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DESIGN AND ANALYSIS OF DISCONTINUOUS MODE SERIES RESONANT CONVERTER

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Abstract - This paper aims at designing a low power, off-line, high efficiency, complex in size, twin-capacitors half-bridge series resonant converter. It gives steps by steps design procedures of the discontinuous mode series resonant converter starting from power stage circuit design to feedback loop control design. The small signal analysis is based on the injected-absorbed current method.

I. INTRODUCTION

In the design process of a low power, off-line, high efficiency converter, the size and component counts are the critical factors which are concerned by many designers. Twin-capacitors half-bridge series resonant converter [1] provides a way in which bulky components can be significantly reduced. This paper aims at the design and analysis of the twin-capacitors half-bridge resonant converter running in the discontinuous mode of operation. Steps by steps design procedures for the power stage and the small signal analysis, which is based on the injected-absorbed current method [2,3], for the loop control are given. Computer simulation and power circuit prototype have been used to verify the process. The captured waveforms will also be discussed.

II. POWER STAGE

Comparing with the traditional half-bridge series resonant converter, Fig. 1, the twin-capacitors half-bridge series resonant converter, Fig. 2, uses two very small size resonant capacitors, Co, at the positions of the input bulk capacitors, Cin. Actually the Thevenin equivalent circuits at position X for both Fig. 1 and Fig. 2 are the same, provided that Cin is much greater than Co. And the equivalent resonant capacitor, Cr, is equal to 2Co.

There are three modes of operation in the full-wave discontinuous mode:

(1) time interval 0<t<To; see Fig. 3
where To = half of resonant period
Taking Laplace transform,
\( \frac{e-u}{2} - V_{Cr}(s) - sLrI_{Lr}(s) = 0 \)  
where \( I_{Lr}(0) = 0 \)

(2) time interval To<t<2To; see Fig. 4
\( (e-u) - V_{Cr}(s) - sLrI_{Lr}(s) = 0 \)  
where \( I_{Lr}(To) = 0 \)
\( I_{Lr}(s) - sC_{r}V_{Cr}(s) - V_{a} \)
where \( V_{Cr}(To) = V_{a} \)

(3) time interval 2To<t<Ts; see Fig. 5
where \( Ts = \) half of the switching period
\( I_{Lr3} = 0 \)
\( V_{Cr3} = V_{m} \)

Solving for equation (1) to (6) and taking the inverse Laplace transform, one can find that:

(1) \( I_{Lr1} = \frac{(e-u)}{2} \sin \omega_{o}t \)
where \( e = \frac{V_{m}}{2} \); \( u = \) reflected output voltage
\( I_{pk1} = \frac{e}{2}(1+M) \)
i.e. peak current at \( \omega_{o}t = \pi/2 \)
\( V_{Cr1} = (e-u) - (e-u-V_{m}) \cos \omega_{o}t \)
\( V_{pk1} = 2e \)
i.e. peak voltage at \( \omega_{o}t = \pi \)
(2) time interval $0 < t < T_o$;
$$i_{L2} = \frac{(e + u - V_o)}{Z_o} \sin \omega_o(t - T_o)$$
$$I_{pk2} = \frac{e}{Z_o(1-M)}$$
i.e. peak current at $\omega_o t = 3\pi/2$
$$V_{CR2} = (e + u) - (e + u - V_o)\cos \omega_o(t - T_o)$$
$$V_{pk2} = 2u$$ i.e. peak voltage at $\omega_o t = 2\pi$
(3) time interval $2T_o < t < T_s$
$$i_{L3} = 0$$
$$V_{CR3} = 2u$$

Fig. 4 shows the computer simulated waveforms.

Fig. 4 Simulated waveforms of half-bridge series resonant converter

III. DC CONVERSION RATIO AND AVERAGE OUTPUT CURRENT

By taking the average resonant inductor current over half switching period, it can be found that:
$$i_o = \frac{1}{T_o} \int_0^{T_o} i_{L1} dt + \frac{1}{T_o} \int_0^{2T_o} i_{L2} dt$$
$$\Rightarrow i_o = \frac{\pi}{2} \frac{e}{Z_o}$$
where $f_n = f_s/f_o$

And substituting $i_o = u/R_o$ into equation (18), one gets
$$M = \frac{e}{u} = \frac{\pi}{2} \beta$$

where $\beta = R_o/Z_o$

Fig. 5 shows the plot of $M$ as a function of $f_n$ with different values of $\beta$. One can put the max. (or min.)
value of line (or load) condition on the plot for design purpose, with $f_s$ less than 0.5 $f_o$.

Fig. 5 Plot of voltage conversion ratio of discontinuous mode full-wave half-bridge SCR

From equation (17), by substituting $V_m = -2u$, it is found that
$$V_o = \frac{u}{4f_s CR_o}$$

Using the same approach of finding $I_o$, one can find the average input current as
$$I_m = \frac{1}{T_s} \int_0^{T_s} i_{L1} dt + \frac{1}{T_s} \int_0^{2T_o} i_{L2} dt$$

By substituting $V_a = 2e$, the value of $V_m$ is
$$V_m = -\frac{u^2}{4f_s CR_o}$$

The values of $V_m$ and $V_a$ are used for determining the small signal in later section.
IV. SMALL SIGNAL ANALYSIS

The output current, io, that is derived from equation (17), is being injected into the output loading, Fig. 6.

\[ i_o = 2fs\left\{ \frac{1}{2\omega_o}[(e - u - Vm)(1 - \cos \omega_o t \alpha) - (e + u - Vm)(1 - \cos(2\pi - \omega_o t \alpha))] \right\} \]

where \( \alpha = \frac{1}{\omega_o} \cos^{-1}\left( \frac{e - u - Vm}{e - u + Vm} \right) \)

The total differentials of the output current \( \frac{di_o}{dt} \) and control signal \( \frac{dV}{dt} \) as

\[ \frac{di_o}{dt} = \frac{\partial i_o}{\partial t} dt + \frac{\partial i_o}{\partial u} du + \frac{\partial i_o}{\partial V} dV \]

Taking the Laplace transform of (25), one gets

\[ I_o(s) = AF_o(s) - BV_o(s) + CE(s) \]

where \( A = \frac{\partial i_o}{\partial t}, B = \frac{\partial i_o}{\partial u}, C = \frac{\partial i_o}{\partial V} \)

The numerical values of \( A, B \) and \( C \) can be obtained by substituting the dc values of \( e, u, f_s, Vm \) and \( V_o \). Now changing (26) into the form of

\[ E_{out}(s) = \frac{A}{B} F_o(s) + \frac{C}{B} E(s) = I_o(s)Z_{out} + V_o(s) \]

where \( Z_{out} = \frac{1}{B} \)

Fig. 7 shows the equivalent circuit of equation (27).

V. DESIGN EXAMPLE

An off-line twin-capacitors half-bridge series resonant converter has been built. The specification of the converter is:

- \( V_{in} = 130 \text{ Vdc} \)
- \( V_o = 5 \text{ V} \)
- \( R_o = 2.5 \text{ ohm} \)
- resonant freq. = 700 KHz
- operating freq. = 130 KHz
- resonant cap. = 12.8 nF
- resonant inductor = 8 \mu H
- gain of VCO = 100 KHz/V

The measured waveforms are shown in Fig. 8.

Fig. 9 shows the simulated Bode plot of the open loop output-to-control transfer function. Fig. 10 and Fig. 11 show the block diagram of vco and converter, and the simulated and measured Bode plot of the open loop output-to-vco transfer function, respectively.
VI. CONCLUSION

The power stage of the discontinuous mode series resonant is easy to analyse. The DC conversion ratio is a linear function of normalized frequency. The small signal transfer function of the converter is in first order. Although the injected-absorbed current method is not an exact method to analyse the small signal of the converter, it gives an enough accurate result, and fast and simple way to handle the complexity of the dynamic analysis.

VII. REFERENCES