}(\Delta \tau)\Psi\right\} = 0,\) and we have

\[
\Psi^H \tilde{A}_G^{\text{II}}(\Delta \tau) \tilde{G}_P \tilde{A}_G^{\text{II}}(\Delta \tau) \Psi = \Psi^H \text{Re} \left\{\tilde{A}_G^{\text{II}}(\Delta \tau)\tilde{G}_P\tilde{A}_G^{\text{II}}(\Delta \tau)\Psi\right\}.
\]

(27)

The operation of taking the real part in (27) is necessary [15]. Thus (26) is further written as

\[
\left(\Delta \tau_i, \tilde{S}\right) = \arg\min_{(\Delta \tau, \Psi)} \left\{\Psi^H \Theta(\Delta \tau) \Psi\right\},
\]

(28)

where \(\Theta(\Delta \tau) = \text{Re} \left\{\tilde{A}_G^{\text{II}}(\Delta \tau)\tilde{G}_P\tilde{A}_G^{\text{II}}(\Delta \tau)\right\}\). The solution to (28) is a general optimization problem [22]. To estimate the time delay by using optimization, some constraints are imposed. Since \(\tilde{S} > 0\), it is reasonable to require \(\Psi > 0\) [15] to avoid pseudo estimates [14], where \(0 = [0 \quad 0 \quad \cdots \quad 0]^T\) is an \(M_P \times 1\) column vector. Then the time delay estimates can be expressed as

\[
\left(\Delta \tau_i, \tilde{S}\right) = \arg\min_{(\Delta \tau, \Psi)} \left\{\Psi^H \Theta(\Delta \tau) \Psi\right\}, \quad \text{s.t.} \quad \Psi^H \Psi = 1, \quad \Psi > 0.
\]

(29)

where \(\Psi^H \Psi = 1\) is a nonlinear constraint and normalization. Since \(\Psi^H \Theta(\Delta \tau) \Psi\) is nonnegative for any real \(\Psi, \Theta(\Delta \tau)\) is a nonnegative definite Hermitian matrix, which ensures the global convergence of the quadratic form \(\Psi^H \Theta(\Delta \tau) \Psi\) [22].

The parameter estimation in (29) can be done in two steps. First, we minimize \(\Psi^H \Theta(\Delta \tau) \Psi\) with respect to \(\Psi\), and yield \(\zeta_{\min}(\Delta \tau)\) as a function of \(\Delta \tau\)

\[
\zeta_{\min}(\Delta \tau) = \min_{\Psi} \left\{\Psi^H \Theta(\Delta \tau) \Psi\right\}, \quad \text{s.t.} \quad \Psi^H \Psi = 1, \quad \Psi > 0.
\]

(30)

Then, a search step is performed on \(\Delta \tau\) to find the minima of \(\zeta_{\min}(\Delta \tau)\)

\[
\Delta \tau_i = \arg\min_{\Delta \tau_i} \left\{\zeta_{\min}(\Delta \tau_i)\right\},
\]

(31)

where \(\Delta \tau_i\), \(i = 1, 2, \cdots, D\), are the estimates of \(\Delta \tau_i\).

\section{Performance and Simulation Results}

In this section, we evaluate the performance [1, 23] of the proposed methods.
A. Transmitted and Received Signals

In our simulations, the transmitted signal is a chirp signal

\[ s(n) = \sin((\beta \cdot n + \alpha) \cdot n + \varphi_0), \]

\[ n = 0, 1, \ldots, K_s - 1, \]

where \( \beta = \pi \cdot (f_2 - f_1)/K_s \), \( f_2 \) and \( f_1 \) are the upper and lower frequencies of \( s(n) \), respectively, \( \alpha = 2\pi \cdot f_1 \cdot \varphi_0 \) is the initial random phase, \( K_s \) is the length of \( s(n) \). The bandwidth of \( s(n) \) is defined as \( B_s = (f_2 - f_1) \), and the correlation time is \( \zeta \triangleq 1/B_s \). In our simulations, \( s(n) \) cannot be considered as having flat spectrum since \( B_s \ll 0.5 \).

In active systems, we assume the received signal \( s(n) \) has two signal components with the same valued amplitudes. Then the received signals are modeled as

\[ r(n) = \lambda \cdot s(n - \tau_1) + \lambda \cdot s(n - \tau_2) + w(n), \]

\[ n = 0, 1, \ldots, K_r - 1, \]

where \( \tau_1 \) and \( \tau_2 \) are the unknown time delays. In the received signal \( r(n) \), both \( s(n - \tau_1) \) and \( s(n - \tau_2) \) are padded with zeros to length \( K_r \) and nonzero values only occupy the range \( \tau_1 \leq n \leq \tau_1 + K_r - 1 \) and \( \tau_2 \leq n \leq \tau_2 + K_r - 1 \), respectively. Let \( \Delta \tau_A = |\tau_2 - \tau_1| \) denote the time delay separation.

In passive systems, we assume the received signals can be measured by PN correlation method,


Fig. 1. CRB and MSE of time delay estimation versus SNR and $\Delta D_A$ in active systems ($K_s = 20$, $\tau_1 = 5$, $f_1 = 0.30$, $f_2 = 0.33$).

Fig. 2. CRB and MSE of time delay estimation versus SNR and $\Delta D_P$ in passive systems ($K_s = 20$, $\tau_21 = 5$, $f_1 = 0.30$, $f_2 = 0.33$).

Fig. 3. CRB and MSE of time delay estimation versus normalized bandwidth in active systems ($K_s = 20$, $K_r = 28$, SNR = 20dB, $M_A= 23$, $f_1 = 0.1$).

Fig. 4. CRB and MSE of time delay estimation versus normalized bandwidth in passive systems ($K_s = 20$, $K_r = 28$, SNR = 20dB, $M_P= 27$, $f_1 = 0.1$).

Fig. 5. Performance comparison of time delay estimation in active systems ($K_s = 20$, $K_r = 28$, SNR = 15dB, $M_A= 23$, $\tau_1 = 5$, $\tau_2 = 8$).

Fig. 6. Performance comparison of time delay estimation in passive systems ($K_s = 20$, $K_r = 28$, SNR = 15dB, $M_P= 27$, $\tau_1 = 5$, $\tau_22 = 8$).