ERROR PROBABILITIES OF SYNCHRONOUS DS/CDMA COMMUNICATIONS OVER MULTIPATH RAYLEIGH FADING CHANNELS *

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***Abstract* — This paper derives error probabilities for BPSK synchronous DS/CDMA communications over multipath Rayleigh fading WSSUS channels using a RAKE receiver. Both cases of perfectly and approximately synchronized signal transmission are considered. Numerical results are presented.**

I. INTRODUCTION

Recently, synchronous DS/CDMA techniques on multipath fading channels have received considerable interest because in addition to exploiting the multipath diversity it is possible to increase the system capacity by reducing the multiple-access interference through employing optimized signature sequences having small crosscorrelations within a small region around the origin [1]. Various sets of such sequences have been proposed, e.g., [2]-[4]. In evaluating the performance and estimating the system capacity, however, analytical techniques in previous published literature have been available only for asynchronous DS/CDMA over multipath fading channels, e.g., [5], and synchronous DS/CDMA on AWGN channels [6], [7]. In this paper, we derive the error probability of BPSK synchronous DS/CDMA communications on multipath Rayleigh fading WSSUS channels using a RAKE receiver for multipath diversity reception. Both cases of perfectly and approximately synchronized transmission are considered. The present work thus enables computation of the performance and estimation of the system capacity for synchronous DS/CDMA using optimized signature sequences.

II. SYSTEM MODEL

The system under consideration consists of K users. Suppose that the receiver is intended to detect the user-L signal. For the user ℓ $(1 \le \ell \le K)$, the transmitted signal is $s_\ell(t) = \mathrm{Re}\{u_\ell(t)\exp(j2\pi f_c t)\}$ where f_c is the carrier frequency and

$$u_\ell(t) = e^{j\varphi_\ell}\sqrt{2P_\ell}\sum_{i=-\infty}^{\infty} b_i^{(\ell)}\Psi_\ell(t - T_\ell - iT_b). \qquad (1)$$

In (1), φ_ℓ is the carrier phase, P_ℓ is the signal power, $b_i^{(\ell)} \in \{+1,-1\}$ is the i th bit, T_b is the bit duration, $T_\ell \in [0, T_b)$ is the time delay between bit boundaries of the user-ℓ and the user-L signals, and $\Psi_\ell(t)$ is the spectral-spreading waveform defined as follows. Let $(a_0^{(\ell)}, a_1^{(\ell)}, \ldots, a_{N-1}^{(\ell)})$ be the user-ℓ signature sequence of length N satisfying $a_n^{(\ell)} \in \mathbb{R}$ and $\sum_{n=0}^{N-1} a_n^{(\ell)2} = N$. Then

$$\Psi_\ell(t) = \sum_{n=0}^{N-1} a_n^{(\ell)}\psi(t - nT_c) \qquad (2)$$

where $T_c = T_b/N$ is the chip duration, and $\psi(t)$, time-limited within $[0, T_c)$ and satisfied $\int_0^{T_c}\psi^2(t)dt = T_c$, is the (real) chip waveform. Since synchronized transmission is considered, it is assumed that bit boundaries of all K signals fall within a time window of $\mathcal{L}T_c$ seconds where $\mathcal{L}$ is a nonnegative integer. In perfectly synchronous DS/CDMA, one gets $\mathcal{L} = 0$ so that $T_\ell = 0 \ \forall \ell$. In approximately synchronized transmission, $\mathcal{L}$ is nonzero, so that (i) $T_L = 0$, and (ii) if $\ell \ne L$, T_ℓ follows triangular distribution over $[-\mathcal{L}T_c, \mathcal{L}T_c]$. For the sake of later use, let $k_\ell \triangleq \lfloor T_\ell / T_c \rfloor$ and $T_o^{(\ell)} \triangleq T_\ell \bmod T_c$. It can be shown that, for $\mathcal{L} \ne 0$ and $\ell \ne L$,

$$\Pr\{k_\ell = n\} = \begin{cases} (2\mathcal{L})^{-1}(2\mathcal{L}-2n-1) & n \in \{0,1,\cdots,\mathcal{L}-1\} \\ (2\mathcal{L})^{-1}(2\mathcal{L}+2n+1) & n \in \{-\mathcal{L},-\mathcal{L}+1,\cdots,-1\} \\ 0 & \text{otherwise} \end{cases}$$

and $T_o^{(\ell)}$ is uniformly distributed over $[0, T_c)$.

The signal $s_\ell(t)$ is transmitted via a multipath Rayleigh fading WSSUS channel with

$$\tfrac{1}{2}E\{h_\ell(\tau)h_\ell^*(\tau')\} = Q_\ell(\tau)\delta(\tau' - \tau) \qquad (3)$$

where $h_\ell(\tau)$ is the low-pass equivalent channel impulse response and $Q_\ell(\tau)$ is the delay power spectrum. It is assumed that (i) all K channels are statistically independent, (ii) all channels are slowly varying relative to T_b, (iii) $Q_\ell(t) = 0$ for $t < 0$ and $t > T_M^{(\ell)}$ where $T_M^{(\ell)}$ is the

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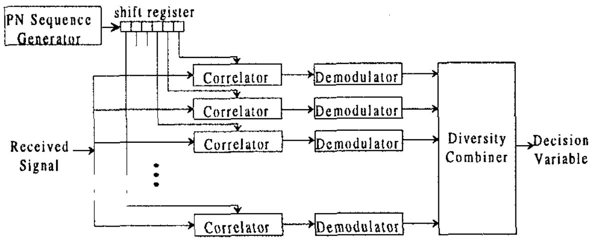

Fig. 1. The RAKE receiver.

maximum delay spread, and (iv) $T_M^{(\ell)}$, $\ell = 1, \ldots, K$, are small compared to T_b. The receiver input is given by $r(t) = \mathrm{Re}\{v(t)\exp(j2\pi f_c t)\}$ where

$$v(t) = n(t) + \sum_{\ell=1}^{K} \int_{0^-}^{T_M^{(\ell)}} u_\ell(t-\tau) h_\ell(\tau) d\tau. \qquad (4)$$

The complex AWGN $n(t)$ has a one-sided power spectral density η, i.e., $\frac{1}{2}E\{n(t)n^*(t+\tau)\} = \eta \cdot \delta(\tau)$.

The RAKE receiver under consideration (Fig. 1) consists of multiple branches followed by a combiner. A set of time-delayed uniformly T_c-spaced user-L signature sequences is used in the correlation, so that each branch output represents signal energy arrived from multipaths within a particular T_c-width time segment of $h_L(t)$ [8]. Let $\mathcal{M}$ be the number of branches. In the q th branch ($0 \le q \le \mathcal{M}-1$), a matched filter matched to $\Psi_L(t)$ delayed by qT_c seconds is used to process $s(t)$. Coherent detection followed by maximal ratio combining (MRC) on all branch outputs is assumed. Without loss of generality, we analyze the case of detecting $b_0^{(L)}$. The q th branch output is $Z_q = \mathrm{Re}\{X_q \cdot \exp(-j\phi_q)\}$ where ϕ_q is some phase introduced due to coherent detection, and

$$X_q = T_c^{-1} \int_{qT_c}^{T_b + qT_c} v(t) \Psi_L(t - qT_c) dt. \qquad (5)$$

After some algebraic manipulations we get (Appendix 1)

$$X_q = \sum_{\ell=1}^{K} \sqrt{2P_\ell} \sum_{m=0}^{M_\ell - 1} g_m^{(\ell)} H_{q,m,\ell} + \sum_{k=0}^{N-1} a_k^{(L)} N_{k+q} \qquad (6)$$

where (i) N_k's are independent zero-mean complex Gaussian random variables with $E\{N_k N_k^*\} = 2\eta/T_c$, (ii) $M_\ell \triangleq 1 + \lceil T_M^{(\ell)}/T_c \rceil$, (iii) $g_m^{(\ell)}$, $m = 0, \ldots, M_\ell - 1$, are jointly zero-mean correlated complex Gaussian random variables conditioned on $T_o^{(\ell)}$ with

$$E\{g_m^{(\ell)} g_n^{(\ell)*}\} = \int_{0^-}^{T_M^{(\ell)}} 2Q_\ell(\tau) W(mT_c - T_o^{(\ell)} - \tau) W(nT_c - T_o^{(\ell)} - \tau) d\tau \qquad (7)$$

but $g_m^{(\ell)}$ and $g_n^{(\ell')}$ are independent if $\ell \ne \ell'$, and (iv)

$$H_{q,m,\ell} = \sum_{i \in \{-1,0,1\}} b_i^{(\ell)} C_{L,\ell}(q - k_\ell - m - iN). \qquad (8)$$

In (7) and (8),

$$W(\tau) = T_c^{-1} \int_0^{T_c} \psi(t) \psi(t+\tau) dt \qquad (9)$$

and

$$C_{L,\ell}(k) = \begin{cases} \sum_{n=0}^{N-1-k} a_{n+k}^{(\ell)} a_n^{(L)} & 0 \le k < N-1 \\ \sum_{n=0}^{N-1+k} a_n^{(\ell)} a_{n-k}^{(L)} & -(N-1) < k < 0 \\ 0 & |k| \ge N \end{cases} \qquad (10)$$

Since $T_M^{(L)}$ is small compared to T_b so that M_L is small relative to N, it follows that $C_{L,L}(q-m+N)$ and $C_{L,L}(q-m-N)$ are small compared to $C_{L,L}(q-m)$. Therefore,

$$X_q \approx \sqrt{2P_L} b_0^{(L)} N g_q^{(L)} + \sum_{m=0, m \ne q}^{M_L - 1} \sqrt{2P_L} b_0^{(L)} g_m^{(L)} C_{L,L}(q-m) + \sum_{\ell=1, \ell \ne L}^{K} \sqrt{2P_\ell} \sum_{m=0}^{M_\ell - 1} g_m^{(\ell)} H_{q,m,\ell} + \sum_{k=0}^{N-1} a_k^{(L)} N_{k+q}. \qquad (11)$$

Since the receiver is intended to recover $b_0^{(L)}$, the first term of (11) is the desired signal contribution so that the q th branch of the RAKE receiver can be viewed as extracting the signal energy associated with "$g_q^{(L)}$". Appendix 1 shows that $g_q^{(L)} = 0$ for $q < 0$ and $q > M_L$ where $M_L = 1 + \lceil T_M^{(L)}/T_c \rceil$; that is, there are at most M_L significant "$g_q^{(L)}$". It is therefore possible to fully utilize the multipath diversity gain by using a RAKE receiver with $\mathcal{M} = M_L$ branches. This particular case is considered in this paper. Since it is assumed that the RAKE receiver performs coherent detection plus MRC, the RAKE receiver output is

$$Z = \mathrm{Re}\{ \sum_{q=0}^{M_L - 1} g_q^{(L)*} X_q \}. \qquad (12)$$

III. Error Performance

For the sake of convenience, let

$$G_\ell = \int_{0^-}^{T_M^{(\ell)}} 2Q_\ell(\tau) d\tau. \qquad (13)$$

It is easy to show that G_ℓ is the average power gain due to transmitting a signal over $h_\ell(\tau)$. Let $g_m^{(\ell)} = \sqrt{G_\ell}(x_m^{(\ell)} + jy_m^{(\ell)})$, so that (i) $x_m^{(\ell)}$ and $y_m^{(\ell)}$ are independent zero-mean Gaussian random variables conditioned on $T_o^{(\ell)}$, and (ii) $x_m^{(\ell)}$ ($y_m^{(\ell)}$), $m = 0, \ldots, M_\ell - 1$, are correlated with $E\{x_m^{(\ell)} x_n^{(\ell)}\}$ ($E\{y_m^{(\ell)} y_n^{(\ell)}\}$) $= (2G_\ell)^{-1} E\{g_m^{(\ell)} g_n^{(\ell)*}\}$. In the derivation of BER, we analyze

$$\tilde{Z} = (G_L \sqrt{2P_L N})^{-1} Z \tag{14}$$

and consider the case of, say, $b_0^{(L)} = +1$ so that the BER, P_e, is given by $P_e = \int_{-\infty}^{0} p(\tilde{Z}) d\tilde{Z}$. The p.d.f. $p(\tilde{Z})$ is obtained by taking the inverse Fourier transform on the characteristic function of $\tilde{Z}$.

Expanding $\tilde{Z}$ gives

$$\begin{aligned}\tilde{Z} = &\sum_{q=0}^{M_L-1} \sum_{k=0}^{N-1} a_k^{(L)} \frac{1}{\sqrt{2\Gamma}} (x_q^{(L)} N_{k+q}^{(\mathrm{Re})} + y_q^{(L)} N_{k+q}^{(\mathrm{Im})}) \\ &+ \sum_{q=0}^{M_L-1} \sum_{m=0}^{M_L-1} (x_m^{(L)} x_q^{(L)} + y_m^{(L)} y_q^{(L)}) \frac{C_{L,L}(q-m)}{\sqrt{N}} \\ &+ \sum_{\ell=1,\ell\neq L}^{K} \sqrt{\frac{G_\ell P_\ell}{G_L P_L}} \sum_{q=0}^{M_L-1} \sum_{m=0}^{M_\ell-1} (x_m^{(\ell)} x_q^{(L)} + y_m^{(\ell)} y_q^{(L)}) \frac{H_{q,m,\ell}}{\sqrt{N}}\end{aligned} \tag{15}$$

where

$$\Gamma = G_L P_L T_b / \eta \tag{16}$$

is the received bit energy to noise ratio, and $N_k = \sqrt{\eta/T_c}(N_k^{(\mathrm{Re})} + jN_k^{(\mathrm{Im})})$. Let $\tilde{x}_q^{(\ell)} \triangleq N^{-1/2} \sum_{m=0}^{M_\ell-1} x_m^{(\ell)} H_{q,m,\ell}$, $\tilde{y}_q^{(\ell)} \triangleq N^{-1/2} \sum_{m=0}^{M_\ell-1} y_m^{(\ell)} H_{q,m,\ell}$, $\tilde{N}_q^{(\mathrm{Re})} \triangleq \sum_{k=0}^{N-1} a_k^{(L)} N_{k+q}^{(\mathrm{Re})}$, and $\tilde{N}_q^{(\mathrm{Im})} \triangleq \sum_{k=0}^{N-1} a_k^{(L)} N_{k+q}^{(\mathrm{Im})}$. Let $\mathbf{x}_L \triangleq [x_0^{(L)}, \cdots, x_{M_L-1}^{(L)}]$, $\mathbf{y}_L \triangleq [y_0^{(L)}, \cdots, y_{M_L-1}^{(L)}]$, $\tilde{\mathbf{x}}_\ell \triangleq [\tilde{x}_0^{(\ell)}, \cdots, \tilde{x}_{M_L-1}^{(\ell)}]$, $\tilde{\mathbf{y}}_\ell \triangleq [\tilde{y}_0^{(\ell)}, \cdots, \tilde{y}_{M_L-1}^{(\ell)}]$, $\tilde{\mathbf{n}}_{\mathrm{Re}} \triangleq [\tilde{N}_0^{(\mathrm{Re})}, \cdots, \tilde{N}_{M_L-1}^{(\mathrm{Re})}]$ and $\tilde{\mathbf{n}}_{\mathrm{Im}} \triangleq [\tilde{N}_0^{(\mathrm{Im})}, \cdots, \tilde{N}_{M_L-1}^{(\mathrm{Im})}]$. Let $\mathbf{C}_{L,L}$ be an $M_L \times M_L$ matrix with the (q,m)th[1] element $N^{-1/2} C_{L,L}(q-m)$. Consequently, $\tilde{Z}$ can be expressed as the sum of bilinear forms in correlated Gaussian random variables:

$$\begin{aligned}\tilde{Z} = &\frac{1}{\sqrt{2\Gamma}} \cdot (\tilde{\mathbf{n}}_{\mathrm{Re}} \mathbf{x}_L^T + \tilde{\mathbf{n}}_{\mathrm{Im}} \mathbf{y}_L^T) + \mathbf{x}_L \mathbf{C}_{L,L} \mathbf{x}_L^T + \mathbf{y}_L \mathbf{C}_{L,L} \mathbf{y}_L^T \\ &+ \sum_{\ell=1,\ell\neq L}^{K} \sqrt{\frac{G_\ell P_\ell}{G_L P_L}} \cdot (\tilde{\mathbf{x}}_\ell \mathbf{x}_L^T + \tilde{\mathbf{y}}_\ell \mathbf{y}_L^T).\end{aligned} \tag{17}$$

[1] Throughout this paper, the upper left element of a matrix is indexed by (0,0).

Although the conditional joint distribution of $\tilde{x}_q^{(\ell)}\ \forall q$ (or $\tilde{y}_q^{(\ell)}\ \forall q$) are jointly zero-mean Gaussian, the unconditional one may not. However, for mathematical tractability in the derivation of P_e, it is assumed that $\tilde{x}_q^{(\ell)}\ \forall q$ (and $\tilde{y}_q^{(\ell)}\ \forall q$) are approximately jointly zero-mean Gaussian with $E\{\tilde{x}_q^{(\ell)} \tilde{x}_{q'}^{(\ell)}\}$ (and $E\{\tilde{y}_q^{(\ell)} \tilde{y}_{q'}^{(\ell)}\}$) averaged over random variables involved in them ($T_o^{(\ell)}$, k_ℓ, $b_{-1}^{(\ell)}$, $b_0^{(\ell)}$ and $b_1^{(\ell)}$). Expressions of $E\{\tilde{x}_q^{(\ell)} \tilde{x}_{q'}^{(\ell)}\}$ and $E\{\tilde{y}_q^{(\ell)} \tilde{y}_{q'}^{(\ell)}\}$ are listed in Appendix 2. It is easy to show that $\tilde{N}_q^{(\mathrm{Re})}$'s ($\tilde{N}_q^{(\mathrm{Im})}$'s) are jointly zero-mean Gaussian with $E\{\tilde{N}_q^{(\mathrm{Re})} \tilde{N}_{q'}^{(\mathrm{Re})}\}$ ($= E\{\tilde{N}_q^{(\mathrm{Im})} \tilde{N}_{q'}^{(\mathrm{Im})}\}$) $= C_{L,L}(q-q')$ and that $\tilde{N}_q^{(\mathrm{Re})}$ and $\tilde{N}_q^{(\mathrm{Im})}$ are independent. Let $\mathbf{S}_\ell$, $\mathbf{N}$ and $\mathbf{S}_L$ be $M_L \times M_L$ matrices with the (q,q')th elements given by $E\{\tilde{x}_q^{(\ell)} \tilde{x}_{q'}^{(\ell)}\}$, $E\{\tilde{N}_q^{(\mathrm{Re})} \tilde{N}_{q'}^{(\mathrm{Re})}\}$ and $(2G_L)^{-1} E\{g_q^{(L)} g_{q'}^{(L)*}\}$, respectively. Since these matrices are covariance matrices, they are positive semidefinite [9, ch. 22.3]. Although these matrices can be singular with zero determinants, in practice we are only interested in the case that $\mathbf{S}_\ell$, $\mathbf{N}$ and $\mathbf{S}_L$ are non-singular, so that they are positive definite. Consequently, we have $p(\tilde{\mathbf{x}}_\ell) = (2\pi)^{-M_L/2} |\mathbf{S}_\ell|^{-1/2} \times \exp(-\frac{1}{2} \tilde{\mathbf{x}}_\ell \mathbf{S}_\ell^{-1} \tilde{\mathbf{x}}_\ell^T)$. Similar expressions exist for $p(\tilde{\mathbf{y}}_\ell)$, $p(\mathbf{x}_L)$, $p(\mathbf{y}_L)$, $p(\tilde{\mathbf{n}}_{\mathrm{Re}})$ and $p(\tilde{\mathbf{n}}_{\mathrm{Im}})$.

The characteristic function of $\tilde{Z}$ is given by

$$\begin{aligned}&E\{\exp(-j\omega\tilde{Z})\} \\ &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp(-j\omega\tilde{Z}) p(\mathbf{x}_L) p(\mathbf{y}_L) p(\tilde{\mathbf{n}}_{\mathrm{Re}}) p(\tilde{\mathbf{n}}_{\mathrm{Im}}) \\ &\quad d\mathbf{x}_L d\mathbf{y}_L d\tilde{\mathbf{n}}_{\mathrm{Re}} d\tilde{\mathbf{n}}_{\mathrm{Im}} \cdot \left[\prod_{\ell=1,\ell\neq L}^{K} p(\tilde{\mathbf{x}}_\ell) p(\tilde{\mathbf{y}}_\ell) d\tilde{\mathbf{x}}_\ell d\tilde{\mathbf{y}}_\ell\right].\end{aligned} \tag{18}$$

Consider

$$\begin{aligned}&E\{\exp(-j\omega\sqrt{\tfrac{G_\ell P_\ell}{G_L P_L}} \tilde{\mathbf{x}}_\ell \mathbf{x}_L^T) \mid \mathbf{x}_L\} \\ &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp(-j\omega\sqrt{\tfrac{G_\ell P_\ell}{G_L P_L}} \tilde{\mathbf{x}}_\ell \mathbf{x}_L^T) p(\tilde{\mathbf{x}}_\ell) d\tilde{\mathbf{x}}_\ell.\end{aligned}$$

Applying (11.12.1a) of [9] gives

$$E\{\exp(-j\omega\sqrt{\tfrac{G_\ell P_\ell}{G_L P_L}} \tilde{\mathbf{x}}_\ell \mathbf{x}_L^T) \mid \mathbf{x}_L\} = \exp\left(-\tfrac{1}{2}\omega^2 \tfrac{G_\ell P_\ell}{G_L P_L} \mathbf{x}_L \mathbf{S}_\ell \mathbf{x}_L^T\right).$$

Similar relationships exist for $E\{\exp(-j\omega\sqrt{\tfrac{G_\ell P_\ell}{G_L P_L}} \tilde{\mathbf{y}}_\ell \mathbf{y}_L^T) \mid \mathbf{y}_L\}$, etc. Consequently, (18) is reduced to

$$\begin{aligned}&E\{\exp(-j\omega\tilde{Z})\} \\ &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \exp(\mathbf{x}_L \mathbf{U} \mathbf{x}_L^T + \mathbf{y}_L \mathbf{U} \mathbf{y}_L^T) \cdot p(\mathbf{x}_L) p(\mathbf{y}_L) d\mathbf{x}_L d\mathbf{y}_L.\end{aligned}$$

where

$$\mathbf{U} = -j\omega \mathbf{C}_{L,L} - \frac{\omega^2}{2}\sum_{\ell=1,\ell\neq L}^{K} \frac{G_\ell P_\ell}{G_L P_L}\mathbf{S}_\ell - \frac{\omega^2}{2}\cdot\frac{1}{2\Gamma}\mathbf{N}.$$

Applying (11.12.2) of [9] yields

$$E\{\exp(-j\omega\tilde{Z})\} = \left|\mathbf{I} + \omega^2\mathbf{A} + 2j\omega\mathbf{B}\right|^{-1} \quad \textbf{(19)}$$

where $\mathbf{I}$ is the $M_L \times M_L$ identity matrix, $\mathbf{A} = \sum_{\ell=1,\ell\neq L}^{K} \frac{G_\ell P_\ell}{G_L P_L}\mathbf{S}_L\mathbf{S}_\ell + \frac{1}{2\Gamma}\mathbf{S}_L\mathbf{N}$, and $\mathbf{B} = \mathbf{S}_L\mathbf{C}_{L,L}$.

Let $\mu_0, \mu_1, \cdots, \mu_{2M_L-1}$ be the $2M_L$ latent roots of $\mathbf{I} - s^2\mathbf{A} + 2s\mathbf{B}$, that is, $|\mathbf{I} - \mu_i^2\mathbf{A} + 2\mu_i\mathbf{B}| = 0\ \forall i$. It is known that [10, ch. 14.1] these latent roots coincide with eigenvalues of the matrix $\begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{A}^{-1} & 2\mathbf{A}^{-1}\mathbf{B}\end{bmatrix}$, and $|\mathbf{I} - s^2\mathbf{A} + 2s\mathbf{B}| = (-1)^{M_L}|\mathbf{A}|\cdot\prod_{i=0}^{2M_L-1}(s-\mu_i)$. It can be shown [11] that μ_i's are nonzero real. In this paper, we consider only the case that μ_i's are distinct. Partial fraction expansion on (19) gives

$$E\{\exp(-j\omega\tilde{Z})\} = \frac{(-1)^{M_L}}{|\mathbf{A}|}\sum_{i=0}^{2M_L-1} \vartheta_i \cdot \frac{1}{j\omega - \mu_i} \quad \textbf{(20)}$$

where

$$\vartheta_i = \frac{1}{(\mu_i-\mu_0)\cdots(\mu_i-\mu_{i-1})(\mu_i-\mu_{i+1})\cdots(\mu_i-\mu_{2M_L-1})}.$$

It is known [12] that $\mathcal{F}^{-1}\{(j\omega-\mu_i)^{-1}\} = \mathrm{sgn}(-\mu_i)\,e^{\mu_i\tilde{Z}} \times U(\mathrm{sgn}(-\mu_i)\,\tilde{Z})$ where $\mathcal{F}^{-1}\{\cdot\}$ is the inverse Fourier transform, sgn(·) is the signum function and $U(\cdot)$ is the unit step function. Since $p(\tilde{Z}) = \mathcal{F}^{-1}\{E\{\exp(-j\omega\tilde{Z})\}\}$ and $P_e = \int_{-\infty}^{0} p(\tilde{Z})d\tilde{Z}$, we get

$$P_e = \frac{(-1)^{M_L+1}}{|\mathbf{A}|}\sum_{\mu_i>0}\frac{\vartheta_i}{\mu_i}. \quad \textbf{(21)}$$

IV. A Numerical Example

Eqn. (21) was used to compute P_e of synchronous DS/CDMA systems using Gold codes[2] and those employing optimized cyclic codes[3] proposed separately in [3] and [4], in order to demonstrate the significant performance improvement resulting from the use of signature sequences that are crosscorrelation-optimized within a small region around the origin. In the computation, it was assumed that $N = 127$, $L = 1$ and $K = 1, 5, 10$. Rectangular chip waveform and equal power transmission were assumed. All K channels were assumed one-sided exponentially decaying with $2Q_\ell(\tau) = \tau_{\mathrm{rms}}^{-1}\exp(-\tau/\tau_{\mathrm{rms}})$, $\tau \geq 0$, where $\tau_{\mathrm{rms}} = 0.5T_c$ (r.m.s. delay spread) was assumed. The multipath spread was truncated at $T_M^{(\ell)} = 2T_c$. Fig. 2 plots the P_e against Γ for systems with $\mathcal{L} = 0$ (perfectly synchronized transmission) and $\mathcal{L} = 3$ (approximately synchronized transmission) using both sets of signature sequences under consideration. It is apparent that P_e values using optimized cyclic codes are lower than those using Gold codes in the presence of multiple access for both cases of $\mathcal{L} = 0$ and $\mathcal{L} = 3$. In particular, the irreducible P_e between systems using the two types of signature sequences is three to four orders of magnitude. The result demonstrates that synchronous DS/CDMA communications using optimized cyclic codes provide significant suppression of the multiple-access interference in the multipath Rayleigh fading environment.

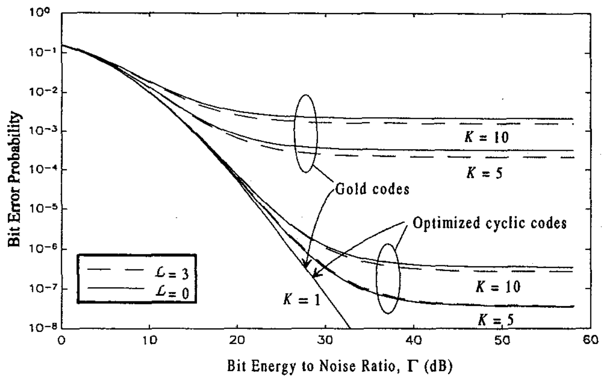

Fig. 2. Bit error probabilities of synchronous DS/CDMA systems ($\mathcal{L} = 0$ and $\mathcal{L} = 3$) using Gold codes and using optimized cyclic codes, under different numbers of users (K).

[2] Although the Gold codes are crosscorrelation-optimized over all shifts, in this example we did not purposedly choose the codes having very small crosscorrelations (–1) within a small region around the origin.

[3] The optimized cyclic codes used in this example were derived from a common m-sequence [3].

Appendix 1

Substituting (2) into (5) yields $X_q = \sum_{k=0}^{N-1} a_k^{(L)}\Xi_{k+q}$ where $\Xi_k = T_c^{-1}\int_{kT_c}^{(k+1)T_c} v(t)\psi(t-kT_c)dt$. Let

$$\Xi_k^{(\ell)} = T_c^{-1}\int_{kT_c}^{(k+1)T_c}\int_{0^-}^{T_M^{(\ell)}} u_\ell(t-\tau)h_\ell(\tau)\psi(t-kT_c)d\tau dt \quad \textbf{(A1)}$$

so that $\Xi_k = N_k + \sum_{\ell=1}^{K}\Xi_k^{(\ell)}$ where $N_k = T_c^{-1}\int_{kT_c}^{(k+1)T_c} n(t)\,\psi(t-kT_c)dt$. It follows that N_k's are i.i.d. zero-mean complex Gaussian random variables with $E\{N_k N_k^*\} = 2\eta/T_c$. Substituting (2) into (1) gives $u_\ell(t) =$

$e^{j\varphi_\ell}\sqrt{2P_\ell}\sum_{m=-\infty}^{\infty} c_{m-k_\ell}^{(\ell)} a_{m-k_\ell}^{(\ell)} \psi(t-T_o^{(\ell)}-mT_c)$ where $a_m^{(\ell)} \triangleq a_{m \bmod N}^{(\ell)}$ and $c_{iN+n}^{(\ell)} \triangleq b_i^{(\ell)}$ for $0 \le n \le N-1$. Rewriting $u_\ell(t)$ as $u_\ell(t) = e^{j\varphi_\ell}\sqrt{2P_\ell}\sum_{m=-\infty}^{\infty} c_{k-k_\ell-m}^{(\ell)} a_{k-k_\ell-m}^{(\ell)} \psi(t-T_o^{(\ell)}-(k-m)T_c)$ and evaluating $\Xi_k^{(\ell)}$ using (A1), we get $\Xi_k^{(\ell)} = \sqrt{2P_\ell}\sum_{m=-\infty}^{\infty} c_{k-k_\ell-m}^{(\ell)} a_{k-k_\ell-m}^{(\ell)} g_m^{(\ell)}$ where

$$g_m^{(\ell)} = e^{j\varphi_\ell}\int_{0^-}^{T_M^{(\ell)}} h_\ell(\tau)W(mT_c - T_o^{(\ell)} - \tau)d\tau \qquad \text{(A2)}$$

and $W(\cdot)$ is given by (9). We should mention that in the derivation we make use of the assumption that $h_\ell(\tau)$ is slowly varying relative to T_b; otherwise $h_\ell(\tau)$ would also be a function of t in (A1) and the expression of $\Xi_k^{(\ell)}$ would be very complicated. Since $h_\ell(\tau)$ and $W(\tau)$ are time-limited within $[0, T_M^{(\ell)}]$ and $[-T_c, T_c]$, respectively, $g_m^{(\ell)}$ is zero if $m < 0$ or $m > M_\ell$ where $M_\ell = 1 + \lceil T_M^{(\ell)}/T_c \rceil$. It is also apparent that $g_m^{(\ell)}$, $m = 0, 1, \ldots, M_\ell - 1$, are jointly zero-mean complex Gaussian random variables conditioned on $T_o^{(\ell)}$ and it is easy to show that $E\{g_m^{(\ell)} g_n^{(\ell)*}\}$ is given by (7). From (7), it is apparent that $E\{g_m^{(\ell)} g_{m+1}^{(\ell)*}\}$ is in general nonzero so that $g_m^{(\ell)}$'s are correlated. Since all K channels are independent, $g_m^{(\ell)}$ and $g_n^{(\ell')}$ are statistically independent if $\ell \ne \ell'$. We now arrive at $\Xi_k^{(\ell)} = \sqrt{2P_\ell}\sum_{m=0}^{M_\ell-1}(c_{k-k_\ell-m}^{(\ell)} a_{k-k_\ell-m}^{(\ell)} g_m^{(\ell)})$. Substituting this expression into Ξ_k and hence X_q gives (6).

APPENDIX 2

It is found that

$$E\{\tilde{x}_q^{(\ell)}\tilde{x}_{q'}^{(\ell)}\} = \begin{cases} \mathcal{F}_0(q,q',\ell) + \mathcal{F}_1(q,q',\ell), & M_\ell \ne 1 \\ \mathcal{F}_0(q,q',\ell), & M_\ell = 1. \end{cases} \qquad \text{(A3)}$$

where $\mathcal{F}_0(\cdot)$ and $\mathcal{F}_1(\cdot)$ are given as follows. Let $f_0(k_\ell, m) = \sum_{i\in\{-1,0,1\}} C_{L,\ell}(q-k_\ell-m-iN)C_{L,\ell}(q'-k_\ell-m-iN)$ and $f_1(k_\ell,m) = \sum_{i\in\{-1,0,1\}}[C_{L,\ell}(q-k_\ell-m-iN)C_{L,\ell}(q'-k_\ell-(m+1)-iN) + C_{L,\ell}(q-k_\ell-(m+1)-iN)C_{L,\ell}(q'-k_\ell-m-iN)]$. Then (i) $\mathcal{L} \ne 0$:

$$\mathcal{F}_0(q,q',\ell) = N^{-1}\sum_{m=0}^{M_\ell-1}\overline{x_m^{(\ell)2}} \cdot \{[\sum_{k_\ell=0}^{\mathcal{L}-1}\frac{2\mathcal{L}-2k_\ell-1}{2\mathcal{L}^2} f_0(k_\ell,m) + \sum_{k_\ell=-\mathcal{L}}^{-1}\frac{2\mathcal{L}+2k_\ell+1}{2\mathcal{L}^2} f_0(k_\ell,m)]\}$$

and

$$\mathcal{F}_1(q,q',\ell) = N^{-1}\sum_{m=0}^{M_\ell-2}\overline{x_m^{(\ell)}x_{m+1}^{(\ell)}}\{[\sum_{k_\ell=0}^{\mathcal{L}-1}\frac{2\mathcal{L}-2k_\ell-1}{2\mathcal{L}^2} f_1(k_\ell,m) + \sum_{k_\ell=-\mathcal{L}}^{-1}\frac{2\mathcal{L}+2k_\ell+1}{2\mathcal{L}^2} f_1(k_\ell,m)]\}$$

where, for $n \in \{m, m+1\}$,

$$\overline{x_m^{(\ell)}x_n^{(\ell)}} = (G_\ell T_c)^{-1}\int_0^{T_c}\int_{0^-}^{\infty} Q_\ell(\tau)W(mT_c - T_o^{(\ell)} - \tau) W(nT_c - T_o^{(\ell)} - \tau)d\tau dT_o^{(\ell)};$$

(ii) $\mathcal{L} = 0$:

$$\mathcal{F}_0(q,q',\ell) = N^{-1}\sum_{m=0}^{M_\ell-1}\overline{x_m^{(\ell)2}} f_0(0,m)$$

and

$$\mathcal{F}_1(q,q',\ell) = N^{-1}\sum_{m=0}^{M_\ell-2}\overline{x_m^{(\ell)}x_{m+1}^{(\ell)}} f_1(0,m)$$

where, for $n \in \{m, m+1\}$,

$$\overline{x_m^{(\ell)}x_n^{(\ell)}} = G_\ell^{-1}\int_{0^-}^{\infty} Q_\ell(\tau)W(mT_c - \tau)W(nT_c-\tau)d\tau.$$

It is easy to show that $E\{\tilde{y}_q^{(\ell)}\tilde{y}_{q'}^{(\ell)}\} = E\{\tilde{x}_q^{(\ell)}\tilde{x}_{q'}^{(\ell)}\}$ as $\overline{x_m^{(\ell)}x_n^{(\ell)}} = \overline{y_m^{(\ell)}y_n^{(\ell)}}$, $n \in \{m, m+1\}$.

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