Blind MIMO Channel Estimation Method Tolerating Order Overestimation

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Abstract— **A new second-order statistics (SOS) based blind algorithm for MIMO channel identification is proposed. It can give estimations of all channel impulse response subject to a scalar matrix ambiguity which is intrinsic to all SOS based blind method. As a generalization of the blind identification algorithm for SIMO system in [6], this method is truly robust to channel order overestimation, i.e., it can accurately estimate the channels when only upper bounds for all MIMO channel orders are known.**

Key words. MIMO, Blind channel identification, order overestimation, second-order statistics algorithms.

I. Introduction

Channel estimation in multiple input multiple output (MIMO) system is usually a difficult task. In order to alleviate the need for training sequences, thereby achieving a much desired bandwidth gain, blind identification, that is, estimating the channel impulse responses using solely their output statistics, is desirable. Research in this area is very active in recent years [1], [2], [3], [4], [5], [6], [7], [9], [10], [11], [13], [14], [15]. Among various known algorithms, second-order statistics (SOS) based algorithms are most attractive due to their special properties [3], [4]. Since the first SOS based method for simple input multiple output (SIMO) system introduced by Tong et al. in [14], there have been a variety of SOS based algorithms [1], [2], [4], [5], [6], [7], [9], [10], [11], [13], [14], [15]. The subspace (SS) method [1], [11], [12], the linear prediction (LP) approaches [2], [10], [13], [15] and the outer product decomposition (OPD) [5], [4] algorithm are most popular among them. The SS method has a simple structure and achieves good performance for SIMO system [11], but it requires precise knowledge of the channel order [12], which is impossible in practice. Also, the extension of it to MIMO channel is not successful because it generally can only estimate the channels subject to a polynomial matrix ambiguity [1]. LP and OPD algorithm can be used for both SIMO and MIMO system and are valid when the channel order is overestimated, but their performance is very sensitive to observation noise. It has been pointed in [8], [7] that LP's claimed robustness to channel order overestimation does not hold when SOS contains estimation errors. It was shown in [2] that the LP algorithm can achieve acceptable performance when the assumed order equals that provided by an order detection criterion (the MDL and the AIC criteria [16]) overestimated by a few (one or two) taps only. This means that the LP algorithm is not fully robust to order overestimation, and therefore, it is still necessary to estimate the channel orders before identifying channel responses. Unlike SIMO case, it is much more difficult to estimate channel orders in MIMO system, because not just one order but many orders are needed to be estimated. Due to noise and roundoff errors, it is impossible to obtain precise channel orders. Usually we can only get upper bounds for them. Therefore, a practical channel estimation method must first tolerate channel overestimations.

In [6], a method robust to channel overestimation is proposed for SIMO channel estimation. In this paper, we generalize the method and present a channel estimation algorithm for MIMO system. As in [6], the proposed algorithm is also based on a shifted version of the correlation matrix and the its associated kernel. The algorithm needs not to compute pseudo-inverse of the correlation matrix, which is required in LP and OPD algorithms. It can accurately estimate the channels subject to a scalar matrix ambiguity when only upper bounds for all MIMO channel orders are known. It is, hence, proved theoretically that identification is possible when the channel order is arbitrarily overestimated. Channel order upper bounds can be obtained from some priori knowledge of propagation conditions in wireless communications.

Some notations are used in the following. Superscript T, $\dagger$ and $*$ stand for transpose, transconjugate, and conjugate, respectively. I_q is the identity matrix of order q and $\otimes$ is the Kronecker product of matrices.

II. MIMO system model

A multi-user multi-antenna /fractionally spaced wireless communication system can be modeled as a MIMO system, as depicted in Figure 1. Assume that there are P users with each user sending a symbol sequence: $s_j(n)$ $(j = 1, 2, \cdots, P)$ and M receivers (antennas) with received signal: $x_i(n)$ $(i = 1, 2, \cdots, M)$. The MIMO system can be described as

$$x_i(n) = \sum_{j=1}^{P} \sum_{k=0}^{N_j} h_{ij}(k) s_j(n-k) + \eta_i(n), \ n = 0, 1, \cdots, \tag{1}$$

where $h_{ij}(k)$ is the channel response from user j to antenna i, N_j is the maximum order of these channels for $i = 1, 2, \cdots, M$ and $\eta_i(n)$ is the channel noise. We

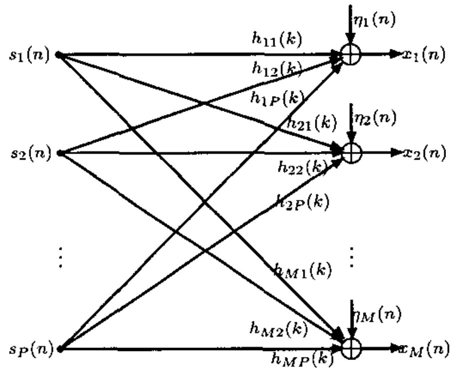

Fig. 1. MIMO system

assume that $M > P$. Let

$$\begin{aligned} x(n) &= (x_1(n), x_2(n), \cdots, x_M(n))^T, \\ h_j(n) &= (h_{1j}(n), h_{2j}(n), \cdots, h_{Mj}(n))^T \\ \eta(n) &= (\eta_1(n), \eta_2(n), \cdots, \eta_M(n))^T \end{aligned} \quad (2)$$

we can express (1) into vector form as

$$x(n) = \sum_{j=1}^{P} \sum_{k=0}^{N_j} h_j(k) s_j(n-k) + \eta(n), \quad (3)$$

We use $w_j(n)$ to represent the inner summation as

$$w_j(n) = \sum_{k=0}^{N_j} h_j(k) s_j(n-k), \; n = 0, 1, \cdots. \quad (4)$$

If we consider L consecutive outputs in the above equation and denote

$$W_j(n) = (w_j^T(n), w_j^T(n-1), \cdots, w_j^T(n-L+1))^T,$$

$$S_j(n) = (s_j(n), s_j(n-1), \cdots, s_j(n-N_j-L+1))^T,$$

and

$$H_j = \begin{bmatrix} h_j(0) & \cdots & \cdots & h_j(N_j) & 0 & \cdots & 0 \\ 0 & h_j(0) & \cdots & & h_j(N_j) & \cdots & 0 \\ & & \ddots & & & \ddots & \\ 0 & 0 & \cdots & h_j(0) & \cdots & \cdots & h_j(N_j) \end{bmatrix} \quad (5)$$

where H_j is a $ML \times (N_j + L)$ block Toeplitz matrix with first block row being $(h_j(0), h_j(1), \cdots, h_j(N_j), 0, \cdots, 0)$, then

$$W_j(n) = H_j S_j(n). \quad (6)$$

By grouping L consecutive output and noise respectively into vector $X(n)$ and $\Upsilon(n)$ as

$$\begin{aligned} X(n) &= (x^T(n), x^T(n-1), \cdots, x^T(n-L+1))^T, \\ \Upsilon(n) &= (\eta^T(n), \eta^T(n-1), \cdots, \eta^T(n-L+1))^T, \end{aligned} \quad (7)$$

from (3), (4) and (6), we get

$$X(n) = \sum_{j=1}^{P} W_j(n) + \Upsilon(n) = HS(n) + \Upsilon(n) \quad (8)$$

where H is a $ML \times (N + PL)$ matrix and $S(n)$ is a $(N + PL) \times 1$ vector defined by

$$\begin{aligned} H &= [H_1, H_2, \cdots, H_p], \\ S(n) &= (S_1^T(n), S_2^T(n), \cdots, S_p^T(n))^T \end{aligned} \quad (9)$$

respectively, where $N = \sum_{j=1}^{P} N_j$.

III. Algorithm development

Before discussing the algorithm, we give the following assumptions for the statistical properties of transmitted symbols $s_j(n)$ and channel noise $\eta_i(n)$.

(A1) Transmitted symbols are independent and identically distributed (i.i.d), that is,

$$\mathbf{E}(s_j(n) s_l^*(m)) = \begin{cases} \sigma_s^2, & (j,n) = (l,m) \\ 0, & (j,n) \neq (l,m) \end{cases}$$

(A2) Noises are white and uncorrelated, that is,

$$\mathbf{E}(\eta_i(n) \eta_l^*(m)) = \begin{cases} \sigma_\eta^2, & (i,n) = (l,m) \\ 0, & (i,n) \neq (l,m) \end{cases}$$

(A3) Noises and transmitted signals are uncorrelated, that is, $\mathbf{E}(\eta_i(n) s_l^*(m)) = 0$.
Here $\mathbf{E}(y)$ means the mathematical expectation of a random variable y.

Now we consider the statistical correlation matrices of $X(n)$. Based on the assumptions (A1-3), we can verify that

$$\begin{aligned} R &= \mathbf{E}(X(n) X^\dagger(n)) = \sigma_s^2 H H^\dagger + \sigma_\eta^2 I_{ML}, \quad (10) \\ \bar{R} &= R - \sigma_\eta^2 I_{ML} = \sigma_s^2 H H^\dagger \quad (11) \end{aligned}$$

and

$$\begin{aligned} Q &= \mathbf{E}(X(n) X^\dagger(n-1)) \\ &= \sigma_s^2 \sum_{j=1}^{P} H_j J_{N_j+L} H_j^\dagger + \sigma_\eta^2 (J_L \otimes I_M) \\ &= \sigma_s^2 H \tilde{J}_{N+PL} H^\dagger + \sigma_\eta^2 (J_L \otimes I_M), \quad (12) \\ \bar{Q} &= Q - \sigma_\eta^2 (J_L \otimes I_M) = \sigma_s^2 H \tilde{J}_{N+PL} H^\dagger \quad (13) \end{aligned}$$

where J_q is a $q \times q$ down shift matrix defined as

$$J_q = \begin{bmatrix} 0 & & & \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & 1 & 0 \end{bmatrix} \quad (14)$$

and $\bar{J}_{N+PL}$ is a $(N + PL) \times (N + PL)$ matrix with block diagonal form defined by

$$\bar{J}_{N+PL} = \mathrm{diag}(J_{N_1+L}, J_{N_2+L}, \cdots, J_{N_P+L}) \qquad (15)$$

If we have some knowledge about the upper bounds of the channel orders N_j, we can choose L, which is called the smoothing factor, large enough to ensure that $ML > N + PL$ because $M > P$. Hence, the matrix H has more rows than columns, that is, it is a tall matrix. Now we give another assumption which is also the condition for the existence of zero-forcing equalizer:

(A4) Matrix H is of full column rank.

This condition is also assumed in all known blind MIMO channel estimation method [1], [2], [4], [5], [13]. Then it is easy to verify that the rank of $\bar{R}$ and $\bar{Q}$ are respectively $N + PL$ and $N + P(L - 1)$. Let $J = ML - (N + PL - P)$, which is the dimension of the left null space of $\bar{Q}$. From (10), we know that σ_η^2 is the least eigenvalue of matrix R. After obtaining σ_η^2, $\bar{R}$ and $\bar{Q}$ can be computed. By computing the singular value decomposition (SVD) of matrix $\bar{Q}$, we can get a basis for the left null space of it, that is, we can find J linear independent vectors, u_i $(i = 1, 2, \cdots, J)$, such that $u_i^\dagger \bar{Q} = u_i^\dagger H \bar{J}_{N+PL} H^\dagger = 0$ or equivalently $H \bar{J}_{N+PL}^\dagger H^\dagger u_i = 0$. Since H is full column rank, we have

$$\bar{J}_{N+PL}^\dagger H^\dagger u_i = 0 \qquad (16)$$

By dividing the vector $H^\dagger u_i$ into blocks as

$$H^\dagger u_i = [v_{i1}^T, v_{i2}^T, \cdots, v_{iP}^T]^T,$$

where v_{ij} is a vector of size $(N_j + L) \times 1$ $(j = 1, 2, \cdots, P)$, we can express (16) into

$$J_{N_j+L}^\dagger v_{ij} = 0, \; j = 1, 2, \cdots, P \qquad (17)$$

It is easy to show that a vector v_{ij} satisfying condition (17) if and onlf if

$$v_{ij} = \alpha_{ij}(1, 0, \cdots, 0)^T, \; j = 1, 2, \cdots, P \qquad (18)$$

where α_{ij} is arbitrary an complex number. For simplicity, we use the notation e_q to express a $q \times 1$ vector whose first entry is 1 and all other entries are zeros. Then we have

$$H^\dagger u_i = \mathrm{diag}(\alpha_{i1} I_{N_1+L}, \alpha_{i2} I_{N_2+L}, \cdots, \alpha_{iP} I_{N_P+L}) \times (e_{N_1+L}^T, e_{N_2+L}^T, \cdots, e_{N_P+L}^T)^T. \qquad (19)$$

From the above equations, we get

$$\begin{aligned} & u_i^\dagger H S(n) \\ &= \alpha_{i1}^* s_1(n) + \alpha_{i2}^* s_2(n) + \cdots + \alpha_{iP}^* s_P(n) \\ &= [\alpha_{i1}^*, \alpha_{i2}^*, \cdots, \alpha_{iP}^*] \begin{bmatrix} s_1(n) \\ s_2(n) \\ \vdots \\ s_P(n) \end{bmatrix} \end{aligned} \qquad (20)$$

Since $J = ML - (N + PL) + P > P$, we can choose P vectors u_i $(i = 1, 2, \cdots, P)$ and define a matrix U by

$$U = [u_1, u_2, \cdots, u_P] \qquad (21)$$

We also define a $P \times P$ matrix α by

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{21} & \cdots & \alpha_{P1} \\ \alpha_{12} & \alpha_{22} & \cdots & \alpha_{P2} \\ & \cdots & \cdots & \\ \alpha_{1P} & \alpha_{2P} & \cdots & \alpha_{PP} \end{bmatrix} \qquad (22)$$

and a $P \times 1$ vector $\bar{s}(n)$ by

$$\bar{s}(n) = (s_1(n), s_2(n), \cdots, s_P(n))^T \qquad (23)$$

Then we have

$$U^\dagger H S(n) = \alpha^\dagger \bar{s}(n) \qquad (24)$$

By properly choosing the P vectors u_i, we may make the matrix α invertible. We will discuss this point further in the next section. Then we get a critical formula as follows:

$$\bar{s}^\dagger(n) = S^\dagger(n) H^\dagger U \alpha^{-1} = Z^\dagger(n) U \alpha^{-1} \qquad (25)$$

where $Z(n) = HS(n) = X(n) - \Upsilon(n)$. We also define $z(n) = x(n) - \eta(n)$.

Now we are ready to consider the estimation of the channels. From (3) we see that

$$h_j(k) = \mathbf{E}(x(n) s_j^*(n-k))/\sigma_s^2.$$

Let

$$h(k) = [h_1(k), h_2(k), \cdots, h_P(k)] \qquad (26)$$

be the MIMO channel matrix. It can be verified that

$$\begin{aligned} & h(k)\sigma_s^2 \\ &= \mathbf{E}(x(n)\bar{s}^\dagger(n-k)) \\ &= \mathbf{E}(x(n) Z^\dagger(n-k) U \alpha^{-1}) \\ &= \mathbf{E}(z(n) Z^\dagger(n-k) U \alpha^{-1}) \\ &= \mathbf{E}(z(n) Z^\dagger(n-k)) U \alpha^{-1} \end{aligned} \qquad (27)$$

Now we define a $M \times M$ matrix $r_z(k)$ as

$$r_z(k) = \mathbf{E}(z(n) z^\dagger(n-k)). \qquad (28)$$

$r_z(k)$ can be computed from $r_x(k) = \mathbf{E}(x(n) x^\dagger(n-k))$ given that the noise variance (power) σ_η^2 is known. In fact,

$$r_z(k) = r_x(k) - r_\eta(k) = \begin{cases} r_x(k), & k \neq 0 \\ r_x(k) - \sigma_\eta^2 I_M, & k = 0 \end{cases}$$

Noticing that

$$Z^\dagger(n-k) = (z^\dagger(n-k), z^\dagger(n-k-1), \cdots, z^\dagger(n-k-L+1)),$$

we get

$$h(k) = (r_z(k), r_z(k+1), \cdots, r_z(k+L-1))U\alpha^{-1}/\sigma_s^2 \quad (29)$$

In (29), only matrix α^{-1} cannot be determined from the statistics of outputs of the MIMO system. That is, the MIMO channel can be estimated from its output second order statistics with ambiguity of a $P \times P$ matrix. This ambiguity matrix is intrinsic for any blind channel estimation method based on second order statistics. That is the best goal we can achieve by using blind channel estimation method. The determination of the matrix needs other informations, such as higher order statistics, or pilot symbols.

IV. Analysis of the method

In order to analyze the algorithm, we summerize the method in the following. Before implementing the algorithm, we assume that an upper bound for all the channel orders, that is, a number N_{max} such that $N_j \le N_{max}$ $(j = 1, 2, \cdots, P)$, is known or estimated. Although it is impossible to precisely know all the true channel orders of a MIMO system in practice, upper bounds are usually achievable. Choose an integer L such that $ML > PN_{max} + PL$.

Algorithm 1: **Blind MIMO Channel Estimation Algorithm**

Step 1. Compute $R = \mathbf{E}(X(n)X^\dagger(n))$ and its least eigenvalue σ_η^2. Let $\bar{R} = R - \sigma_\eta^2 I_{ML}$.

Step 2. Compute $Q = \mathbf{E}(X(n)X^\dagger(n-1))$ and $\bar{Q} = Q - \sigma_\eta^2(J_L \otimes I_M)$.

Step 3. Compute the SVD or eigenvalue decomposition of matrix $\bar{Q}$ and hence get a basis for the left null space of $\bar{Q}$. Choose P vectors u_i $(i = 1, 2, \cdots, P)$ from the left null space such that $u_i^\dagger \bar{R} u_i \neq 0$.

Stpe 4. Compute $r_z(k) = \mathbf{E}(z(n)z^\dagger(n-k))$ $(k = 0, 1, \cdots, N_{max} + L - 1)$.

Step 5. Form a matrix

$$G = \begin{bmatrix} r_z(0) & r_z(1) & \cdots & r_z(L-1)) \\ r_z(1) & r_z(2) & \cdots & r_z(L)) \\ & \cdots & \cdots & \\ r_z(N_{max}) & r_z(N_{max}+1) & \cdots & r_z(N_{max}+L-1) \end{bmatrix} U \quad (30)$$

where $U = (u_1, u_2, \cdots, u_P)$. The MIMO channel matrix is then

$$\begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(N_{max}) \end{bmatrix} = G\beta \quad (31)$$

where β is a $P \times P$ matrix to be determined.

A. Robust to order overestimation

Only an upper bound is assumed for all the orders in the algorithm. No exact order is needed. If the true channel order for user j is N_j and in the algorithm it is overestimated into $N_{max} > N_j$, we can prove that the overestimated channel is only a zero padding of the real channel. In fact, our computation of channel is based on

$$h_j(k) = \mathbf{E}(x(n)s_j^*(n-k))/\sigma_s^2, \ k = 0, 1, \cdots, N_{max}.$$

Based on the assumption (A1) and (A3), it is easy to prove that

$$\begin{aligned} & \mathbf{E}(x(n)s_j^*(n-k)) \\ = & \mathbf{E}(\sum_{l=1}^{P}\sum_{m=0}^{N_l} h_l(m)s_l(n-m)s_j^*(n-k) \\ & +\eta(n)s_j^*(n-k)) \\ = & \mathbf{E}(h_j(k)s_j(n-k)s_j^*(n-k)) \\ = & \begin{cases} \sigma_s^2 h_j(k), & 0 \le k \le N_j \\ 0, & k > N_j \end{cases} \end{aligned}$$

that is, the estimated channel response $h_j(k)$ should be zero theoretically for $k > N_j$.

B. About the matrix α

For any u_i belonging to the left null space of $\bar{Q}$, from (11) and (19) we know that

$$u_i^\dagger \bar{R} u_i = \sigma_s^2 ||H^\dagger u_i||^2 = \sigma_s^2 \sum_{j=1}^{P} |\alpha_{ij}|^2 \quad (32)$$

So, $u_i^\dagger \bar{R} u_i \neq 0$ if and only if u_i is not in the left null space of H. The left null space of H is of dimension $J' = ML - (N + PL)$, while that of $\bar{Q}$ is of dimension $J = J' + P$. The former is a nontrivial subspace of the latter. It is obvious that the probability of an arbitrarily chosen nonzero vector in a linear space belonging to its a nontrivial subspace is zero. This means that the probability of $\sum_{j=1}^{P} |\alpha_{ij}|^2 = 0$ is zero. So, we have enough vectors u_i to construct the matrix α such that any column of the α is nonzero. However, this cannot assure that the matrix α is invertible.

We use α_j to represent the jth column of the matrix α. Let λ_j $(j = 1, 2, \cdots, P)$ be P complex numbers. We use ξ and v to denote the linear combinations respectively as

$$\xi = \lambda_1\alpha_1 + \lambda_2\alpha_2 + \cdots + \lambda_P\alpha_P,$$

$$v = \lambda_1 u_1 + \lambda_2 u_2 + \cdots + \lambda_P u_P.$$

v is in the left null space of $\bar{Q}$. Let $\xi = (\xi_1, \xi_2, \cdots, \xi_P)^T$. Then

$$\begin{aligned} H^\dagger v = & \operatorname{diag}(\xi_1 I_{N_1+L}, \xi_2 I_{N_2+L}, \cdots, \xi_P I_{N_P+L}) \\ & \times (e_{N_1+L}^T, e_{N_2+L}^T, \cdots, e_{N_P+L}^T)^T \end{aligned} \quad (33)$$

It is obvious that $\xi = 0$ if and only if v belongs to the left null space of H. Again we know that the probability of $\xi = 0$ is zero. In a whole, we can almost assure that the matrix α is invertible.

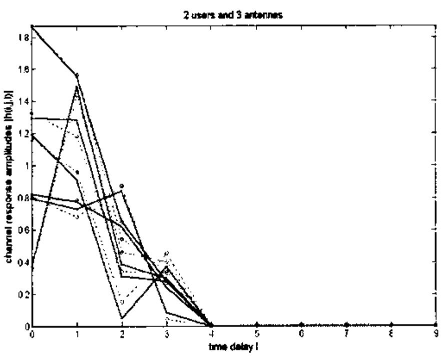

Fig. 2. Channel response amplitudes

V. Simulations

In practice, the auto-correlation matrix R and the cross-correlation matrix Q can only be computed from the received signal samples. We use the following approximations for them: $R \approx \frac{1}{L_s}\sum_{n=L}^{L+L_s-1} X(n)X^\dagger(n)$, and $Q \approx \frac{1}{L_s}\sum_{n=L}^{L+L_s-1} X(n)X^\dagger(n-1)$, where L_s is the number of samples used. The noise variance σ_η^2 is approximated by averaging over all the eigenvalues of R which are not greater than 5 times of the least eigenvalue. Simulations show that the algorithm is truly robust to channel overestimation. An example is given below.

In a 2 user and 3 antenna MIMO system, the true channel orders are 3 which are overestimated to be 9. The channel responses are generated randomly. The transmitted baseband signals are coded as 16-QAM, and the received signal-noise-ratio (SNR) is 15.38. The smooth factor L is chosen to be 21 and the number of samples for computing the SOS is 800. By using the proposed algorithm, the ambiguity matrix β cannot be resolved. For verification of the algorithm, we resolve the matrix β by comparing the first coefficients of estimated channel with those of the true channels, that is, $\beta = (\hat{h}^\dagger(0)\hat{h}(0))^{-1}\hat{h}^\dagger(0)h(0)$, where $\hat{h}$ is the estimated channel response. Then the ambiguity matrix is eliminated from the estimated channels. Simulation results are: the maximum absolute error and mean square error (MSE) between the estimated and true channel responses are 0.1670 and 0.0914 respectively. The amplitudes of the channel responses are shown in Figure 2, where dashed lines are for estimated channels and solid lines for true channels.

VI. Conclusions

For MIMO system, it is practically impossible to precisely obtain the channel orders, which makes the algorithms depending on true channel orders as a priori useless. However, channel order upper bounds can be obtained from some priori knowledge of propagation conditions in wireless communications. The proposed SOS based method in this paper can accurately estimate the channels subject to a scalar matrix ambiguity when only upper bounds for all MIMO channel orders are known. This, hence, proves theoretically that identification is possible when the channel order is arbitrarily overestimated.

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