

Novel filter algorithm for removing impulse noise in digital images

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ABSTRACT

In this paper, we present a novel filter algorithm that is more capable in removing impulse noise than some of the common noise removal filters. The philosophy of the new algorithm is based on a pixel identification concept. Rather than processing every pixel in a digital image, this new algorithm intelligently interrogates a sub-image region to determine which are the "corrupted" pixels within the sub-image. With this knowledge, only the "corrupted" pixels are eventually filtered, whereas the "uncorrupted" pixels are untouched. Extensive testing of the algorithm over a hundred noisy images shows that the new algorithm exhibits three major characteristics. First, its ability in removing impulse noise is better visually and has the smallest mean-square error compared with the median filter, averaging filter and sigma filter. Second, the effect of smoothing is minimal. As a result, edge and line sharpness is retained. Third, the new algorithm is consistently faster than the median filter in all our test cases. In its current form, the new filter algorithm performs well with impulse noise.

Keywords: noise removal, filtering, pixel identification, impulse noise, digital images, mean-square error

1. INTRODUCTION

Practical digital images are often degraded in some manner to some extend that require algorithmic steps to reduce or eliminate the degradation effect. In essence, the objective of removing such degradation effect is to determine the output image $\hat{f}(x, y)$ such that it resembles the input image $f(x, y)$ as closely as possible^{1, 2}. In order to achieve this objective, image restoration algorithms must be appropriately chosen to deal with the type of degradation introduced at the input channels, transmission medium, sensor and/or digitizer. Common types of degradation are blurring, distortion, additive random noise such as Gaussian white noise and salt-and-pepper impulse noise, signal-dependent noise such as speckle, film grain noise and quantization noise³.

In general, image restoration algorithms are mathematical and complex. Within this category, there is a subset of algorithms that are simple and heuristic, and has been widely employed to perform deblurring and noise removal in both the spatial and frequency domains. Most of these algorithms are designed to deal with the more popular and practical types of signal-independent noise, e.g. Gaussian white noise, impulse noise, burst channel errors and noise with a geometric structure⁴.

Mathematically, the image degraded by additive random noise can be represented by the following equation:

$$g(x, y) = f(x, y) + \eta(x, y) \quad (1)$$

where $g(x, y)$ is a degraded image consisting of $f(x, y)$, the input image and an additive noise term $\eta(x, y)$. Therefore, the process of noise removal may be interpreted as given $g(x, y)$ and the knowledge of the statistical nature of $\eta(x, y)$, an approximation, $\hat{f}(x, y)$ of the input image can be determined. If $f(x, y)$ is also known, the difference between $f(x, y)$ and $\hat{f}(x, y)$ may be optimised in terms of minimum mean-square error or other form of error representation.

A good noise removal filter in theory would remove the additive noise distributions exactly, restoring the original image from the noisy image completely. To do this, the filtering algorithm must be specially designed to remove a particular noise distribution. In reality, no matter how well a noise removal filter is designed, the restored image always exhibits a certain degree of deviation in its pixel values from the original image, $f(x, y)$. Excessive deviation often renders the restored image useless. In other words, the restored image may be visually unacceptable if subjected to human inspection, or some of the original features may have been lost or distorted which significantly affect the success of the subsequent feature extraction

processes. Therefore, the problem of practical image restoration in the case of additive random noise is reduced to minimizing a chosen error function such that the restored image resembles as closely as possible to the original image objective and subjectively.

Over the years, many filtering algorithms have been developed in the spatial domain for removing strong, spikelike impulse noise. A good example is the median filter. Median filtering is a successful and effective technique for removing such spikelike components in a noisy image^{5, 6, 7, 8}. Apart from being able to remove impulse noise effectively, this non-linear filter algorithm has a known advantage of preserving most of the edge information when compared with some other filters such as averaging filter. In fact, although noise suppression is mostly achieved, a degree of signal distortion is still apparent⁸. This manifests itself as a small degree of edge blurring in the restored image. There are many forms of median filters^{7,8}. The following equation depicts a general form and the broad aspects of the algorithm. In general, a two-dimensional $(2N+1)$ by $(2N+1)$ median filter is defined by

$$\hat{f}(x, y) = \text{median}\{g(x+i, y+j) | i = -N, \dots, -1, 0, 1, \dots, N, j = -N, \dots, -1, 0, 1, \dots, N\} \quad (2)$$

where $\hat{f}(x, y)$ is the restored pixel at (x, y) which is defined as the median of the pixel values enclosed in a two-dimensional window of size $(2N+1)$ by $(2N+1)$ centred at $g(x, y)$. The restored image is obtained by applying equation (2) to all the pixels. If the image size of M by M , equation (2) is applied M^2 times to complete the filtering process. This philosophy of "blind" processing is not solely used in median filtering and has been proven effective in removing impulse noise and indeed other types of noise but is also capable of introducing a smoothing or blurring effect to the restored image. The reason being the filtering algorithm does not consider which high spatial frequency component is noise and which is not. All pixels are considered and treated in exactly the same way. This effect is not entirely undesirable if fine details in the image are to be removed before feature extraction and object segmentation, or small gaps in lines or curves are to be filled. However, the distortion may be unacceptable and may reduce the sharpness of lines, edges and boundaries which may eventually affect the accuracy of the subsequent feature extraction and recognition processes. Furthermore, it is well-known that only 20-30% of pixels in the image carry useful information. Blindly processing the whole image wastes a significant amount of computing resources and becomes unattractive in speed critical applications. Obviously, if only a selected sub-set of pixels is processed, considerable saving in computation would be expected given the algorithmic overhead for determining the selected sub-set is small enough. Keeping this overhead at a small enough level is critical to the success of this type of algorithms.

Apart from the median filter, filters such as averaging, sigma, minimum, maximum, box, and etc. are all employing the same strategy of processing all pixels in the image. Their major and only difference is in the way $\hat{f}(x, y)$ is calculated over the $(2N+1)$ by $(2N+1)$ window.

In this paper, we present a novel filter algorithm that is more capable in removing impulse noise than some of the common noise removal filters, yet without any speed penalty. The philosophy of the new algorithm is based on a pixel identification concept. Rather than processing every pixel in a digital image as in a normal "blind" filtering algorithm, this new algorithm intelligently interrogates a sub-image region to determine which are the "corrupted" pixels within the sub-image. With this knowledge, only the "corrupted" pixels are eventually filtered, whereas the "uncorrupted" pixels are untouched. The principle of the pixel identification algorithm evolves from three observations. Firstly, noise effect should be considered as a local effect rather than a global effect. In other words, the identification of pixel types must be performed locally (in terms of sub-image region) rather than globally (whole image). Secondly, each sub-image region must have its local criterion to differentiate noise "corrupted" pixels and "uncorrupted" pixels, and this criterion could be different between sub-image regions. Thirdly, the sub-image region size should not be fixed. Based on the assumption that the "corrupted" pixels are in minority, a pixel identification algorithm was developed, realised and evaluated over more than one hundred digital images corrupted by impulse noise at different signal-to-noise (SNR) ratio. Extensive testing of the algorithm over these noisy images shows that the new algorithm exhibits three major characteristics. First, its ability in removing impulse noise is better visually and has the smallest mean-square error compared with the median filter, averaging filter and sigma filter. Second, the effect of smoothing is minimal. As a result, edge and line sharpness is retained. Third, the new algorithm is consistently faster than the median filter in all our test cases. In its current form, the new filter algorithm performs well with impulse noise.

This paper is organized in the following manner: Section 2 - Overviews the concept and philosophy of the algorithm. Section 3 - Gives details of the new algorithm through a number of steps. The pixel identification algorithm will be described in full including the assumptions, the algorithmic steps and the signal flow of the algorithm. The realisation of the algorithm will also

be discussed in this section. Section 4 - Presents a performance evaluation of the new algorithm in terms of removing impulse noise from heavily degraded images and the smoothing effect of the algorithm. The results using average filter, median filter and sigma filter will be included for comparison. Their computing resource requirements will also be studied aiming to determine the overall cost-performance effectiveness of the new algorithm with respect to the other three filter algorithms. Section 5 - Concludes the merits and drawbacks of the new algorithm.

2. OVERVIEW OF THE NEW ALGORITHM⁹

The term "blind" mentioned in the previous section refers to the algorithms that process every single pixel in the digital image without giving due consideration to the nature of the pixel. The only difference between majority of the existing filtering algorithms is how each pixel is being processed. This class of techniques has two major drawbacks. Firstly, the algorithms operate on the "uncorrupted" pixels as well as the "corrupted" pixels creating undesirable deviations from their original pixel values. Secondly, it is well-known that only 20-30% of pixels of the image carry useful information, processing the whole image wastes a significant amount of computing resources. If only a selected sub-set of pixels are processed, considerable saving in computation would be possible.

The motivation of this research is based on the above observations and two further hypotheses. Our first hypothesis is that if we can identify the "corrupted" pixels, then we no longer need to process every pixel in the image. By not processing the "uncorrupted" pixels, useful information is preserved. The second hypothesis is that if the "corrupted" pixels are in minority and the algorithmic/computing overhead needed to determine the a priori knowledge of the pixel nature is not more than the processing time required for filtering the "uncorrupted" pixels, then we are likely to have a reduction in computing time.

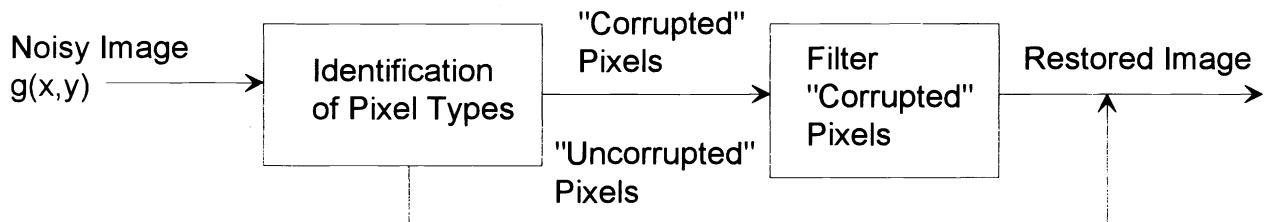


Figure 1 : Conceptual diagram of the new algorithm

Based on these two hypotheses, a number of strategies were studied. Since the noise distribution we are dealing with is strong impulse noise components that are higher than the image components, the simplest method for identifying a "corrupted" pixel is to threshold an image into a binary image by choosing an appropriate global fixed threshold value. In this case, pixels that are white (black) are classified as "corrupted" and pixels of the other value are classified "uncorrupted". Of course, this method assumes the noise distribution is mostly at one end of the gray level spectrum and the original image distribution is mostly at the other end of the spectrum, and there is a clear distinction between the two. In reality, noisy image seldom behave like this. In addition, if salt-and-pepper noise is considered instead of just white impulse or black impulse, global thresholding can only remove half of the noise content, leaving the other half untouched.

Due to the inherent deficiency of the global thresholding technique, a new algorithm was developed based on the following observations:

- *Localised noise effect*
Noise effect should be considered a local effect rather than a global effect. In other words, the pixel identification process must be focussed on the local variation of noise.
- *Variable local criterion*
When local noise effect is concerned, each local sub-image should have its appropriate threshold to differentiate "corrupted" or "uncorrupted" pixels, and this threshold could be different between different sub-images.
- *Variable sub-image size*
There should not be fixed sub-image size. The size of sub-image should be determined by the gray level distribution within that sub-image.

Derived from these observations, a new algorithm was formulated as a pixel identification process preceding the actual filtering. Figure 1 depicts the conceptual diagram of the new algorithm. The noisy image $g(x,y)$ is first pre-processed by a pixel identification algorithm. This algorithm aims to identify the "uncorrupted" and "corrupted" pixels. Details of this identification process will be discussed in the next section. After this process, every pixel in $g(x,y)$ would have a tag indicating which of the two classes it belongs to. The information concerning the locations of "corrupted" pixels is then sent to a standard filter operator which according to this information, process the "corrupted" pixels. This standard filter operator can be any of the common filter algorithms. In our test, median filtering has been used. There is no reason why for example sigma filter cannot be used as well. Eventually the restored image $\hat{f}(x,y)$ is determined by combining the results of the filtered pixels and the original "uncorrupted" pixels. Clearly, the performance of the algorithm relies on how the pixel type identification is performed.

3. IDENTIFICATION OF PIXEL TYPES

Assume that the "corrupted" pixels are in minority, the pixel identification algorithm is outlined in the following. First of all let us define the following terms:

- MIS: Maximum intensity spread is the maximum allowable intensity spread within a sub-image region.
- $S_i(m,n)$: Sub-Image i of size m by n where both m and n must be greater than 1.
- $I_i(m,n)$: Intensity spread within $S_i(m,n)$
- $S(m_0,n_0)$: Minimum sub-image is the minimum allowable sub-image size (m_0,n_0) . This value should not be smaller than the mask size of the filter.
- $M_i(m,n)$: Mean intensity of $S_i(m,n)$.

Assume MIS and $S(m_0,n_0)$ are chosen initially such that for a sub-image $S_i(m,n)$ with origin at (x_i, y_i) , $I_i(m,n)$ is given by

$$I_i(m,n) = \max_{\substack{y=0 \\ x=0 \\ y=n-1 \\ x=m-1}} \{g(x+x_i, y+y_i)\} - \min_{\substack{y=0 \\ x=0 \\ y=n-1 \\ x=m-1}} \{g(x+x_i, y+y_i)\} \quad (3)$$

If $I_i(m,n)$ is greater than MIS and $S_i(m,n)$ is greater than $S(m_0,n_0)$ then divide $S_i(m,n)$ into two equal but smaller sub-images using the following criterion:

C1: If $m \geq n$ then $S_{i+1}(m/2, n)$ else $S_{i+1}(m, n/2)$.

The above condition determines the intensity spread of $S_i(m,n)$ to indicate whether a further sub-division is necessary. The argument is that if $I_i(m,n)$ is larger than our chosen MIS, such large variation will not enable a correct decision of which are the "corrupted" pixels to be made. This is based on the assumption that the noise pixel intensity is different from the image intensity of the sub-image by MIS/2. The only exception in this case is when $S_i(m,n)=S(m_0,n_0)$ meaning that $S_i(m,n)$ should not be subdivided any further. The size of $S(m_0,n_0)$ corresponds to the filter size in the next processing stage simply because of the nature of the filtering operation.

From the above argument, if $I_i(m,n)$ is either less than MIS or $S_i(m,n)$ is equal to $S(m_0,n_0)$ then the algorithm proceeds to search for the "corrupted" pixels. This is achieved by determining the mean intensity $M_i(m,n)$ of $S_i(m,n)$ by equation (4), the thresholded $\bar{g}(x+x_i, y+y_i)$ of $S_i(m,n)$ by equation (5) and the "corrupted" pixels by equation (6).

$$M_i(m,n) = \frac{1}{2} \left\{ \max_{\substack{y=0 \\ x=0 \\ y=n-1 \\ x=m-1}} \{g(x+x_i, y+y_i)\} + \min_{\substack{y=0 \\ x=0 \\ y=n-1 \\ x=m-1}} \{g(x+x_i, y+y_i)\} \right\} \quad (4)$$

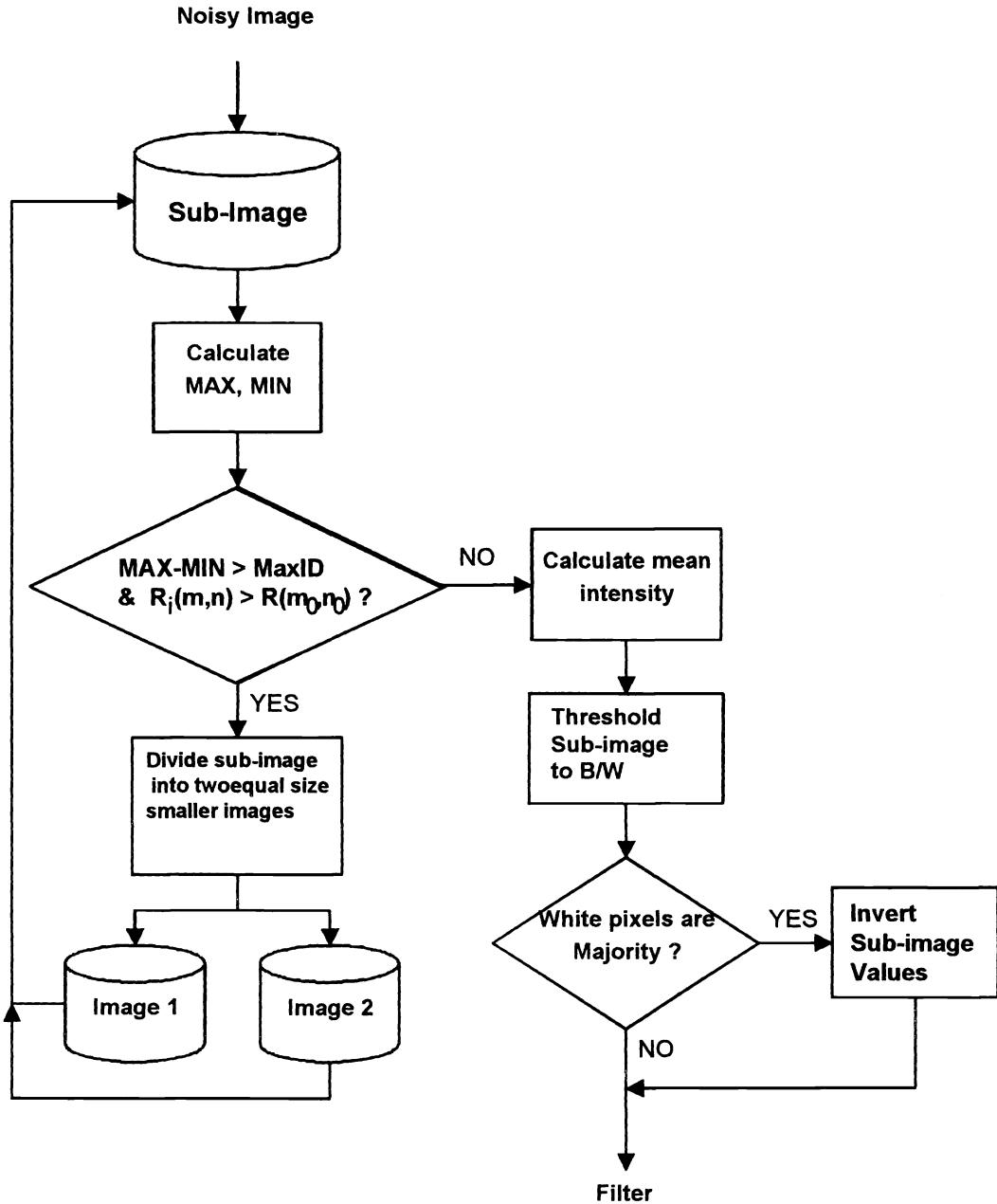


Figure 2 : Signal Flow Graph of the Pixel Identification Algorithm

$$\bar{g}(x+x_i, y+y_i) = \begin{cases} 1 & g(x+x_i, y+y_i) > M_i(m, n) \\ 0 & g(x+x_i, y+y_i) \leq M_i(m, n) \end{cases} \quad \text{for } x=0, \dots, m-1 \text{ and } y=0, \dots, n-1. \quad (5)$$

$$\text{Corrupted Pixels} = \begin{cases} g(x+x_i, y+y_i) \text{ where } \bar{g}(x+x_i, y+y_i) = 0 \text{ when } \sum_{y=0}^{n-1} \sum_{x=0}^{m-1} \bar{g}(x+x_i, y+y_i) \geq \frac{mn}{2} \\ g(x+x_i, y+y_i) \text{ where } \bar{g}(x+x_i, y+y_i) = 1 \text{ when } \sum_{y=0}^{n-1} \sum_{x=0}^{m-1} \bar{g}(x+x_i, y+y_i) < \frac{mn}{2} \end{cases} \quad (6)$$

The thresholding of the sub-image region by equations (4) and (5) is a standard technique. Equation (6) aims to pick-out the "corrupted" pixels which could be black or white determined by which type is the minority in the sub-image. This reflects our assumption of noise distribution is in minority. For most common filters, this assumption applies. However, we have also tested the algorithm with images that are so heavily corrupted that the noise pixels are no longer in minority. The new algorithm performed reasonably well even under this condition, except for some sub-image region where noise pixel concentration was high, unwanted "bright" clusters appeared to remain in the restored image. It should be noted that under this extreme condition, all the other common filters studied here failed to remove such "bright" clusters, and the severity of these noise clusters varies from algorithm to algorithm.

The new algorithm was realised in C++ and MS Windows environment using the following algorithmic steps. The signal flow of the algorithm is depicted in Figure 2.

1. Set the initial $S_i(m,n)$ to the size of the noisy image ($m=N$, $n=N$ & $i=0$);
2. Determine the maximum (MAX) and minimum (MIN) intensity levels within $S_i(m,n)$;
3. If $MAX-MIN > MIS$ and $S_i(m,n) > S(m_0, n_0)$ then goto Step 4, otherwise goto Step 5;
4. Divide the sub-image into two sub-images of equal size. For each sub-image, repeat Steps 2 and 3;
5. Calculate the mean intensity within the sub-image. Convert the pixels within the sub-image to black and white using the mean intensity as a threshold;
6. Count the number of black pixels and the number of white pixels in the sub-image.
7. Whichever type of pixels is in minority, mark them as "corrupted".
8. Spatial information of the "corrupted" pixels are passed to a median filter.

4. PERFORMANCE EVALUATION

The evaluation of algorithmic performance discussed in this section is based on measuring the mean-square error between the restored image and the original "uncorrupted" image. The images used in the evaluation all have 256 gray levels and in the case of the noisy image, it is degraded by impulse noise at -50 dB signal-to-noise ratio. Essentially, the evaluation is focussed on the effect of

- filtering heavily degraded images;
- smoothing;
- computational requirement.

Comparison is made between the new filter algorithm and median filter, average filter and sigma filter. A window size of 5×5 is used in all cases, and a 5×5 median filter is used as the filter core of the new algorithm. For the new algorithm, an $MIS=32$ is used throughout.

4.1 Heavily degraded images

Table 1 depicts the mean-square errors of the images restored by the various filter algorithms with respect to the original. The original, noisy and restored images are depicted in Figure 3.

SNR (dB)	Noisy Image	Averaging filter	Median filter	Sigma filter	New filter (5,32)
-50	6243.88	1139.73	982.95	747.10	477.59
	5.47	1	0.86	0.65	0.42

Table 1 : Mean-square errors of "Mickey" image degraded by impulse noise and restored by various filters

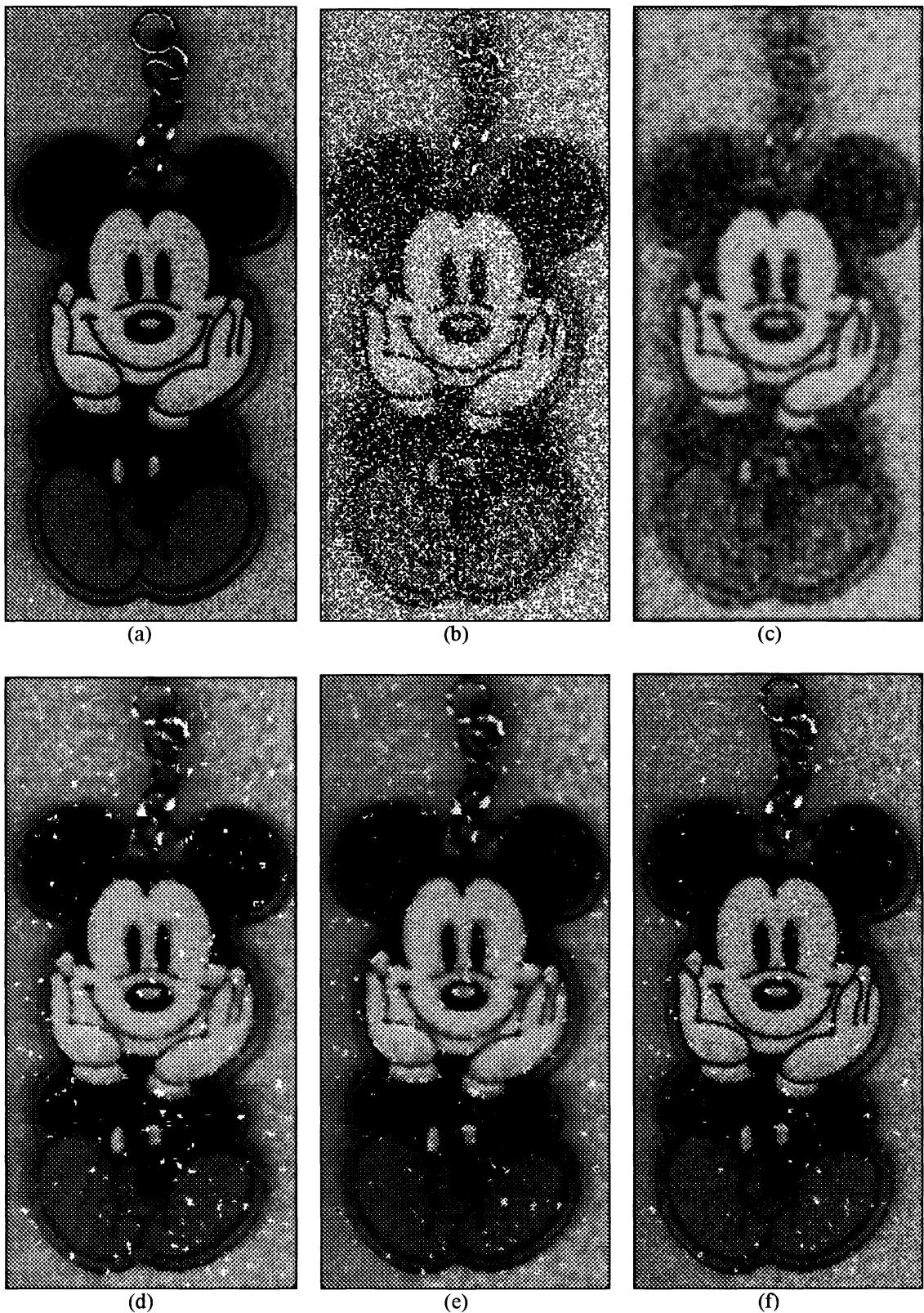


Figure 3 : Comparison of restored "Mickey" images by different filters; (a) original image; (b) image heavily degraded by impulse noise; (c) image restored by averaging filter; (d) image restored by median filter; (e) image restored by sigma filter; (f) image restored by new filter.

The mean-square errors shown in the first row of Table 1 are objective evaluation of the intensity differences between corresponding pixels of the original image and the restored image in each case. The noisy image obviously has the largest mean-square error. In general, all the filtering algorithms satisfy the requirement of removing noise from the noisy image. Out of the four filter algorithms tested, averaging filter gives the largest mean-square error, followed by the median filter and sigma filter in this order. The interesting point of this result is that although median filter is designed to remove impulse noise, whereas sigma filter is designed to remove Gaussian noise, the mean-square error here shows that sigma filter is in fact more suitable to remove impulse noise. This result is consistent in our other tests. The new filter has the smallest mean-square error, roughly only two-third of the sigma filter, half of the median filter and only 0.42 of the average filter. The second row of Table 1 depicts the value of the mean-square errors relative to the mean-square error of the averaging filter. It clearly shows that the new algorithm is most suitable for removing impulse noise.

From Figure 3, we can observe that if we simply perform a visual subjective measurement of the restored images, the results agreed with the objective measurement as given in Table 2. Figure 3(a) depicts an image of a Mickey key-ring on a relatively smooth background. Lines and edges of this image are sharp and clearly defined. Figure 3(b) shows the "Mickey" image corrupted by impulse noise. Figure 3(c)-(f) present the restored images by average filter, median filter, sigma filter and new filter respectively.

In the case of the averaged image, the restored image is severely distorted (Figure 3(c)). Although most of the noise components are removed, for instance on the top left-hand corner, the region is reasonably smooth after filtering. However, the edge and line sharpness is reduced at the same time to such an extend that the edges and lines in the bottom half of the image are almost indistinguishable from its smooth areas. This result agrees with the high mean-square error of averaging as given in Table 1.

In the case of the image restored by median filter (Figure 3(d)), the restored image is almost acceptable except a large number of "bright" clusters remain in the image. These "bright" clusters correspond spatially to the high concentration of noise pixels in the noisy image. Such result can be explained that when the median filter calculates the median pixel value of a 5×5 window, the median of the local window is actually a noise pixel. This could be due to two reasons: first, unevenly high local concentration of noise pixels; and second, the overall noise pixel is in majority. Since the noise pixels are normally distributed, it is more likely that the latter is the case.

For the image restored by the sigma filter (Figure 3(e)), the overall visual appearance is acceptable except that a number of "bright" clusters again remain in the image. However, when this is compared with the median filter case in Figure 3(d), the effect is not as serious. This probably accounts for the reason why the sigma filter has a lower mean-square error than the median filter. Apart from that, both the median and sigma filter caused a small degree of edge/line distortion.

In the case of the new filter, a even smaller number of "bright" clusters remain. In addition, the edge/line distortion is the smallest, which can be detected visually upon close comparison with the original in Figure 3(a). This corresponds to the fact that the new filter algorithm gives the smallest mean-square error. Similar results hold in the case when the original image is lightly corrupted by impulse noise. In this case, the mean-square error difference between the median filter and sigma filter is minimal and their visual appearances look similar. Besides, the "bright" clusters that are apparent in the heavily corrupted case do not exist in all those cases, and all of them resemble closely to the original image except for the averaged image⁹.

4.2 Smoothing effect

This evaluation aims to identify how much distortion or smoothing a filter will introduced when undertaking the noise filtering process. The image used for this evaluation purpose is the original "Mickey" image without any noise added. The mean-square errors for the filter algorithms used are given in Table 2, and the resultant images are depicted in Figure 4.

Averaging filter	Median filter	Sigma filter	New filter (5,32)
406.62	291.45	293.49	144.03
1	0.71	0.72	0.35

Table 2 : Mean-square errors of filtering the uncorrupted "Mickey" image by various filters

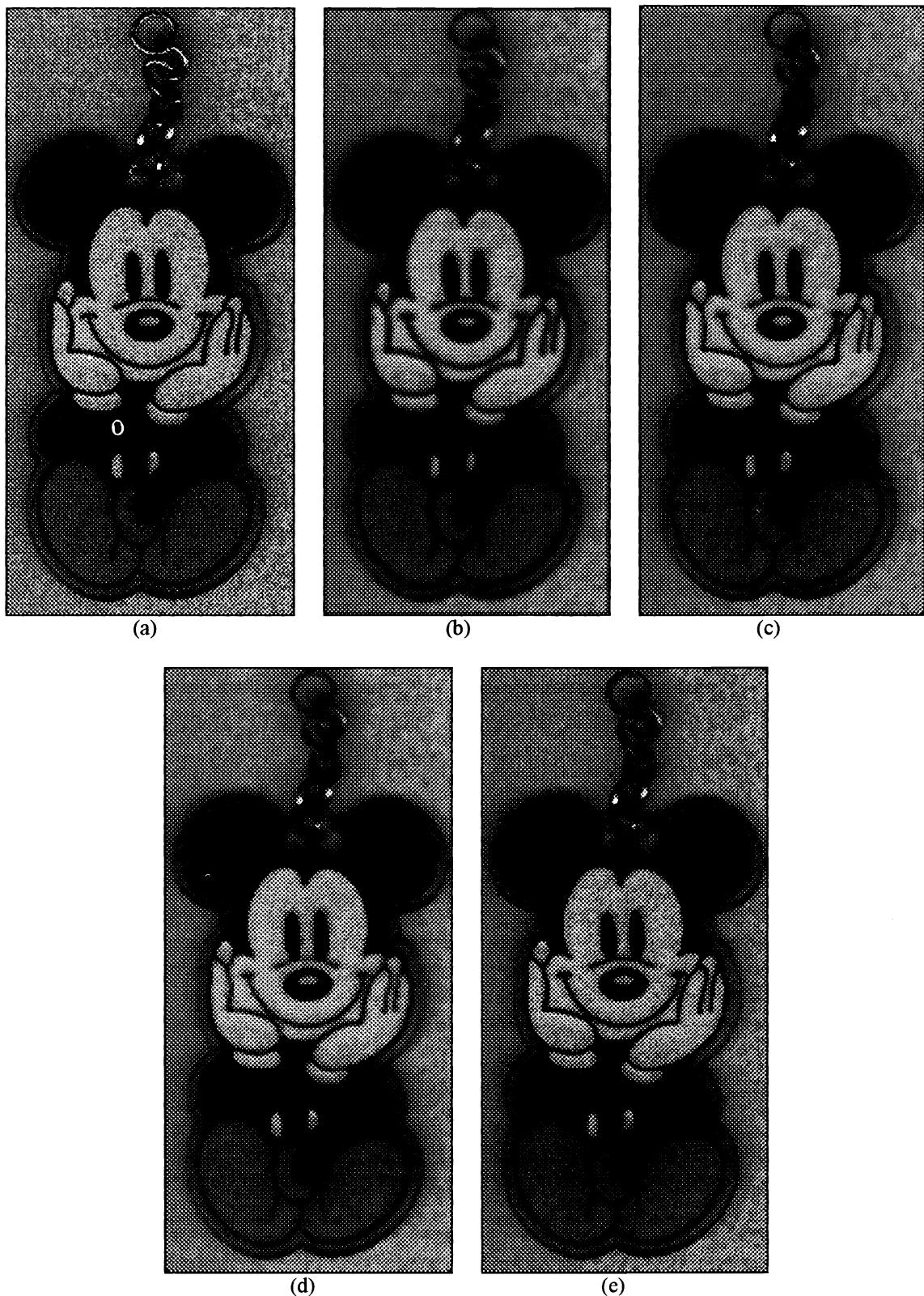


Figure 4 : Comparison of restored "Mickey" images by different filters; (a) original image; (b) image filtered by averaging filter; (c) image filtered by median filter; (d) image filtered by sigma filter; (e) image filtered by new filter.

From Table 2, we can observe that all the filter algorithms tested introduce a certain degree of smoothing to the image itself. Obviously, the degree of smoothing depends on the algorithm. When the smoothing effect of each filter algorithm is evaluated objectively, the new filter still gives the smallest mean-square error. This indicates the smoothing effect of the new algorithm is the least. The small difference between the median and sigma filter ranks them equal in this case. Their actual mean-square error is twice that of the new algorithm as given by the relative figures depicted in the second row of the table. The mean-square error of the averaging filter is almost three times higher than the new algorithm. From this table, we can deduced that the smoothing effect of the new filter is the smallest, whereas the effect of the median and sigma filter are similar but worse than the new filter. Again, the averaging filter comes out worst.

Subjective evaluation of the smoothing effect caused by filtering useful image information seems to agree with the above deduction. From Figure 4, the smoothing effect is rather obvious in the averaging case. The edge and line sharpness is lost in this case, but other areas appear to be smoother than before. However, trying to differentiate the smoothing effect caused by the other three algorithms is a slightly more difficult task. The smoothing effect of the median and sigma filter is almost indistinguishable. Both filtered images show a small degree of smoothing but not severe. In the case of the new algorithm, the actual visual difference between the original image and the filtered image is small. Difference in edge sharpness can only be detected upon close inspection. This is apparent in features like Mickey's mouse and hands. As a result, we can conclude that the smoothing effect of the new algorithm is minimal and its also the smallest among the filter algorithms discussed here. This can be explained by the inherent nature of the algorithm of which only "corrupted" pixels are processed, as such, majority of the pixels are not filtered and therefore the major features retained.

4.3 Computing resource evaluation

For the averaging, median or sigma type of filters, the number of computations required to completely filtering an M by M image is proportional to M^2 and a function of $(2N+1)$ if $(2N+1)$, which is the size of the filter window. For example in the case of median filtering, this figure is $M^2((2N+1)\ln(2N+1))$ if the median is determined by a quick sort. In the case of the new filtering algorithm, as only the "corrupted" pixels are processed, the dependency between the number of computations and the size of the image is no longer the same. A lightly degraded image will have less noise pixels overall, and hence less number of "corrupted" pixels will be identified and processed. A heavily degraded image on the other hand will have a larger number of "corrupted" pixels for identification and processing. Therefore, the computing time of the new algorithm depends on the number of "corrupted" pixels rather than the overall image size. In other words, if the noise pixels are in minority, this number cannot be larger than $\frac{1}{2}M^2$. However, the new algorithm has a computing overhead in the identification of pixel types. As long as this computing time is less than half of the total computing time of an equivalent "blind" filter, the new algorithm will have a computing advantage over these filter algorithms, otherwise, the performance gain will be offset by the slower identification time.

By quoting the result without detail proof⁹, the total computing time for the new algorithm using a $(2N+1)$ by $(2N+1)$ median filter core is given by equation (7).

$$\begin{aligned} f(2N+1) &= \left\{M^2 - 2^{k+1}\right\}\left\{\frac{1}{2}((2N+1)\ln(2N+1)) - 1\right\} - 3 \cdot 2^{k+1} + 4 \\ &= 2^k \left\{ \left[\frac{1}{2}((2N+1)\ln(2N+1)) - 1 \right] \left[(2N+1)^2 - 2 \right] - 6 \right\} + 4 \end{aligned} \quad (7)$$

$$\frac{M^2}{2^k} \leq (2N+1)^2 \quad (8)$$

where k is the number of iterations given by equation (8). Equation (7) is always positive, as long as $(2N+1)$ is larger than 3, implying that the worst case computing requirement of the new algorithm is always better than the median filter for $(2N+1) > 3$. Similar results are expected when comparing with the sigma filter, but not with the averaging filter. The computing requirement of the averaging filter is much less than the median and sigma filters. Although similar relationship is expected to hold with the averaging filter, the identification process of the new algorithm will be expect to be longer than calculating the average of $(2N+1)^2$ pixels over $\frac{1}{2}M^2$ pixels.

In practice, after extensive experimentation with over one hundred digital images and measuring the actual computing time, the following relationship between the averaging filter, median filter, sigma filter and new filter is established for $S(m_0, n_0) = 5$ ($2N+1=5$) and $MIS=32$. In the worst case, using the averaging filter as a reference again, the median filter is 5.25 times slower than the averaging filter, the sigma filter is 4.43 times slower than the averaging filter and the new filter is 2.75 times slower than the averaging filter. The difference between lightly and heavily corrupted images is minimal.

5. CONCLUSION

The new filter algorithm is no doubt the best performed filter among all those considered in this paper. In general, we can conclude that the new filter algorithm is more capable in removing impulse noise than averaging, median and sigma algorithms. The new algorithm consistently gives the smallest mean-square error among the group of algorithms in question. Subjective evaluation also shows that the new algorithm has a slightly better visual appearance due to the fact that noise pixels are more effectively removed. In terms of computing resource requirement, the computing speed of the new algorithm is determined by the signal-to-noise ratio rather than the overall image size. In theory, the total computing resource required for identifying pixel types and filtering the "corrupted" pixels is always less than that of median filter for $(2N+1) > 3$. This is under the condition that if the filtering core in the new algorithm is also a median filter. Extensive tests on digital images of different SNR levels show that in the worst case in practice, the new algorithm is about twice as fast as the median filter. Although the averaging algorithm is the fastest in this case, its poor filtering accuracy excludes it from being used practically. Furthermore, the new filter algorithm has the unique characteristics of being able to retain edge and line sharpness while others are unable to do so. This is due to the inherent selective property of the new algorithm. As "uncorrupted" pixels are not processed at all, the new algorithm gives the least distorted restored image in all the test cases. In its current form, the new algorithm is proven to be quite effective in removing impulse noise. Further detailed tests are still required to test the behaviour of the algorithm when the image is corrupted by Gaussian white noise. Initial tests have shown that the new algorithm is equally promising in the case of Gaussian white noise. Furthermore, the method of interrogating local sub-image pixel value can be studied in terms of understanding the more exact effect of heavily corrupted cases where the noise distribution is no longer in minority. So far, the indication from our results is that most traditional filter algorithms will fail when this is the case, however, the new algorithm seems to be least affected by this as shown in Section 4.1. Detail investigation of this aspect may improve the performance of the algorithm under this condition.

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