International Conference on Communication Technology
ICCT'98 October 22-24, 1998 Beijing, China

Performance sensitivity of quasi-synchronous, multicarrier DS-CDMA systems due to carrier frequency disturbance

Kun-Wah Yip (kwyip@eee.hku.hk) and Tung-Sang Ng (tsng@eee.hku.hk)

Dept. of Electrical and Electronic Engineering
The University of Hong Kong
Pokfulam Road, Hong Kong
Fax: (852) 2559 8738. Tel.: (852) 2857 8406

Abstract — **The multiple access interference (MAI) of a quasi-synchronous, multicarrier DS-CDMA system can be substantially reduced by using sequences having low crosscorrelation at small shifts around the origin. This paper shows that the time-frequency crosscorrelation rather than the usual (time-domain) crosscorrelation determines the MAI when the system is operated in the presence of carrier frequency offset (CFO) which arises due to frequency-accuracy limit of the oscillator. Analysis on the time-frequency crosscorrelation properties reveals that (i) a system using Walsh codes or Suehiro-Hatori polyphase sequences can be driven into outage in the presence of CFO as a result of significant worst-case MAI, and (ii) it is possible to minimize the MAI for systems using preferentially phased Gold codes, cyclic-shift *m*-sequences or Lin-Chang sequences only if the product of chip period and maximum frequency deviation is less than around 0.01.**

I. INTRODUCTION AND MOTIVATION

Direct-sequence (DS-) code division multiple access (CDMA) communications using quasi-synchronous (QS-) signal transmission and crosscorrelation-optimized sequences have received considerable interest in past years because of the potential in supporting higher multiple access capacity [1]-[3]. In a QS DS-CDMA system, the signals of all users are time-aligned to within a small synchronization window which is one or a few chip periods in length. The multiple access interference (MAI) is therefore determined by crosscorrelation at small shifts around the origin. By using sequences that have low crosscorrelation at small shifts around the origin, the MAI can be substantially reduced. Sets of sequences that possess such crosscorrelation-optimized property include, for example, the preferentially phased Gold code [1], the Walsh code [2], the set of cyclic-shift *m*-sequences [3], polyphase sequence sets proposed by Suehiro and Hatori [4] and by Suehiro [5], and the set of sequences recently proposed by Lin and Chang [6]. Despite the advantage of reducing the MAI, one of the challenges in the implementation of a QS DS-CDMA system is to time-synchronize signals of all users if the synchronization window is small. To make implementation easier, one can use multicarrier signaling to increase the chip period while maintaining the same data rate [2]. A longer chip period makes the synchronization window longer.

In a wireless communication system, the carrier frequency of a transmitted signal is usually slightly off from the desired frequency because of the limit on the frequency accuracy of the oscillator. To date, typical oscillators have a frequency accuracy of around ±10ppm. In the presence of carrier frequency offset (CFO), it can be shown that the MAI of a QS multicarrier DS-CDMA system is determined by the *time-frequency crosscorrelation* (see (9)) rather than the usual (time-domain) crosscorrelation. The time-frequency crosscorrelation at zero time shift is reduced to the spectral crosscorrelation. The spectral crosscorrelation was introduced by Popović [7] in the study of multicarrier CDMA, a spread spectrum technique that spreads the signal in the frequency domain. Popović [7] also showed that for a set of sequences the behavior of spectral crosscorrelation is very different from that of the time-domain one. In [1]-[6], the selection of optimized sequences was entirely based on minimization of the time-domain crosscorrelation. Since the CFO is inevitable in a QS multicarrier DS-CDMA system, it is important to know whether these optimized sequences can still minimize the MAI despite the presence of CFO. This knowledge is, however, not known in the previous literature. In this paper, we investigate the MAI-minimization capabilities of these sets of optimized sequences in the presence of CFO. Our results are obtained by analyzing the time-frequency crosscorrelation properties of various sets of sequences. Analysis based on the error performance of a communication system is given in a fuller version of this paper [8].

II. SYSTEM MODEL AND ANALYSIS

The system under consideration consists of K users. We consider multicarrier signaling that employs M spectrally disjoint subcarriers with subcarrier spacing Δf. The bit stream of each user is first serial-to-parallel converted into M substreams. Each substream is used to modulate a separate subcarrier using binary-phase-shift keying, so that a total of M bits are transmitted at a time. These M bits are spread by the same signature sequence of length N. Let the bit transmission rate of *each substream* be $1/T_b$ bits per second. The chip period, T_c, is given by $T_c = T_b/N$. The complex envelope of the k th-user transmitted signal is given by

$$s_k(t)=\sqrt{2P_b}\,e^{j(2\pi v_k t+\theta_k)}\sum_{n=-\infty}^{\infty}\sum_{m=0}^{M-1}b_{m,n}^{(k)}\cdot e^{j2\pi m\Delta f(t-\tau_k)}\phi_k(t-nT_b-\tau_k) \quad (1)$$

where P_b is the signal power *per subcarrier*, v_k is the CFO of the k th-user signal, θ_k is the carrier phase which is modeled as a uniform random variable over $[0,2\pi)$, $b_{m,n}^{(k)} \in \{+1,-1\}$ is the n th bit transmitted on the mth subcarrier of the k th-user signal, τ_k is the time offset of the k th-user signal, and

$$\phi_k(t)=\sum_{\ell=0}^{N-1}c_\ell^{(k)}\psi(t-\ell T_c) \quad (2)$$

is the k th-user spectral spreading waveform. In (2), $(c_0^{(k)}, \ldots, c_{N-1}^{(k)})$ is the k th-user signature sequence and $\psi(t)$ is the chip waveform satisfying $\int_{-\infty}^{\infty}|\psi(t)|^2dt = T_c$. We assume that $\left|c_\ell^{(k)}\right| = 1$ and that the chip waveform is a square-root-raised-cosine pulse with a rolloff factor α. Since the subcarriers are not spectrally overlapped, we set

$$\Delta f=(1+\alpha)T_c^{-1}. \quad (3)$$

Quasi-synchronized signal transmission is considered. We model that τ_k is uniformly distributed over $[-\Delta_\tau T_c, \Delta_\tau T_c]$ (so that $2\Delta_\tau T_c$ is the size of the synchronization window) and that τ_k's, $k = 1,\cdots,K$, are mutually independent. The CFO is modeled as follows. It is assumed that v_k is uniformly distributed over $[-\Delta_v, \Delta_v]$, where Δ_v is the maximum frequency deviation from the desired carrier frequency, and that v_k's are statistically independent. We consider an additive white Gaussian noise (AWGN) channel with a two-sided power spectral density $N_0/2$. The complex envelope of the received signal is given by

$$r(t)=n(t)+\sum_{k=1}^{K}s_k(t) \quad (4)$$

where $n(t)$ is the complex envelope of the AWGN. The receiver is intended to demodulate the Lth-user signal. It is assumed that the receiver oscillator is able to frequency-synchronize at the Lth-user signal and a phase reference can be derived so that coherent detection is possible. The received signal is processed by a bank of matched filters each of which detects the desired signal transmitted on a particular subcarrier. Without loss of generality we consider detection of $b_{\tilde{m},0}^{(L)}$. The matched filter output for decision making is given by

$$\hat{b}_{\tilde{m},0}^{(L)}=\frac{1}{\sqrt{2E_bT_b}}\mathrm{Re}\left\{\int_{-\infty}^{\infty}r(t)e^{-j\theta_L-j2\pi[v_Lt+\tilde{m}\Delta f(t-\tau_L)]}\phi_L^*(t-\tau_L)dt\right\} \quad (5)$$

where $(\cdot)^*$ denotes complex conjugate and $E_b = P_bT_b$ is the bit energy.

We limit ourselves to consider only the case that $2\Delta_v$, the maximum frequency difference between the carrier frequency of the desired signal and that of the other-user signal, is less than Δf. It follows that in the detection of the desired signal transmitted on the $\tilde{m}$th subcarrier, the interference due to the other-user signal is contributed only from its $\tilde{m}$th subcarrier signal as well as from one of its adjacent subcarriers. In the analysis, we do not consider the cases $\tilde{m} = 0$ and $\tilde{m} = M - 1$. In these two cases, it is possible that there is only one other-user subcarrier signal that contributes to the MAI; in other cases, there are two. Therefore, these two cases may yield slightly optimistic results.

Define the time-frequency crosscorrelation function as

$$C_{k,L}(\ell;\mu)=\begin{cases}\sum_{i=0}^{N-1-\ell}c_i^{(k)}c_{i+\ell}^{(L)*}e^{j2\pi i\mu} & 0\le\ell\le N-1\\ \sum_{i=-\ell}^{N-1}c_i^{(k)}c_{i+\ell}^{(L)*}e^{j2\pi i\mu} & -(N-1)\le\ell<0\\ 0 & |\ell|\ge N\end{cases}. \quad (6)$$

This function reduces to the spectral crosscorrelation function [7] and the time-domain one in the special cases of $\ell = 0$ and $\mu = 0$, respectively. Note that $C_{k,L}(\ell;\mu)$ is periodic in μ with a period of 1. Let

$$R(\tau;\xi)=\frac{1}{T_c}\int_{-\infty}^{\infty}\psi(t)\psi^*(t+\tau)e^{j2\pi\xi t}dt. \quad (7)$$

By taking Fourier transform on $R(\tau;\xi)$ in the variable τ, and since a square-root-raised-cosine pulse is considered, it can be shown that $R(\tau;\xi) = 0$ for $|\xi| \ge (1 + \alpha)T_c^{-1}$. Let n_R be a positive integer such that, for $|\xi| < (1 + \alpha)T_c^{-1}$, $R(\tau;\xi)$ is negligible when $|\tau| > n_RT_c$. Substituting (4) into (5) yields

$$\hat{b}_{\tilde{m},0}^{(L)}=\tilde{n}+b_{\tilde{m},0}^{(L)}+\sum_{k=1,k\ne L}^{K}J_k \quad (8)$$

where $\tilde{n}$ is a zero-mean Gaussian random variable with a variance $(2E_b/N_0)^{-1}$, and J_k is the MAI due to the k th user. After some algebraic manipulations, J_k can be expressed as

$$J_k=\mathrm{Re}\left\{e^{j(\theta_k-\theta_L)+j2\pi[w_k\tau_k-\tilde{m}(\Delta f)\lambda_k]}\sum_{i=-1}^{1}\sum_{n=-1}^{1}b_{i+\tilde{m},n}^{(k)}\cdot e^{j2\pi(w_k+i\Delta f)nT_b}\sum_{q=-n_q}^{n_q}\frac{1}{N}C_{k,L}\left(nN+q;(w_k+i\Delta f)T_c\right)\cdot R\left(\lambda_k-qT_c;w_k+i\Delta f\right)\right\} \quad (9)$$

where $w_k = v_k - v_L$, $\lambda_k = \tau_k - \tau_L$, and $n_q = \lceil n_R + 2\Delta_\tau \rceil - 1$. Notice that w_k and λ_k follow triangular distributions over the ranges $[-2\Delta_v, 2\Delta_v]$ and $[-2\Delta_\tau T_c, 2\Delta_\tau T_c]$, respectively. From (9), it is clear that the MAI is characterized by the time-frequency crosscorrelation.

III. TIME-FREQUENCY CROSSCORRELATION PROPERTIES

We consider the following sets of sequences: preferentially phased Gold codes [1] ($N = 31$), Walsh codes ($N = 32$), cyclic-shift m-sequences [3] ($N = 31$), Suehiro-Hatori polyphase sequences [4] ($N = 30$) and Lin-Chang sequences [6] ($N = 63$). We do not consider the set of polyphase sequences proposed by Suehiro [5] because of the similarity of this sequence set with the set proposed in [4].

A. *Average crosscorrelation*

Let

$$\Xi(\ell;\mu) = \left[\frac{1}{N-|\ell|}\mathcal{E}_{\text{code}}\left(\left|C_{k,L}(\ell;\mu)\right|^2\right)\right]^{1/2}, \quad |\ell| \neq N, \quad \textbf{(10)}$$

where $\mathcal{E}_{\text{code}}(\cdot)$ denotes averaging over all possible combinations of sequences, $k \neq L$, of the sequence set under consideration. In the special case that the sequences are binary-valued and randomly generated, it can be shown that $\Xi(\ell;\mu) = 1$. Therefore, if the $\Xi(\ell;\mu)$ value computed for the optimized sequence set under consideration is well below 1 over a given range of μ, the optimizing property and hence the MAI-minimization capability of the sequence set are preserved in the presence of CFO. On the other hand, if $\Xi(\ell;\mu)$ is close to unity, the sequences under consideration behave like random sequences in the presence of CFO and the capability to minimize the MAI due to optimized crosscorrelation is lost.

Fig. 1a plots the $\Xi(0;\mu)$ against μ for $|\mu| \leq 0.1$. The deep notch around $\mu = 0$ is a result of the crosscorrelation-optimized property that leads to substantial MAI reduction. However, the capability that the MAI can be substantially reduced diminishes rapidly if $|\mu|$ is away from the origin. The $\Xi(0;\mu)$ values of the Walsh code set, the sets of preferentially phased Gold codes and cyclic-shift m-sequences approach 1 for $|\mu|$ as low as 0.02, and for the Lin-Chang sequence set, $|\mu| = 0.01$. For Suehiro-Hatori sequence set, although $\Xi(0;\mu)$ is relatively low for $\mu < 0$, $\Xi(0;\mu)$ is higher than 1 for positive μ. In Fig. 1b, we plot $\Xi(1;\mu)$ against μ. It is apparent that $\Xi(1;\mu) \approx 1$ for the set of preferentially phased Gold codes, a result that arises because these codes are optimized only for $C_{k,L}(0,0)$ but not $C_{k,L}(1,0)$ [1]. It is also noticed that the $\Xi(1;\mu)$ values for all code sets except the set of Suehiro-Hatori sequences are close to unity for $|\mu| \geq 0.02$, indicating that the MAI-reduction ability is close to that of random sequences if $|\mu|$ is greater than 0.02. Despite $\Xi(1;\mu)$ stays below 0.2 for Suehiro-Hatori sequence set, the characteristic of its $\Xi(0;\mu)$ does not make this set useful for MAI minimization in the presence of CFO. Also notice that Lin-Chang sequences are more sensitive to the CFO than Walsh codes, preferentially phased Gold codes and cyclic-shift m-sequences in the MAI-minimization capability as the Lin-Chang sequences have narrower notches in $\Xi(0;\mu)$ and $\Xi(1;\mu)$ around $\mu = 0$.

From the results, we also observe that to preserve the MAI-minimization capability of optimized sequences in the presence of CFO, one needs to operate a system with $2\Delta_\nu T_c$ (maximum possible value of $|\mu|$) less than around 0.02.

B. *Worst-case crosscorrelation*

The worst-case MAI can be characterized by

$$\Lambda(\ell;\mu) = \frac{1}{N-|\ell|}\sup_{k\neq L}\left|C_{k,L}(\ell;\mu)\right|, \quad \ell \neq N, \quad \textbf{(11)}$$

where the supremum is obtained by examining all possible combinations of sequences, $k \neq L$, of the sequence set that is considered. Fig. 2 plots the $\Lambda(0;\mu)$ for the sequence sets under consideration. The most important observation is that $\Lambda(0;\mu)$ attains a value of 1 at some values of μ for Walsh codes and Suehiro-Hatori polyphase sequences. (This result for the Walsh code set has also been reported by Popović [7] in the study of spectral crosscorrelation properties.) We also plot $\Lambda(1;\mu)$, not presented here for saving space, and find that $\Lambda(1;\mu)$ shows a similar result of achieving a maximum value of 1 at some μ values. Since $C_{k,L}(\ell;\mu)$ is given by (6), our observation indicates that the Lth-user sequence becomes *identical* to signature sequences of some of the other users after frequency- and time-shifting due to the CFO and quasi-synchronized signal transmission, respectively. Consequently, at some combinations of parameters the receiver may not be able to distinguish the desired signal against the MAI if Walsh codes or Suehiro-Hatori sequences are used. In this situation, the system is driven into *outage*.

IV. IMPLICATIONS TO SYSTEM IMPLEMENTATION

It has been shown that the system using Walsh codes or Suehiro-Hatori polyphase sequences can be driven into outage at some combinations of parameters. The occurrence of outage is likely to result in a loss of communication link, which may lead to serious consequences for the system performance. Although Walsh codes and Suehiro-Hatori polyphase sequences show good time-domain crosscorrelation properties which have drawn much attention for applications to single-carrier and multicarrier QS DS-CDMA [2], [4], the systems implemented using these sequences are vulnerable to the presence of CFO.

It has been indicated that it is possible to maintain the MAI-minimization capability of an optimized sequence set in the presence of CFO only if the system can be operated at a $\Delta_\nu T_c$ value less than around 0.01. Currently, the frequency accuracy of a typical frequency synthesizer is around ±10ppm. Let us consider a system operating at a desired carrier frequency of 1GHz. It follows that Δ_ν = 10kHz. One can utilize the MAI-minimization capability of the optimized sequences only if the system can be operated at $T_c < 1\mu$s. A higher carrier frequency results in a shorter T_c. However, a chip period of much longer than $1\mu s$ is generally desired for QS multicarrier DS-CDMA communications because of the consideration on the complexity involved in the time synchronization of all users' signals. Alternatively, one can operate a system with a tighter frequency control. For example, consider a system using $T_c = 100\mu s$ and a carrier frequency of 1GHz. Operating at $\Delta_v T_c < 0.01$ requires that $\Delta_v < 100$Hz. Maintaining an oscillator running at a frequency of 1GHz with a frequency error less than 100Hz is currently a challenging task.

V. CONCLUSIONS

We have shown that the MAI of a QS multicarrier DS-CDMA communication system can be characterized by the time-frequency crosscorrelation function. Properties of this function have been analyzed for a number of sequence sets that are optimized in the time-domain crosscorrelation. It has been found that the Walsh code set and the set of Suehiro-Hatori polyphase sequences yield normalized worst-case crosscorrelation values equal to 1. Thus, the system implemented using either of these two sequence sets can be driven into outage at some combinations of parameters. Analysis on the average time-frequency crosscorrelation has revealed that the optimized code sets we consider can maintain their MAI-minimization capabilities only if $\Delta_v T_c$ is less than around 0.01. Discussion results have revealed that maintaining this requirement on $\Delta_v T_c$ is currently a challenging task.

ACKNOWLEDGMENTS

This work was supported by the Hong Kong Research Grants Council and by the University Research Committee of The University of Hong Kong, Hong Kong.

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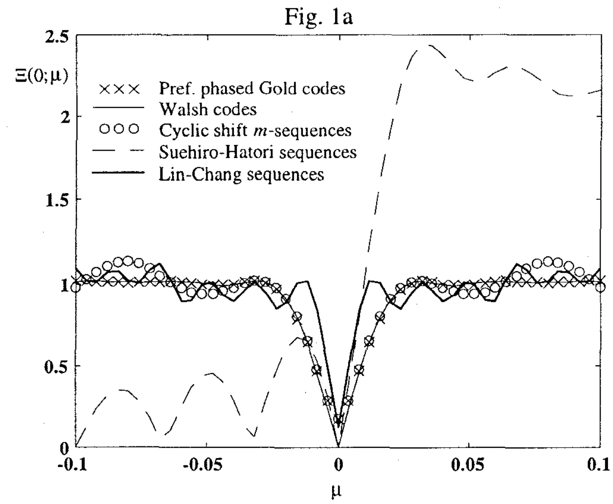
Fig. 1. Average time-frequency crosscorrelation: (a) $\Xi(0;\mu)$; (b) $\Xi(1;\mu)$.



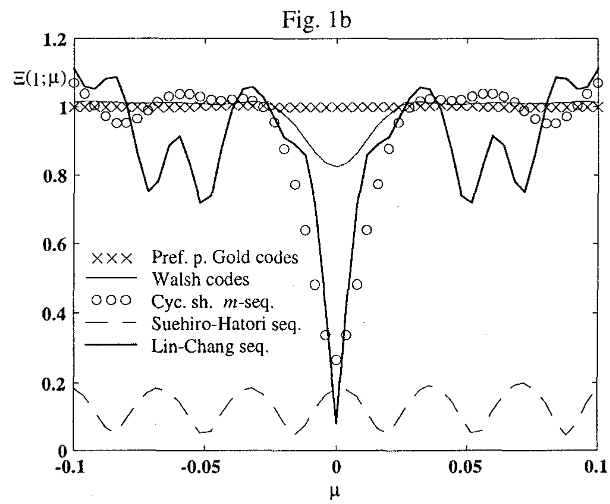


Fig. 2. Worst-case time-frequency crosscorrelation $\Lambda(0;\mu)$.