# Performance Comparison of Asynchronous Orthogonal Multi-Carrier CDMA in Frequency Selective Channel


*X. Gui and T.S. Ng*

Department of Electrical and Electronic Engineering
The University of Hong Kong
Pokfulam Road, Hong Kong



## Abstract

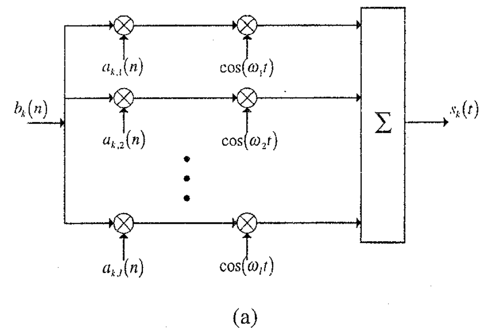
Bit error rate (BER) performance of an asynchronous multi-carrier code division multiple access (MC-CDMA) system for the uplink of the mobile communication system with equal gain combining and maximum ratio combining are obtained. Given a fixed bandwidth frequency selective channel, performance of MC-CDMA is compared with that of conventional CDMA and MC-DS-CDMA in numerical results.


## 1. Introduction

Orthogonal frequency division multiplexing (OFDM) spread spectrum communication schemes seem to be a good candidate for high rate wireless data transmissions and have drawn a lot of attention in recent years [1,2]. Among these schemes, a system called MC-CDMA [3,4] has been proposed and analyzed for both uplink and downlink transmissions in wireless radio networks. However, the analyses are based on the assumptions of perfect synchronization among users and independent fading at carriers. While in practice, perfect time synchronization is difficult to achieve in the uplink of a mobile communication system and the fading of the subcarriers are usually correlated due to insufficient frequency separation between the carriers. In this paper, we analyze the asynchronous MC-CDMA system with correlated fading among carriers. The paper is organized as follows: Section **2** describes the asynchronous MC-CDMA system and the frequency selective fading channel model. Section **3** presents the performance analysis. Section **4** compares the performance of MC-CDMA with that of conventional CDMA and MC-DS-CDMA [9] in numerical example. Finally, Section **5** gives the conclusion.

## 2. MC-CDMA System and Channel Model

The MC-CDMA system proposed in [4] transmits $L$ chips of a data symbol in parallel on $L$ different carriers, one chip per carrier, where $L$ is the total number of chips per data bit or processing gain (PG). Thus the chip duration of the MC-CDMA system is the same as the bit duration and is denote by $T_s$. We assume the use of random spreading sequence throughout this paper. The frequency separation between the neighboring carriers is $F/T_b$ Hz where $F$ is an integer. As mentioned in [4], this scheme is similar to performing OFDM on a DS SS signal for $F$=1. Since we are considering orthogonal MC-CDMA here, we fix $F$ to be equal to 1 in the rest of this paper. The block diagrams of the transmitter and receiver are depicted in Fig. 1. In order



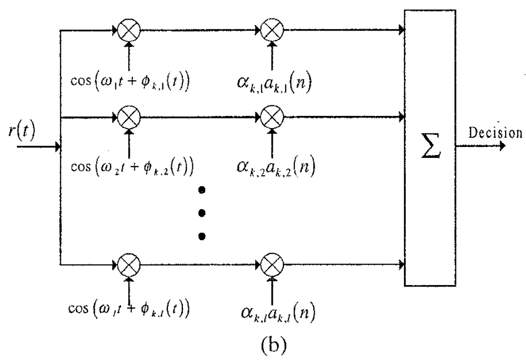




**Fig.1**. (a) MC-CDMA transmitter.
(b) MC-CDMA receiver.

to compare the MC-CDMA and conventional CDMA, we fix the pass-band null-to-null bandwidth and data transmission rate of the system. Thus given a conventional CDMA system with chip duration $T_c$ and bit duration $T_b$, its PG is $L_1 = T_b/T_c$, the pass-band null-to-null bandwidth is $2/T_c$ assuming rectangular waveform and the data rate is $1/T_b$. For the MC-CDMA system, its bandwidth is given by $(L+1)(1/T_s)$ and data rate is $1/T_s$. Hence we have

$$(L+1)(1/T_s) = 2/T_c \ , \tag{1}$$

and

$$1/T_s = 1/T_b \ . \tag{2}$$

From (1) and (2) we find

$$T_s = T_b \quad , \tag{3}$$

and

$$L = 2L_1 - 1 \; . \tag{4}$$

That is, the chip duration of MC-CDMA is $L_1$ times as long as that of DS CDMA and the PG of MC-CDMA is nearly twice as high as that of DS CDMA, which reflects the 50% spectral overlapping in the MC-CDMA system. Due to the fact that the channel multipath spread $T_m$ is usually several times as long as $T_c$ and much less than $T_b$, we have frequency non-selective fading on each carrier. However, the fading at carriers are not independent due to the spectral overlapping and insufficient frequency separation between carriers.

Given a frequency selective multipath fading channel for conventional CDMA with multipath intensity profile (MIP) $\phi_c(\tau)$, we can find the spaced-frequency correlation function $\phi_c(\Delta f)$ of the channel by taking the Fourier transform of $\phi_c(\tau)$ [5]. Thus, we can determine the correlation among the fading of carriers when MC-CDMA is used in the channel. We further assume that each carrier is subject to identical frequency no-selective Rayleigh fading, which is constant over the duration of at least $T_s$ seconds. The complex low-pass impulse response of the channel for carrier $i$ of user $k$ is assumed to be

$$g_{k,i}(t) = \beta_{k,i}(t) e^{j\varphi_{k,i}(t)} \quad , \tag{5}$$

which is a complex Gaussian random variable (r.v.) with zero mean and variance $\sigma^2$. Here we imply that each user experiences identical independent fading channel.

## 3. Performance Analysis

Now we proceed to investigate the performance of MC-CDMA with asynchronous users and correlated fading among carriers. The transmitted BPSK signal of user $k$ can be written as

$$s_k(t) = \sum_{n=-\infty}^{+\infty} \sqrt{2S} \sum_{l=1}^{L} b_k(n) a_{k,l}(n) u_{T_s}(t - nT_s) \cos(\omega_l t + \theta_{k,l}) \, , \tag{6}$$

where $u_{T_s}(t)$ is the rectangular waveform defined as

$$u_{T_s}(t) = \begin{cases} 1 & 0 \le t \le T_s \\ 0 & \text{elsewhere} \end{cases} .$$

The transmission power on different carriers is the same and is denoted by $S$, and $b_k(n)$ is the $n$th data bit of user $k$, $a_{k,l}(n)$ is the $l$th chip of $n$th data bit in the data stream of user $k$, $\omega_l$ is the $l$th carrier frequency in radians. The random phase $\theta_{k,l}$ is uniformly distributed over $[0, 2\pi)$ and is independently identically distributed (i.i.d.) for different $k$ and $l$. Assuming $K$ asynchronous CDMA users in the system, all using the same set of carriers and PG $L$, and the same transmission power in the uplink, the received signal at the base station is given by

$$r(t) = \eta(t) + \sum_{n=-\infty}^{+\infty} \sqrt{2S} \sum_{k=1}^{K} \sum_{l=1}^{L} \beta_{k,l}(t) b_k(n) a_{k,l}(n) \cdot u_{T_s}(t - nT_s - \zeta_k) \cos(\omega_l t + \phi_{k,l}(t)) \quad , \tag{7}$$

where $\phi_{k,l}(t) = \theta_{k,l} + \varphi_{k,l}(t) - \omega_l \zeta_k$, $\zeta_k$ is the time misalignment of user $k$ with respect to the reference user at the receiver which is i.i.d. for different $k$ and uniformly distributed in $[0, T_s)$, and $\eta(t)$ is the additive white Gaussian noise with zero mean and unilateral power spectral density $N_0$. With coherent reception and user 1 as the reference user whose $\zeta_0$ is zero, the decision variable $U$ of the $0^{\text{th}}$ data bit of the $p$th data stream of user 1 is given by

$$U = \sum_{l=1}^{L} \int_0^{T_s} r(t) a_{1,l}(0) \cos(\omega_l t + \phi_{1,l}(0)) \alpha_{1,l} dt \; . \tag{8}$$

Due to slow fading, the path gain and phase shift variables are considered to be constant over the time interval $[0, T_s]$ and is denoted by $\beta_{1,l}(0)$ and $\varphi_{1,l}(0)$, respectively. Depending on the choice of $\{\alpha_{1,l}\}$, there are two ways of combining the chips of the same data bit: Equal Gain Combining (EGC) and Maximum Ratio Combining (MRC). Both will be studied in the following.

### A. *Equal Gain Combining*

With EGC, $\alpha_{1,l} = 1$ for all $l$. Equation (8) can be rewritten as

$$U = D + \eta + I + J \; . \tag{9}$$

The term $D$ is the desired output :

$$D = \sqrt{\frac{S}{2}} T_s b_1(0) \sum_{l=1}^{L} \beta_{1,l}(0) \, , \tag{10}$$

and $\eta$ is the interference term due to Gaussian noise with zero mean and variance $N_0 L T_s / 4$. In (9), the term $I$ is the same carrier interference from other users and $J$ is other carrier interference from other users. Due to the assumption that $\theta_{k,l}$ is i.i.d. for different $k$ and $l$, it is easy to show that all terms in $I$ and $J$ are uncorrelated. Hence both $I$ and $J$ are approximately Gaussian. By averaging $I$ and $J$ over $\beta_{k,l}$, $\theta_{k,l}$ and $\zeta_k$, we find that both $I$ and $J$ have zero mean and variance

$$\mathrm{Var}(I) = (K-1) L S \sigma^2 T_s^2 / 3 \, , \tag{11}$$

and

$$\mathrm{Var}(J) = \frac{(K-1) S \sigma^2 T_s^2}{4\pi^2} \sum_{l=1}^{L} \sum_{\substack{j=1 \\ l \ne j}}^{L} \frac{1}{(l-j)^2} \, , \tag{12}$$

respectively. From (10)-(12), assuming a "one" is transmitted, the mean and variance of $U$ are given, respectively, by

$$\mathrm{E}(U) = \sqrt{\frac{S}{2}} T_s \sum_{l=1}^{L} \beta_{1,l}(0) \tag{13}$$

and

$$\mathrm{Var}(U) = N_0 L T_s/4 + (K-1) S\sigma^2 T_s^2 \left(L/3 + Q/4\pi^2\right) \quad (14)$$

where

$$Q = \sum_{l=1}^{L} \sum_{\substack{j=1 \\ l \neq j}}^{L} \frac{1}{(l-j)^2} \,. \quad (15)$$

We see $U$ is conditional Gaussian conditioned on $\{\beta_{1,l}(0)\}$. The r.v. set $\{\beta_{1,l}(0)\}$ in (10) consists of $L$ correlated Rayleigh random variables and is identical for different data bit and the choice of the $0^{th}$ data bit as the reference data bit is in fact arbitrary. Hence for simplicity, we use $\{\beta_l,\ l=1,2,\ldots,L\}$ to denote $\{\beta_{1,l}(0)\}$ in what follows. Therefore the probability of error conditioned on $\{\beta_l\}$ is simply given by

$$P\left[e|\{\beta_l\}\right] = \frac{1}{2}\mathrm{erfc}\left(\frac{\mathrm{E}(U_p)}{\sqrt{2\mathrm{Var}(U_p)}}\right). \quad (16)$$

Then the bit error rate (BER) is obtained via averaging $P\left[e|\{\beta_l\}\right]$ over $\{\beta_l\}$ :

$$\mathrm{BER}_p = \int_0^\infty P\left[e|\{\beta_l\}\right] p(\beta_1, \beta_2, \ldots, \beta_L) d\beta_1 d\beta_2 \cdots d\beta_L \,, \quad (17)$$

where $p(\beta_1, \beta_2, \ldots, \beta_L)$ is the joint probability density function of $\{\beta_l\}$.

The average signal-to-noise ratio (SNR) is defined by [9]

$$\mathrm{SNR} = \frac{ST_s}{N_0 L} \mathrm{E}\left[\left(\sum_{l=1}^{L} \beta_l\right)^2\right]. \quad (18)$$

The evaluation of (17) and $\mathrm{E}\left[\left(\sum_{l=1}^{L} \beta_l\right)^2\right]$ in (18) can be done by Monte Carlo integration [6,9].

### *B. Maximum Ratio Combining*

With MRC, $\alpha_{1,l} = \beta_l$. The decision variable $U$ takes the same form as that given in (9), with the variance of $\eta$ change to $(N_0 T_s/4)\sum_{l=1}^{L}\beta_l^2$, and

$$D = \sqrt{\frac{S}{2}} T_s b_1(0) \sum_{l=1}^{L} \beta_l^2 \,. \quad (19)$$

The interference terms $I$ and $J$ from other users are still approximately Gaussian with zero mean and variance

$$\mathrm{Var}(I) = \frac{(K-1)S\sigma^2 T_s^2}{3} \sum_{l=1}^{L} \beta_l^2 \,, \quad (20)$$

and

$$\mathrm{Var}(J) = \frac{(K-1)S\sigma^2 T_s^2}{4\pi^2} \sum_{l=1}^{L} \beta_l^2 \sum_{\substack{j=1 \\ l \neq j}}^{L} \frac{1}{(l-j)^2} \,, \quad (21)$$

respectively. Using (16)-(17), we can obtain the system BER with MRC, with $\mathrm{E}(U)$ equal to $D$ given in (19), $\mathrm{Var}(U)$ equal to the sum of $\mathrm{Var}(I)$ and $\mathrm{Var}(J)$ given by (20) and (21) ), and the variance of $\eta$.

## 4. Numerical Results

To make a fare comparison of system performance, the fading channel parameters, the system bandwidth and data rate are fixed in this section. For the conventional CDMA, its processing gain $L_1$ is set equal to 60. The channel is a multipath channel modeled as a finite tapped delay line with $N$=4 Rayleigh fading paths and two types of multipath intensity profiles, i.e., uniform and exponential MIPs, which are the same as those used in [9]. Therefore if MC-CDMA is used in this channel, from (3) and (4) we obtain $T_s = 60T_c$ and $L$=119. Since the multipath spread of the channel is $T_m = 4T_c$, each carrier of MC-CDMA has frequency non-selective fading, i.e., there are not multipaths at each carrier. However, the fading at carriers are correlated and the frequency correlation function $\phi_c(\Delta f)$ can be obtained by taking the Fourier transform of the uniform MIP as discussed in Section **2**. Using Monte Carlo integration, we can generate correlated complex Gaussian random sequence[7] corresponding to $\phi_c(\Delta f)$. The Rayleigh r.v. set $\{\beta_l\}$ can then be readily obtained based on the generated complex Gaussian random sequence. In the evaluation of the BER performance of MC-CDMA, $\{\beta_l\}$ is generated 10,000 times. As shown in Fig. 2, the BER performance of

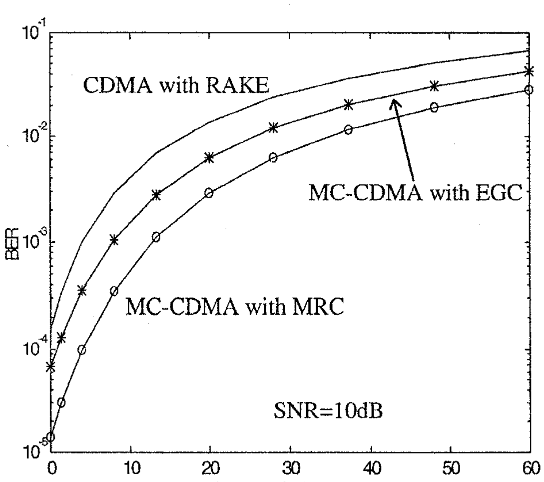



**Fig.2.** Performance of asynchronous MC-CDMA and conventional CDMA with RAKE.

MC-CDMA with EGC and MRC are plotted against the number of other users, where the average SNR is set to be 10dB and uniform MIP is used. Also shown in the figure is the performance of conventional CDMA, which is evaluated using the analytical results of [8], with a RAKE receiver using all the four paths with maximum ratio combining. It is clear that MC-CDMA outperforms the conventional CDMA, where MC-CDMA with MRC performs the best.

Next, we compare the performance of MC-CDMA, MC-DS-CDMA and conventional CDMA under uniform MIP in Fig.3. The BERs are plotted against the average SNR, and the total number of users is fixed at 10. The BERs of MC-DS-CDMA and conventional CDMA are taken from Fig.4 of reference [9], where the BER of MC-DS-CDMA is obtained under the assumption that only the Rayleigh

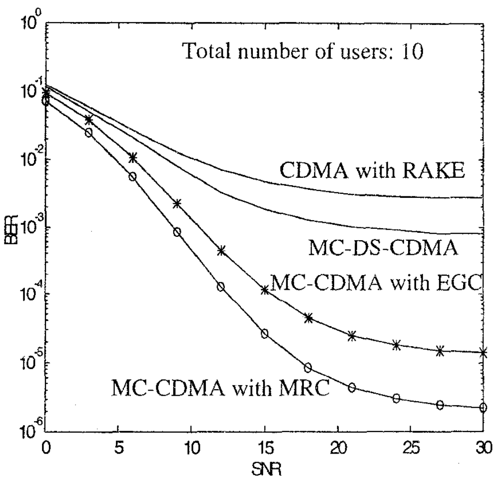



**Fig.3.** Performance comparison of MC-CDMA, MC-DS-CDMA and CDMA under uniform MIP.

envelops of successive carriers are correlated with coefficient 0.25 and the number of carriers is 6, and RAKE receiver with EGC are employed in the conventional CDMA to take advantage of all the 4 paths. It is clear that MC-CDMA with MRC performs the best, followed by MC-CDMA with EGC, MC-DS-CDMA, and the conventional CDMA. We also compare the performance of these three systems in Fig.4 with exponential MIP, and the results are quite similar to that of the case of uniform MIP.

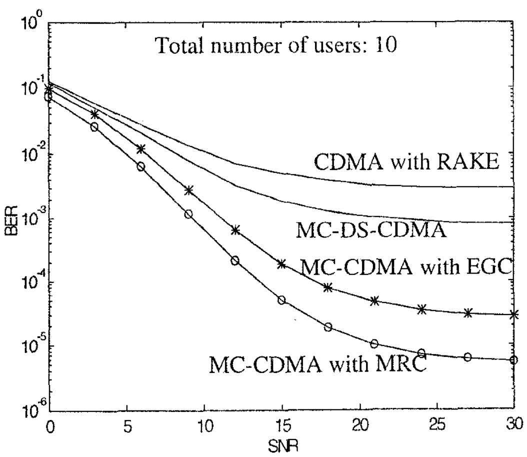



**Fig.4.** Performance comparison of MC-CDMA, MC-DS-CDMA and CDMA under exponential MIP.

## 5. Conclusions

BER performance of an asynchronous orthogonal MC-CDMA system with EGC and MRC has analyzed in the uplink of the mobile communication system. Numerical results that the best BER performance of such a system is achieved by using maximum ratio combining and the system performs better than the conventional CDMA system as well as the MC-DS-CDMA under the same channel conditions.

## 6. References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## Acknowledgment


This work was supported by the Hong Kong Research Grants Council and the CRCG of The University of Hong Kong.