# A ROBUST SUBSPACE TRACKING ALGORITHM FOR SUBSPACE-BASED BLIND MULTIUSER DETECTION IN IMPULSIVE NOISE

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*Abstract*—**Subspace tracking is an efficient method to reduce the complexity in estimating the signal subspace required in subspace-based multiuser detection algorithm. Recursive least square (RLS)-based subspace tracking algorithms such as the PAST [1] algorithm can be used to estimate the signal subspace adaptively with relatively low computational complexity. However, it is shown in this paper that subspace estimation using conventional autocorrelation matrix is very sensitive to impulse noise. A new robust correlation matrix, based on robust statistic [8], is proposed to overcome this problem. Moreover, a new robust PAST algorithm is developed, again using robust statistics, for robust subspace tracking. A new restoring mechanism is also proposed to handle long burst of impulses, which sporadically occur in communications systems. Simulation results show that the proposed robust subspace tracking-based blind multiuser detector performs better than the conventional approach, especially under consecutive impulses. The adaptation of the proposed scheme in dynamic multiple access channel, where users may enter and exit the shared mobile channel, is also found to be satisfactory.**

## 1. INTRODUCTION

Multiuser detection (MD) using adaptive signal processing techniques is very effective in suppressing interference in direct sequence code division multiple access (DS-CDMA) communications systems. They have the potential to offer the anticipated MD performance gains with an implementation complexity that would be manageable for third generation wireless systems. For a recent review of different multiuser detection algorithms, see [5]. An attractive method is the subspace-based blind MD [2], which requires only prior knowledge of the signature sequence of the user of interest. Furthermore, the number of parameters to be estimated is significantly reduced with minimal loss in performance. RLS-based subspace tracking algorithms, such as the PAST algorithm [1], are frequently used to reduce the complexity in estimating the signal subspace. The problem of robust multiuser detection in non-Gaussian channels was first studied in [3], where the effect of the impulse noise on the detector is suppressed by minimizing the *M*-estimators instead of the commonly used least-squares (*LS*) estimate. From simulation, we notice that although the *M*-estimator-based detector is robust to impulse noise, the underlying subspace tracking algorithm is not. In fact, the PAST algorithm, which is based on a recursive least-squares (*RLS*) type of algorithm, is extremely vulnerable to impulse noise. Simulation results, to be followed in Section 4, show that the estimation errors of the *RLS*-based PAST and PASTd [1] algorithms increase significantly when the ambient noise is corrupted by impulses. The performance of the blind subspace multiuser detector is therefore significantly degraded. Any other multiuser detection schemes, which employ similar *RLS*-based subspace tracking algorithm, are very likely to suffer from the same problem. This motivates us to consider in this paper the problem of robust subspace tracking under impulse noise. First of all, it is shown that subspace estimation using conventional autocorrelation matrix is very sensitive to impulse noise. Then, a new robust correlation matrix, based on robust statistic [8], is proposed to overcome this problem and a new robust PAST algorithm, similar to the robust statistic based adaptive filters in [4,6,7,8], is developed. Basically, those impulse-corrupted data vectors are detected using robust *M*-estimator and are prevented from corrupting the subspace estimate. To handle long burst of impulsive interference, a restoring mechanism is also devised so that the tracking algorithm can recover more quickly from the hostile effect of the impulses. These new mechanisms not only prevent the impulse noise from spoiling the fragile subspace tracking process, but also stabilize the required iterations in the robust *M*-estimate decorrelating multiuser detector. Simulation results show that the proposed robust subspace multiuser detector can quickly recover from the adverse effect of the impulse and re-converge much more rapidly than it's conventional counterpart.

The layout of the paper is as follows: Section 2 is a brief description of the blind multiuser detection scheme in [3] and the subspace tracking algorithm it used [1]. The proposed robust subspace tracking algorithm and its corresponding multiuser detection scheme are introduced in Section 3. Simulation results and comparison with the conventional scheme in [3] are presented in Section 4. The conclusions are drawn in Section 5.

## 2. BLIND MULTIUSER DETECTION AND ROBUST CORRELATION MATRIX ESTIMATION

Consider a baseband DS-CDMA system with $K$ active users using a coherent BPSK modulation scheme. Since an asynchronous system with $K$ users can be regarded as being equivalent to a synchronous system of $2K-1$ users, we only consider the synchronous case in this paper. The signaling waveform of the $k$-th user is

$$s_k(t)=\sum_{j=0}^{N-1}\beta_j^k\varphi(t-jT_c)\,,\ t\in[0,T]\,, \tag{1}$$

where $(\beta_0^k,\beta_1^k,\ldots,\beta_{N-1}^k)$ is the spreading sequence of the $k$-th user consisting of $\pm1$, $N$ is the processing gain, and $\varphi$ is the normalized chip waveform of duration $T_c$. The received signal during the $i$-th signaling interval, $t\in[iT,(i+1)T]$ is given by

$$r(t)=\sum_{k=1}^{K}A_kb_k(i)s_k(t-iT)+n(t)\,, \tag{2}$$

where $A_k$, $b_k(i)$, and $n(t)$ are, respectively, the received amplitude, data bit, and additive white Gaussian noise (AWGN) with power spectral density of $\sigma^2$. The received signal, $r(t)$, is then filtered by a chip-matched filter and is sampled at the chip rate to obtain a $N$-dimensional data vector at the $i$-th symbol interval $t=[iT,(i+1)T]$,

$$\underline{r}(i)=[r_0(i)\quad r_1(i)\quad\cdots\quad r_{N-1}(i)]^T=\sum_{k=1}^{K}A_kb_k(i)\underline{s}_k+\underline{n}(i)\,, \tag{3}$$

where $\underline{s}_k=(1/\sqrt{N})[\beta_0^k\beta_1^k\ldots\beta_{N-1}^k]^T$ is the normalized signature waveform vector of the $k$-th user, and $\underline{n}(i)=\left[n_0(i)\quad n_1(i)\quad\cdots\quad n_{N-1}(i)\right]^T$ is the sampled noise vector with zero mean and covariance matrix $\sigma^2I_N$, where $I_N$ is the $(N\times N)$ identity matrix. Without loss of generality, the $K$ users' signature waveforms $\{\underline{s}_k,k=1,\ldots,K\}$ are assumed to be linearly independent. Denote $S=[\underline{s}_1\quad\underline{s}_2\quad\ldots\quad\underline{s}_K]$, then (3) can be written as

$$\underline{r}(i)=S\underline{\theta}(i)+\underline{n}(i)\,, \tag{4}$$

where $\underline{\theta}(i)=[\theta_1(i),\ldots,\theta_K(i)]^T=[A_1b_1(i),\ldots,A_Kb_K(i)]^T$. The $LS$ solution is readily shown to be

$$\hat{\underline{\theta}}_{LS}(i)=(S^TS)^{-1}S^T\underline{r}(i)\,, \tag{5a}$$

$$\hat{A}_k=\left|\hat{\theta}_k(i)\right|,\ \text{and}\ \hat{b}_k(i)=\mathrm{sgn}(\hat{\theta}_k(i))\,, \tag{5b}$$

This requires the knowledge of all the signature waveforms. The blind subspace method relies on the eigendecomposition of the auto-correlation matrix, $C$, of the data vector $\underline{r}(i)$ and is given by

$$C = E\left[\underline{r}(i)\cdot \underline{r}^T(i)\right] = \sum_{k=1}^{K} A_k^2 \underline{s}_k \underline{s}_k^T + \sigma^2 I_N = SAS^T + \sigma^2 I_N , \quad (6)$$

where $A = diag(A_1^2,...,A_K^2)$. Here, it is assumed that the noise and the data bits themselves are uncorrelated. It can be seen that the signal and noise subspaces can be separated by performing an eigen-decomposition of the matrix $C$. Let $\lambda_i$ and $\underline{u}_i$ be the eigenvalue and its eigenvector of $C$. If the eigenvalues are arranged in descending order of their magnitudes such that $\lambda_1 \geq ... \geq \lambda_K \geq \lambda_{K+1} = ... = \lambda_n = \sigma^2$, then the corresponding column span of eigenvectors: $U_s = [\underline{u}_1,...,\underline{u}_K]$ and $U_n = [\underline{u}_{K+1},...,\underline{u}_N]$ constitute, respectively, the signal subspace and noise subspace, and we have

$$C = U\Sigma U^H = [U_s \quad U_n]\cdot\begin{bmatrix}\Lambda_s & \\ & \Lambda_n\end{bmatrix}\cdot\begin{bmatrix}U_s^T \\ U_n^T\end{bmatrix}, \quad (7)$$

where $U = [\underline{u}_1,...,\underline{u}_N]$, $\Sigma = diag(\lambda_1,...,\lambda_N)$, $\Lambda_s = diag(\lambda_1,...,\lambda_K)$, and $\Lambda_n = diag(\lambda_{K+1},...,\lambda_N)$. In the robust blind multiuser detector [3], the $M$-estimator is minimized instead of the conventional $LS$ estimator in the signal subspace, so as to avoid excessive error in estimating $\underline{\theta}(i)$ when the received signal vector $\underline{r}(i)$ is corrupted by impulses. This leads to the following iterative equations for solving the robust estimate $\hat{\underline{\zeta}} = [\hat{\zeta}_1, \hat{\zeta}_2,...,\hat{\zeta}_K]^T$

$$\underline{z}^{(l)} = \psi(\underline{r} - \hat{U}_s \hat{\underline{\zeta}}^{(l)}) ; \; \hat{\underline{\zeta}}^{(l+1)} = \hat{\underline{\zeta}}^{(l)} + \frac{1}{\vartheta}\hat{\underline{U}}_s^T \cdot \underline{z}^{(l)}, \; l = 0,...., \quad (8)$$

where $\hat{U}_s$ is some estimate of the signal subspace, $\psi(\cdot)$ is the derivative of the *M-estimator* penalty function, and $\vartheta$ is a step-size parameter. The iteration typically converges in less than 10 iterations, and it yields the desired estimate $\hat{\underline{\zeta}}$, from which the corresponding robust estimate of $\underline{\theta}$ can be calculated [3] as follows

$$\hat{\theta}_k = \sum_{j=1}^{K} \frac{\hat{\underline{u}}_j^T \underline{s}_k}{\hat{\lambda}_j} \hat{\zeta}_j , \; k = 1,...,K , \quad (9)$$

where $\hat{\lambda}_i$ is the estimated eigenvalues. The data bit and the amplitude of the $k$-th user can then be estimated using (5b), which requires only the prior knowledge of the signature waveform of the user of interest. In order to efficiently estimate the subspace parameters, subspace tracking algorithms such as the PAST and it's deflation version PASTd [1] were employed. The PAST algorithm [1] is a *RLS-based* algorithm which can be summarized as follows

| **PAST Algorithm** |
|---|
| Initialize $P(0)$ and $W(0)$ |
| **FOR** $i = 1,2,....$ **DO** |
| $\underline{y}(i) = W^H(i-1)\underline{r}(i)$, $\underline{h}(i) = P(i-1)\underline{y}(i)$, |
| $\underline{g}(i) = \underline{h}(i)/[\beta + \underline{y}^H(i)\underline{h}(i)]$, |
| $P(i) = \frac{1}{\beta}Tri\{P(i-1) - \underline{g}(i)\underline{h}^H(i)\}$, |
| $\underline{e}(i) = \underline{r}(i) - W(i-1)\underline{y}(i)$, |
| $W(i) = W(i-1) + \underline{e}(i)\underline{g}^H(i)$. |

The superscript $^H$ denotes Hermitian transpose and the operator $Tri\{\cdot\}$ indicates that only the upper (or lower) triangular part of the matrix argument is calculated and its Hermitian transposed version is copied to the lower (or upper) triangular part. For each input vector $\underline{r}(i)$, the algorithm computes a new estimate of the signal subspace $W(i)$ ($\hat{U}_s$) from the previous estimate $W(i-1)$. $\beta$ is the forgetting factor. As mentioned earlier, the performance of this algorithm, like the *RLS* algorithm, is extremely sensitive to the impulse noise. Suppose that $n(i)$ is modeled as a contaminated Gaussian noise given by $\underline{n}(i) = \underline{n}_g(i) + b(i)\cdot\underline{n}_i(i)$, where $\underline{n}_g(i)$ and $\underline{n}_i(i)$ are uncorrelated zero mean white Gaussian processes with covariance matrices $\sigma^2 I_N$ and $\sigma_i^2 I_N$, respectively. $\underline{n}_i(i)$ represents the impulsive component with $\sigma_i >> \sigma$. $b(i) \in \{0,1\}$ is a random binary sequence independent of $\underline{n}_i(i)$, which indicates the presence (absence) of an impulse at time $i$ if $b(i) = 1$ (0). It can be shown that the correlation matrix $C$ in (6) becomes $C = SAS^T + \sigma^2 I_N + E[b^2(i)]\sigma_i^2 I_N$. Any subspace tracking or eigen-decomposition methods for estimating the subspaces from $C = E[\underline{r}(i)\underline{r}^T(i)]$ will be significantly affected by the impulsive component $E[b^2(i)]\sigma_i^2 I_N$. Here, we define the robust correlation matrix $C_\rho = E[\rho_r \cdot \underline{r}(i)\cdot\underline{r}^T(i)]$, where $\rho_r$ is a weight function which should ideally be zero when an impulse is detected in vector $\underline{r}(i)$ and 1 otherwise. Under this assumption, $C_\rho \approx SAS^T + \sigma^2 I_N$, which stabilizes the subspace estimation. The definition of $C_\rho$ can be justified more formally using maximum likelihood estimation. The details are omitted here due to page limitation. We shall show in next section that the weight function $\rho_r$ can be derived from the error in the PAST algorithm so that a more robust algorithm against impulse noise can be developed [9].

## 3. ROBUST SUBSPACE TRACKING AND MULTIUSER DETECTION ALGORITHMS

We see in section 2 that the conventional correlation matrix and hence the PAST algorithm is extremely sensitive to impulse noise in the data vector $\underline{r}(i)$. In the PAST algorithm, the measure $J(W) = E\|\underline{r}(i) - WW^H\underline{r}(i)\|^2 = E\|\underline{e}(i)\|^2$ is minimized using the *RLS* algorithm. It can be seen from the PAST algorithm given in the previous section that $\underline{y}(i)$, $\underline{h}(i)$, $\underline{g}(i)$, $P(i)$, $\underline{e}(i)$, and $W(i)$ will be affected in turn by an impulse in $\underline{r}(i)$. The corrupted matrices, $P(i)$ and $W(i)$, will be used to compute the new $P(i)$'s and $W(i)$'s, causing hostile effects on the subspace estimate and requires many iterations to recover (see Fig. 1), especially when $\beta$ is close to one. We now consider the proposed robust PAST algorithm using robust statistic. First of all, we note that the purpose of $\rho_r$ in the robust correlation matrix estimate $C_\rho$ is to de-emphasis the impulse-corrupted observation $\underline{r}(i)$. A similar approach can be applied to the PAST algorithm by defining a robust distortion measure $J_\rho(W) = E\|\rho(\|\underline{e}(i)\|_F - \mu_e)\underline{e}(i)\|^2$, where $\rho(\cdot)$ is the weight function of an M-estimator [8]. For the Huber $M$-estimate that will be used in this paper, $\rho(e) = 1$ when $|e| < \Gamma$ and 0 otherwise, where $\Gamma$ is a threshold to be estimated continuously. $\mu_e$ is the robust location or mean estimator of $\|\underline{e}(i)\|_F$. It can be seen that if $\underline{r}(i)$ is corrupted by impulses, the Frobenius norm of the error vector $\underline{e}(i)$, $\|\underline{e}(i)\|_F$, will become very large. $\rho(\|\underline{e}(i)\|_F - \mu_e)$ will become zero and the impulse-corrupted measurement is prevented from entering into the minimization. A similar approach has been successively applied to develop robust adaptive filters under impulse noise [4,6,7]. We now consider the estimation of the threshold $\Gamma$ and robust mean estimator $\mu_e$ (for simplicity, the subscript $e$ in $\mu_e$ will be dropped in the subsequent discussion). Though the exact distribution of $\|\underline{e}(i)\|_F$ is unknown, for simplicity, it is assumed to be Gaussian distributed but corrupted by additive impulse noise (note also that $\|\underline{e}(i)\|_F$ is always positive). Specifically, the probability that $|\Delta e_{\mu}(i)| = \left| \|\underline{e}(i)\|_F - \hat{\mu}(i) \right|$ is greater than a given threshold $\Gamma(i)$ is

$$p_{\Gamma} = P_r\{|\Delta e_{\mu}(i)| > \Gamma(i)\} = erfc(\Gamma(i)/\hat{\sigma}(i)), \quad (10)$$

where $erfc(r) = (2/\sqrt{\pi})\int_r^{\infty} e^{-x^2}dx$ is the complementary error function. $\hat{\mu}(i)$ and $\hat{\sigma}(i)$ are the estimated mean and standard deviation of the Frobenius norm of the "impulse free" error vector. Using different threshold parameters $\Gamma(i)$, we can detect the presence of the impulse noise with different degrees of confidence. In this work, $p_{\Gamma}$ is chosen to be 0.05 so that we have 95% confidence in saying that the current error vector is corrupted by impulse noise. The corresponding threshold parameter $\Gamma(i)$ is determined to be $\Gamma(i) = 1.96 \cdot \hat{\sigma}(i)$. A commonly used estimate for $\hat{\sigma}^2(i)$, $\hat{\mu}(i)$ are respectively: $\hat{\sigma}^2(i) = \lambda_{\sigma}\hat{\sigma}^2(i-1) + (1-\lambda_{\sigma})(\Delta e_{\mu}(i))^2$ and $\hat{\mu}(i) = \lambda_{\mu}\hat{\mu}(i-1) + (1-\lambda_{\mu})\|\underline{e}(i)\|_F$, where $\lambda_{\mu}$ and $\lambda_{\sigma}$ are some forgetting factors. It is, however, not robust to impulse noise. In fact, a single impulse with large amplitude can substantially increase the value of $\hat{\sigma}(i)$ and $\hat{\mu}(i)$, and hence the values of $\Gamma(i)$. Better estimates for $\hat{\sigma}^2(i)$ and $\hat{\mu}(i)$ are [6]

$$\hat{\sigma}^2(i) = \lambda_{\sigma}\hat{\sigma}^2(i-1) + 1.483\left(1+\frac{5}{N_w - 1}\right)(1-\lambda_{\sigma})\,\mathrm{med}\left(A((\Delta e_{\mu}(i))^2)\right) \quad (11a)$$

and $$\hat{\mu}(i) = \lambda_{\mu}\hat{\mu}(i-1) + (1-\lambda_{\mu})\,\mathrm{med}\left(A(\|\underline{e}(i)\|_F)\right), \quad (11b)$$

where $A(x(i)) = \{x(i), \cdots, x(i-N_w+1)\}$, $N_w$ is the length of the estimation window, and med(.) is the median operation. $\lambda_{\mu}$ and $\lambda_{\sigma}$ are the forgetting factors. In practice $N_w$ varies from 5 to 11 so that the operations required by the median operations are quite reasonable. For large values of $N_w$, the pseudo median instead of the median can be computed to reduce the arithmetic complexity. Therefore, the arithmetic complexity of the proposed robust PAST algorithm is comparable to that of the conventional PAST algorithm. Our robust PAST algorithm updates $\Gamma(i) = 1.96 \cdot \hat{\sigma}(i)$, $\hat{\sigma}^2(i)$ and $\hat{\mu}(i)$ at each iteration. If $|\Delta e_{\mu}(i)| > \Gamma(i)$, both signal subspace $\boldsymbol{W}(i)$ and the intermediate matrix $\boldsymbol{P}(i)$ will not be updated, preventing the impulse from affecting the subspace estimate. Using the weight function: $\rho(|\Delta e_{\mu}(i)|) = 1$ when $|\Delta e_{\mu}(i)| < \Gamma(i)$, and 0 otherwise, the robust PAST algorithm that minimizes $J_{\rho}(\boldsymbol{W})$ is obtained as follows.

**Robust PAST Algorithm**

Initialize $\boldsymbol{P}(0)$, $\boldsymbol{W}(0)$, $\hat{\sigma}^2(0)$, and $\hat{\mu}(0)$

**FOR** $i = 1,2,\ldots$ **DO**

$\underline{y}(i) = \boldsymbol{W}^H(i-1)\underline{r}(i)$, $\underline{h}(i) = \boldsymbol{P}(i-1)\underline{y}(i)$,

$\underline{g}(i) = \underline{h}(i)/[\beta + \underline{y}^H(i)\underline{h}(i)]$,

$\underline{e}(i) = \underline{r}(i) - \boldsymbol{W}(i-1)\underline{y}(i)$,

$\boldsymbol{P}(i) = (1-\rho(|\Delta e_{\mu}(i)|))\boldsymbol{P}(i-1) + \rho(|\Delta e_{\mu}(i)|)\frac{1}{\beta}Tri\{\boldsymbol{P}(i-1) - \underline{g}(i)\underline{h}^H(i)\}$,

$\boldsymbol{W}(i) = \boldsymbol{W}(i-1) + \rho(|\Delta e_{\mu}(i)|)\underline{e}(i)\underline{g}^H(i)$,

update $\hat{\sigma}^2(i)$ and $\hat{\mu}(i)$ using (11),

$\Gamma(i) = 1.96 \cdot \hat{\sigma}(i)$,

**END**

There is however one problem remains unsolved, which occurs when there is a long burst of impulses. In this case, due to length of the median filter $N_w$, the system might misinterpret the series of error vectors with large Frobenius norm as being created from a sudden system change in the signal subspace, e.g., when users enter or leave the system in dynamic channel traffic. To solve this problem, the differences in statistical properties of $\underline{e}(i)$ during sudden system change and a series of impulses are exploited. For the former case, if the system continues to adapt, the Frobenius norm of the error vector will continue to decrease reaching a steady state when the algorithm converges. While for a long burst of consecutive impulse noise, the impulses will also produce a sequence of error vector $\underline{e}(i)$ with large Frobenius norm. However, it remains at a certain level without a deterministic trend of decreasing in its magnitude. Therefore, the following buffering mechanism is adopted to distinguish between the different situations of sudden system changes and corruption by consecutive impulse noise.

Suppose that at $i = i_0$, $|e_{\mu}(i)| > \Gamma(i_0)$, which indicates that the input vector might be corrupted by an impulse. $\boldsymbol{P}(i_0-1)$, $\boldsymbol{W}(i_0-1)$ and $\hat{\mu}(i_0-1)$ will be buffered, and the system continues to adapt. After an observation window of length $L_w$, which is chosen as a certain fraction of the initial convergence time of the tracking system to provide a sufficient decrease in $\|\underline{e}(i)\|_F$ in case of a system change, $\hat{\mu}(i_0+L_w-1)$ is compared to $\hat{\mu}(i_0-1)$. If $\hat{\mu}(i_0+L_w-1)$ is close to $\hat{\mu}(i_0-1)$, this means that there is a system change or the system has started to recover from the impulses. The restoring mechanism will not be invoked and the system will continue to adapt as normal. On the other hand, if $\hat{\mu}(i_0+L_w-1)$ is much greater than $\hat{\mu}(i_0-1)$, consecutive impulse noise is expected and $\boldsymbol{P}(i_0+L_w)$ and $\boldsymbol{W}(i_0+L_w)$ will be reinitialized to $\boldsymbol{P}(i_0-1)$ and $\boldsymbol{W}(i_0-1)$, respectively. It might happen, though very rare, that many users suddenly entered the system during the observation window, after a series of impulses, and give rise to a relatively high $\|\underline{e}(i)\|_F$. To avoid the restoring mechanism from disturbing this normal adaptation, we suggest to disable the restoring mechanism for a certain period of time, say 100 symbols, after its last activation. The robustness of the system to very long burst of impulse is therefore weaken. But simulation result shows that this scheme causes very little degradation in dynamic channel traffic and is able to suppress the adverse effect of long burst of impulse by period re-initialization. To differentiate the two situations at the end of the observation window, the relative discrepancy $\chi = \frac{\hat{\mu}(i_0+L_w-1) - \hat{\mu}(i_0-1)}{\hat{\mu}(i_0-1)}$ is adopted as a measure. If $\chi < \chi_s$, a certain threshold, it is recognized as a system change. Otherwise, it will be treated as the consecutive noise case. $\chi_s$ is chosen as 2 in the simulation section, which means that the restoring mechanism will be invoked if $\hat{\mu}(i_0+L_w-1) > 3\hat{\mu}(i_0-1)$. As for the number of active users in the channel, using the estimated subspace, it can be estimated from the estimated eigenvalues using the Akaike information criterion (AIC) or the minimum description length (MDL) criterion.

## 4. SIMULATION RESULTS

A synchronous DS-CDMA system with processing gain $N = 31$ and $K = 6$ users are assumed. The first user is the user of interest. The remaining 5 users produce multiple access interference (MAI) with relative power of 10, 10, 10, 10, and 20 dB. Background noise is assumed to be AWGN with power 20dB lower than that of user of interest. The median filter length $N_w$ is set to be 11, and the forgetting factors $\lambda_{\mu}$, $\lambda_{\sigma}$, and $\beta$ are all set equal to 0.99. The initial value $\boldsymbol{P}(0)$, $\boldsymbol{W}(0)$ are chosen to be identity matrices or their leading submatrices. Both $\hat{\sigma}^2(0)$, and $\hat{\mu}(0)$ are set to be 10, a relatively large number to their normal value, to initialize system adaptation and prevent buffering from happening at the beginning. $L_w, \chi_s$ are chosen to be 30 and 2 according to the experimental measurement of system convergence properties. The output signal-to-interference ratio ( $SIR$ ) of the correlator is adopted as performance measure, which is defined as $SIR \equiv E^2\{|\hat{\theta}_1(i)|\} / Var\{|\hat{\theta}_1(i)|\}$, where the expectation is with respect to the data bit of MAI's and the noise. The performance of the proposed robust subspace tracking based multiuser detector and its conventional counterpart is compared in Figures 1 to 4. In this case, the individual and consecutive impulse noise, both of which are modeled as a Gaussian noise with a power 15dB higher than that of the user of interest, intrudes the DS-CDMA channel at 300th symbol and during the time interval from the 600th symbol to the 605th symbol, respectively. Fig.1 and Fig.2 show, respectively, the simulation result of the *SIR* and *BER* of the conventional blind subspace-based multi-user detector [3] in impulsive noise. It is evident from Fig.1 and Fig.2 that this type of scheme is significantly affected by the impulsive noise. The subspace tracking process is substantially interfered and perturbed to an unconverged state. A very long time is needed for the algorithm to re-converge again after the impulsive noise has passed off. The output *SIR* and *BER* also take a long time to return to the ordinary levels. In contrast, Fig.3 and Fig.4, which show respectively the simulation result of the *SIR* and instant *BER* of the proposed scheme in impulsive noise, suggest that it is much more robust to the presence of impulsive noise than the conventional approach. The tracking process re-converges very rapidly and the *SIR* recover almost right after the impulsive noise disappeared. Though the instant *BER* is also high, due to the impulse noise in the channel, it returns to its normal value soon after the impulsive noise passed away. For the dynamic channel traffic case, the simulation starts with five users with a 10dB MAI each in the channel; at the 1000th symbol, a user with a relative power of 20dB enters the channel; at 2000th symbol, the user with a 20dB MAI and the other three with 10dB exit the channel. The *SIR* result of the proposed algorithm is shown in Fig.5. It clearly demonstrates the fast adaptation capability of the proposed scheme in dynamic channel traffic. The *SIR* is reduced when the new user entered the system at $i$=1000. The system very quickly adapts to this new user and the *SIR* returns to its optimal value. From $i$ = 2000 and onwards, the *SIR* increases due to the reduced MAI after the four users left the system. The data and those presented in Fig.1 to 5 are generated by averaging over 100 independent runs.

## 5. CONCLUSION

A new robust PAST algorithm for subspace tracking is presented in this paper. A systematic method, using the robust statistic concept, is used to detect the impulse in the input data vector and prevent them from corrupting the signal subspace for further tracking. A new restoring mechanism is also proposed to handle long burst of impulses, which sporadically occur in communications systems. Simulation results using the blind subspace-based multiuser detector show that the proposed subspace multiuser detector, using the robust subspace tracking algorithm, performs better than the conventional approach, especially under long burst of impulses. The adaptation of the proposed scheme in dynamic multiple access channel, where users may enter and exit the shared mobile channel, is also found to be satisfactory.

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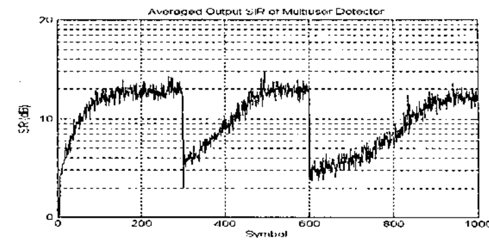


Fig. 1. *SIR* of conventional subspace blind MD

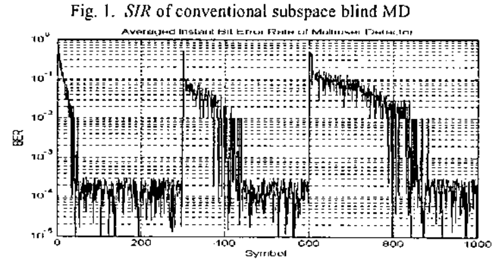


Fig. 2. *BER* of conventional subspace blind MD

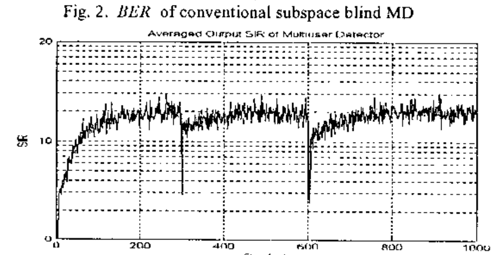


Fig. 3. *SIR* of proposed subspace blind MD

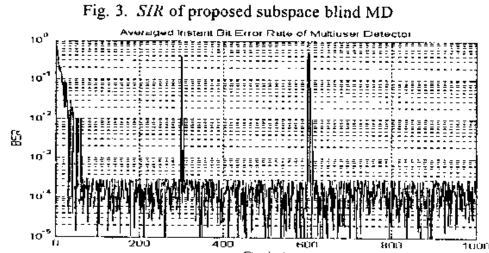


Fig. 4. *BER* of proposed subspace blind MD

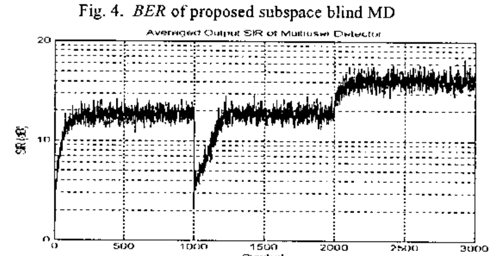


Fig. 5. *SIR* of proposed subspace blind MD in dynamic channel traffic.