

A Heuristic Algorithm for Channel Assignment in Cellular Mobile Systems

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Abstract -- This paper describes an efficient heuristic algorithm for the channel assignment problem (CAP) in cellular mobile systems. The channel assignment scheme proposed here is based on repetitive ordering of requirements in sequences. The performance of the proposed algorithm is verified by several benchmark problems and found to be superior than other existing methods. The study show that the proposed algorithm yields optimal assignment in most of the cases and near-optimal assignment in other cases.

I. INTRODUCTION

The recent demand in mobile telephone services has increased rapidly. The efficiency in spectrum utilization becomes an important issue in frequency planning. In the channel assignment problem, a set of nominal channels has to be assigned to each cell. Channel assignment schemes have widely been investigated in recent literature [1-11] and they can be classified into two categories namely, the general optimization methods and the problem orientated methods. The general optimization methods consist of the neural network algorithms [10], the genetic algorithms (GA) [8] and the simulated annealing (SA) [7]. On the other hand, the heuristic channel assignment technique [1], the sequential ordering algorithms [5], the adaptive local search algorithm [6], graph coloring algorithms and pattern based optimization algorithms are classified into the problem oriented methods

For instance, Lai [8] has employed the genetic algorithm to solve the CAP, where each string in the population represents a particular assignment. For each iteration, increasingly improved populations are generated using three procedures, namely, the selection, the mutation and the crossover. The best assignment is subsequently selected from the improved population. Other researchers [7] had employed the simulated annealing algorithm for CAP. Both the demand traffic and constraints are represented by a cost function which is reduced to zero when all constraints are satisfied. However, both the genetic algorithm and the simulated annealing method can easily be trapped in the local minima which requires substantial iterations to converge to the optimum solution.

In the heuristic channel assignment scheme [1], Box used the idea of repeated ordering of the calls and assigning the channels from both the top and the bottom ends of a given

range of frequencies. Denied calls in each attempt are moved to the top of the list in a random manner and other remaining calls are moved to the bottom of the list. Furthermore, Sivaraman [5] employed a similar approach in his sequential ordering algorithms and presented four algorithmic orderings of calls with two strategies for assigning frequencies. Recently, a local search technique [6] is employed to search for an improved ordering of calls. The key is to swap any pair of calls in the list in a random fashion in order to generate a new ordering of calls. In this discourse, a new methodology for the repetitive ordering of calls with determined assignment difficulties is proposed.

II. PROBLEM STATEMENT

The CAP is a classic optimization problem which involves allocating required channels to cells under the following electromagnetic compatibility constraints,

1. *Cochannel constraint*: Same channel cannot be assigned to certain cells simultaneously.
2. *Adjacent channel constraint*: Adjacent channels cannot be assigned to adjacent radio cells simultaneously.
3. *Co-site constraint*: Channels which are separated from a small distance in frequency domain cannot be assigned to a cell simultaneously.

These constraints can be represented by minimum channel separation between any pair of channels assigned to a pair of cells or a cell itself. The channel separation is described by a symmetric matrix C to reflect the above three constraints. The demand traffic is represented by a vector M which consists of the number of required channels for each cell. If there are n number of radio cells serving a system, then the compatibility matrix C will be an $n \times n$ symmetric matrix of which an element c_{ij} indicates the minimum channel separation required between a channel assigned to a call in the i^{th} cell and a channel assigned to a call in the j^{th} cell. If f_{ik} indicates that k^{th} channel is assigned to the i^{th} cell, then the electromagnetic compatibility constraints are represented by

$$|f_{ik} - f_{jl}| \geq c_{ij}$$

for all integer values of k, l, i, j where

$$1 \leq k \leq m_i, 1 \leq l \leq m_j \text{ for } 1 \leq i \leq n \text{ and } 1 \leq j \leq n \\ i \neq j \text{ when } k = l \text{ or } k \neq l \text{ when } i = j ;$$

The aim of channel assignment is to allocate carriers to cells to meet the system demand compatible with above constraints, with minimum number of channels. The CAP is formulated as a minimum span problem.

The channel assignment problem can be modeled as a graph coloring problem [4]. Since the graph coloring problem is known to be NP-complete, channel assignment problem is also NP-complete where computation complexity of searching for the optimum solution grows exponentially with problem size.

III. THE HEURISTIC ALGORITHM

Basic idea of the proposed algorithm is to list the calls in different ordering at each attempt and to employ frequency exhaustive strategy to assign channels to the ordered list of calls. Different ways of ordering calls have been proposed in the literature [1],[2],[5],[6].

In *Frequency exhaustive strategy* [1],[5],[6], channels are assigned to the call list starting from the minimum channel. First call in the list is assigned with minimum possible channel. If the first channel can not be assigned to the next call, second channel is tried. Likewise minimum possible channel is assigned to that call. Similarly minimum possible channel is assigned to each call in the list starting from the top.

The key of the technique is the procedure for changing the assignment order at each iteration. Calls are ordered in descending order of the determined assignment difficulty of each call at each iteration. The assignment difficulty is measured in unit of integers ranging from 0 to D_{max} where D_{max} is an arbitrary value and indicates the most difficult call to assign. In each attempt of assignment iteration, calls assigned with channels above a certain frequency f_c , are said to be *difficult calls*, where f_c is called the *control frequency*. A certain portion of the calls are identified as difficult calls and their difficulties are incremented at each iteration. The f_c must be less than the required maximum channel f_{max} . However when f_c is significantly small, calls become unnecessarily *difficult*. Therefore, f_c value is selected as the simple first level lower bound of the problem given by,

$$f_c = \left[\left(\max_i m_i \right) - 1 \right] \times C_{ii} \quad (1)$$

where $\left(\max_i m_i \right)$ is the maximum demand per cell.

For the *difficult calls*, the assignment difficulty of the i^{th} call is increased by $(f_i - f_c)$ in the next iteration. Note that the subscript i denotes the identity of each call, but not the position of the call in the particular order in an iteration. For the remaining calls, the assignment difficulty is unchanged.

At each iteration, calls are ordered in descending order of their assignment difficulty and thus channels are assigned to the ordered call list. Similarly, when the i^{th} call is assigned with a channel f_i less than f_c , the assignment difficulty of that call is unchanged. For $f_i > f_c$, the assignment difficulty of the i^{th} call is increased by $(f_i - f_c)$. Therefore, the assignment difficulty of each call is updated at each iteration in a recursive manner as described above. Note that there is a possibility for some calls in the top of the list which can be easily assigned in this frequency exhaustive strategy. As the assignment difficulty is increased in each iteration, difficult calls are moved to the top of the list after a few iterations. When a superior assignment is achieved in any attempt, the assignment difficulties of all calls are set to zero and hence the convergence to the optimal solution is enhanced. In the following discussion, let:

f_{max} be the minimum number of channels needed for the assignments in first $(\gamma - 1)$ iterations;

f_m be the minimum number of channels needed for the assignment in γ^{th} iteration;

D_i^γ be the assignment difficulty of i^{th} call for the assignment in γ^{th} iteration;

f_i^γ be the channel assigned to i^{th} call in γ^{th} iteration;

f_c be the control frequency;

N be the total number of calls in the call list;

then we can write the following after γ^{th} iteration,

$$(A) \text{ If } f_m < f_{max} \text{ then,} \quad (2)$$

$$(a) D_i^\gamma = 0 \text{ for all } i = 0 \dots N$$

$$(b) f_{max} = f_m$$

$$(B) D_i^{\gamma+1} = D_i^\gamma + X_i^\gamma, \quad 1 \leq i \leq N \quad (3)$$

where

$$X_i^\gamma = \begin{cases} f_i^\gamma - f_c & \text{if } f_i^\gamma > f_c \\ 0 & \text{Otherwise} \end{cases}, \quad 1 \leq i \leq N$$

Subsequently the initialization procedure of the call list is discussed, as follows.

Initialization

It can be observed that an appropriate initial ordering of the call list can improve the convergence to the optimal solution [6]. To ensure that the solution point does not significantly deviate from the optimum point, an algorithmic ordering, namely the column wise node-degree ordering [5], is used for the initial ordering of the call list. We adopt the notations by Sivarajan [5], who measure the *difficulty* to assign channels to a cell, as degree of the cell given by,

$$d_i = \left[\sum_{j=1}^n m_{ij} \times C_{ij} \right] - C_{ii} \quad (4)$$

In the initialization procedure, cells are ordered in descending order of their degrees. Calls are filled in a $(n \times M)$ matrix as described in [5], where M is the maximum demand per cell and n is the number of cells. The initial call list is prepared by the column wise- ordering of that matrix.

The procedure for the proposed algorithm is described as follows.

Algorithm

1. Using the Column wise node-degree ordering described above, prepare the initial call list.
2. Assign channels to the call list using frequency exhaustive strategy.
3. Assign assignment difficulties to all calls as follows:
after γ^{th} iteration,

$$D_i^{\gamma+1} = D_i^\gamma + X_i^\gamma$$
where $X_i^\gamma = \begin{cases} f_i^\gamma - f_c & \text{if } f_i^\gamma > f_c \\ 0 & \text{Otherwise} \end{cases}$
and $D_i^0 = 0$ for all i .
4. Rearrange the calls in descending order of assignment difficulties calculated (In the term D_i^γ , i does not indicate the position of the call in the list at each iteration. It actually is the call-identity).
5. Assign channels to the ordered list of calls using frequency exhaustive strategy and find the maximum frequency (f_m) needed to assign all calls given by:

If $f_m < f_{\max}$ then ,

- (a) $D_i^\gamma = 0$ for all i ,
- (b) Select the assignment to the *best assignment*,
- (c) $f_{\max} = f_m$,
- (d) *Iteration No* = 0 .

6. Increment the *Iteration No*.
7. If terminating conditions are not reached then go to step 3.

In the study the terminating condition is given by:
Stop if $f_{\max} = c_{ii} \times (M - 1)$ or *Iteration No* = 100

Obviously the value of $c_{ii} \times (M - 1)$ is the simple first level lower bound [3] of the given CAP.

IV. IMPLEMENTATION

For the purpose of performance comparison, several bench mark problems in the literature are used. The proposed algorithm is examined with other five well-known algorithms in the literature including the local search technique-CAP3 [6], the eight sequential ordering (SO) algorithms [5], the heuristic technique [1], the simulated annealing [7] and the genetic algorithm [8]. The configuration of the comparison study of the five major algorithms is described below.

The eight sequential ordering algorithms proposed by Sivarajan [5], CCF, CCR, CRF, CRR, DCF, DCR, DRF and DRR were examined for each benchmark problem, and the best solution in these eight algorithms is selected as shown in the column SO in the table 3. For the heuristic technique proposed by Box [1], all the problems were examined with 300 iterations per trial. Starting with possible f_{\max} above the lower bound [3],[11], available channel number is gradually reduced in each trial and at the limiting conditions, out of 10 trials the best solution (minimum f_{\max}) for each problem is selected. The local search algorithm [6], CAP³ was examined as stated in [6] and was tested for 10 trials.

The simulated annealing approach described in [7], was implemented with simple flip flop move generation procedure and logarithmic cooling schedule. The genetic algorithm for CAP [8] was examined with population size = 100, probability of mutation $\alpha_m = 0.01$, and the probability of cross over $\alpha_x = 0.6$. For the genetic algorithm and the simulated annealing algorithm, The best results obtained in 10 trials were recorded for performance comparison.

In the algorithms in [5],[6] and [1] minimum number of channels needed (Minimum f_{\max}) can be directly found. But for the genetic algorithm (GA) and the simulated annealing (SA), for the given number of available channels, possible assignment is searched. Therefore for these two algorithms (GA and SA), taking the minimum f_{\max} obtained in the proposed algorithm as available number of channels for each problem, the percentage of number of assigned channels to the total number of calls, is listed for each problem in the last two columns of the table 4.

Thirteen CAP problems have been employed for performance evaluation. The first ten problems have been employed by many researchers [3],[5],[6],[10] as typical benchmark problems to test their channel assignment schemes. The last three problems were generated by increasing the adjacent and co-site separation constraints. All these problems correspond to a 21-cell system shown in fig. 1 and adjacent channel interference extends up to second layer cells. These problems impose different frequency separation constraints and inhomogenous demand traffic. In the table 2, a indicates the co-site separation and b indicates the adjacent cell separation needed. The Lower bound values for each problem were calculated using the methods in [3] and [11]. Initially Gamst[3] proposed the method in detail to determine the lower bound for the channel assignment problem. Further, a tighter lower bound was achieved for the problem 7 in [11]. In our discussion, except for the 7th bench mark problem, lower bound values for all other problems were determined using the methods in [3].

V. RESULTS AND DISCUSSION

Table 1 gives the three different demand traffics of the network and the problem specifications are shown in table 2. The table 3 compares the required minimum channel numbers obtained in four major algorithms while the average values of percentage of assignment for SA and GA are shown

in the table 4. The table 5 compares the relative computational time of different algorithms and the table 6 shows the effect of variation of the control frequency in the proposed algorithm.

Observe that four algorithms have achieved the lower bound, for the first six bench mark problems. One can observe that the lower bounds of these problems are the first level lower bounds given by $c_{ii} \times (M - 1)$ which depends only on the maximum demand per cell, M and the co-site separation, c_{ii} . However for these problems, both the simulated annealing algorithm and the genetic algorithm yield relatively poor results compared to the other four algorithms.

In order to test the above algorithms further, the bench mark problems 7..10 for which adjacent cell separation had been increased to 2, were used. In this case, most algorithms do not achieve the lower bound except the proposed algorithm. In the most stringent conditions described by the 10th bench mark problem, observe that the proposed algorithm outperforms all other algorithms but fails to achieve the lower bound by 11 channels as shown in the table 3.

In order to investigate the performance of all the algorithms even further, the adjacent cell separation, b is increased to 3 for the last three bench mark problems which reflect severe interference cases. Observe that all the algorithms fails to achieve the lower bounds but the proposed algorithm outperforms all the others.

Moreover, the computational complexity of the proposed algorithm is comparatively less. Table 5 compares the relative computational times of the six algorithms for the typical bench mark problem 8. The simulated annealing algorithm consumed the most time, while Sivarajan's non iterative sequential ordering algorithms consumed the least as shown in the table 5.

The other considerations for achieving the optimum solution in the proposed algorithm, is to investigate the effect of control frequency f_c . In all the cases studied above, the control frequency f_c was kept constant. When it is varied in a small range about the first level lower bound, improved results can be achieved. For instance, the table 6 shows the variation of f_{max} versus f_c for the proposed algorithm for a typical problem (No. 8). When f_c is equal to the first level lower bound value of 381, the f_{max} is 428. When f_c is increased to 400, the lower bound value of 427 is achieved. Therefore one can improve the performance by varying the values of f_c slightly around the first level lower bound.

VI. Conclusion

An efficient heuristic channel assignment algorithm employing a new approach in repetitive ordering of calls has been presented. The minimum channel numbers needed in the algorithm for the benchmark problems are more close to the lower bound values than the other existing algorithms. Simulation results for the bench mark problems show its ability to achieve the optimal or near optimal solutions in comparatively less computational time.

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Appendix

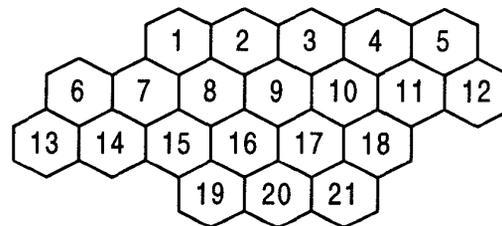


Figure 1 : A 21-cell system

Table 1: Three sets of Typical Demand Traffic for the cell system shown in Fig-01

Cell	NO.	1	2	3	4	5	6	7
Demand Traffic	d_1	8	25	8	8	8	15	18
	d_2	5	5	5	8	12	25	30
	d_3	10	11	9	5	9	4	5

Table 1 Continued

Cell	NO.	8	9	10	11	12	13	14
Demand Traffic	d ₁	52	77	28	13	15	31	15
	d ₂	25	30	40	40	45	20	30
	d ₃	7	4	8	8	9	10	7

Cell	NO.	15	16	17	18	19	20	21
Demand Traffic	d ₁	36	57	28	8	10	13	8
	d ₂	25	15	15	30	20	20	25
	d ₃	7	6	4	5	5	7	6

Table 2 : Problem Specifications

Prob. No.	a	b	Demand Traffic	LB [3],[11]
1	5	1	d ₁	381
2	7	1	d ₁	533
3	5	1	d ₂	221
4	7	1	d ₂	309
5	7	2	d ₂	309
6	7	2	d ₁	533
7	5	2	d ₁	427
8	5	2	d ₂	258
9	5	2	d ₃	71
10	7	2	d ₃	58
11	7	3	d ₂	337
12	7	3	d ₁	599
13	7	3	d ₃	85

where

a = Co-site channel separation;

b = Channel Separation between adjacent cells;

LB = Lower bound for the Minimum Channel Number.

Table 3 : Performance comparison of four major Algorithms with Minimum Channel Numbers needed

Prob. No.	Proposed Algo.	CAP ₃ [6]	SO [5]	Box [1]
1	381	381	381	381
2	533	533	533	533
3	221	221	222	221
4	309	309	309	309
5	309	309	310	309
6	533	533	533	533
7	428	433	447	445
8	258	263	270	263
9	71	75	78	72
10	69	69	74	70

Table 3 Continued

11	370	380	376	373
12	617	634	661	629
13	95	97	100	96

Table 4 : Performance Comparison with Genetic Algorithm and Simulated Annealing (Percentage of Assignment)

Prob. No.	No. of Available Channels	% of Assignment	
		Genetic Algo. [8]	Simulated Ann. [7]
1	381	64.4	28.1
2	533	68.0	24.1
3	221	55.3	26.3
4	309	59.6	23.8
5	309	52.1	21.3
6	533	64.9	21.2
7	428	57.6	24.1
8	258	50.2	21.0
9	71	50.1	38.3
10	69	54.1	35.1
11	370	48.1	22.4
12	617	57.6	23.1
13	95	59.8	32.8

where *Percentage of Assignment* = Number of assigned calls/ total number of calls to be assigned.

Table 5 : The relative computational time for various algorithms in solving a Typical Problem (No. 8)

CAP Algo.	Proposed Algo.	CAP ³ [6]	Box [1]	SO [5]	GA [8]	SA [7]
t	12.4	107.1	10.6	0.5	94.9	671.5

where t = Relative computational time in minutes

Table 6 : Variation of *f_{max}* versus *f_c* for the proposed algorithm in a Typical Problem (No. 8)

<i>f_c</i>	300	325	350	375	381	400	414	420
<i>f_{max}</i>	430	428	428	428	428	427	427	428

where *f_c* = Control Frequency;

f_{max} = Minimum value of maximum channel needed.