# NEW APPROXIMATE QR-LS ALGORITHMS FOR MINIMUM OUTPUT ENERGY (MOE) RECEIVERS IN DS-CDMA COMMUNICATION SYSTEMS

H. Cheng, Y. Zhou, and S. C. Chan
Department of Electrical & Electronic Engineering
The University of Hong Kong, Pokfulam Road, Hong Kong
{huicheng,yizhou,scchan}@eee.hku.hk

*Abstract*—**This paper proposes a new family of approximate QR-based least-squares (LS) adaptive algorithms called *p*-A-QR-LS for blind minimum output energy (MOE) detection in CDMA communications systems. It extends the A-QR-LS algorithm by retaining different numbers of diagonal plus $p-1$ off-diagonals of the triangular factor of the augmented data matrix. For $p=1$ and $N$ ($N$ is the length of the weighting vector), it reduces to the A-QR-LS and the QR-RLS algorithms, respectively. It not only provides a link between the QR-LMS-type and the QR-RLS algorithms through a well-structured family of algorithms, but also offers more freedom in the complexity-performance tradeoffs for practical receiver design in communication systems. The performance of the proposed algorithm is verified by computer simulations.**

## I. INTRODUCTION

One major impediment to the performance of code-division multiple access (CDMA) systems is multiple access interference (MAI), which arises from the users simultaneously using the same frequency band. The linear MMSE multi-user detector (MUD) [1] has been proposed as an effective and relatively simple technique to mitigate MAI in CDMA systems. In [2], Honig et al proposed a blind MMSE receiver, called the minimum output energy (MOE) detector for MAI suppression in CDMA systems. The MOE detector minimizes the output energy of the receiver while preserving the signals of desired users. Under ideal conditions, it achieves performance close to that of the optimal MMSE receiver at high signal-to-noise ratio (SNR) [3]. The generalized sidelobe canceller (GSC) [4] is a popular implementation of the MOE detector. An adaptive realization of the GSC has been proposed using least mean squares (LMS) and recursive least squares (RLS) techniques.

The RLS algorithm generally converges faster than the LMS algorithm but with a high complexity of $O(N^2)$ (where $N$ is the length of the adaptive filter). The QR-RLS based on the QR decomposition (QRD) exhibits better numerical properties due to the direct application of QRD to the data matrix [5]. The QRD-based algorithms usually consist of two parts: 1) recursive updating of the triangular factor of the data matrix and 2) back-solving of the filter parameters. Since the back-solving step requires $O(N^2)$ operations, the entire algorithm still needs at least $O(N^2)$ arithmetic operations. In order to reduce the complexity of the back-solving step in the QRD, Liu proposed an approximate QR-LS algorithm (A-QR-LS) [6], which combines the recursive updating of the triangular matrix and the back-solving of the parameters. In this paper, we propose a new QR-based GSC implementation of the MOE receiver. It is based on a new adaptive filtering algorithm called the *p*-A-QR-LS algorithm [7]. By retaining a different number of diagonal plus $p-1$ off-diagonals ($1 \le p \le N$) during the QRD, the triangular factor of the data matrix can be approximated to different levels according to the value of $p$. Coupling with the back-solving step as in [8], the new QR-LS algorithms of complexity $O(Np)$ present a different complexity/performance tradeoff with various values of $p$. For $p=1$ and $p=N$, it reduces respectively to the A-QR-LS and the QR-RLS algorithms. Other values of $p$ generate a series of new QR-based algorithms with complexity-performance tradeoff between the two well-known families of algorithms. Therefore, it improves the design freedom in practical receiver design by choosing different values of $p$.

The rest of this paper is organized as follows: Section II briefly describes the CDMA system model and the MOE detection. The proposed *p*-A-QR-LS algorithm is presented in Section III. In Section IV, experimental results are given. Finally, conclusions are drawn in Section V.

## II. MOE DETECTION FOR DS-CDMA SYSTEMS

### A. System Model

Consider a $Kth$ user asynchronous DS-CDMA system over a frequency-selective fading channel. The channel impulse response of the $kth$ user is expressed as

$$g_k(t) = \sum_{l=1}^{L} \alpha_{k,l}(t)\delta(t-\tau_{k,l}) , \tag{1}$$

where $L$ is the number of taps, $\alpha_{k,l}(t)$ is the $lth$ path gain which is an independent zero-mean, complex Gaussian random process, and $\tau_{k,l}$ is the propagation delay for the $lth$ path. The received baseband signal can be modeled as

$$y(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} \alpha_{k,l}(t)x_k(t-\tau_{k,l}) + v(t) , \tag{2}$$

where $x_k(t)$ is the transmitted signal of the $kth$ user, and $v(t)$ represents a complex additive white Gaussian noise with zero mean and variance $\sigma_v^2$. The transmitted signal $x_k(t)$ is given as

$$x_k(t) = \sum_{i=-\infty}^{\infty} \sqrt{E_k}\, b_k(i) c_k(t-iT_s) , \tag{3}$$

where $E_k$ is the chip energy, $b_k(i)$ denotes the $ith$ data symbol, and $c_k(t)$ is the code waveform for the $kth$ user, respectively; $T_s$ is the symbol interval with $T_s = L_c T_c$, where $T_c$ is the chip interval, and $L_c$ is the spreading gain.

The received signal is sampled at the chip rate. Due to the delay spread of the multipath channel, the data vector of length $N = L_c + L - 1$ is collected from the output samples of the chip-matched filter. Assuming a coarse synchronization between the transmitter and receiver, only two adjacent symbols contribute to the data vector $\mathbf{y}(i)$. Also for simplicity, it is assumed that the propagation delay $\tau_{k,l}$ is a multiple of the sampling rate, i.e., $\tau_{k,1} = p_k T_c$, where $p_k \in \{0,1,\cdots,L_c - 1\}$. Then the vector $\mathbf{y}(i) \in \mathbf{C}^{N\times 1}$ can be expressed as

$$\mathbf{y}(i) = \sum_{k=1}^{K} \sqrt{E_k}\,[\mathbf{C}_k \mathbf{s}_k(i) + \widetilde{\mathbf{C}}_k \mathbf{s}_k(i-1)] + \mathbf{v}(i)\,, \quad (4)$$

where $\mathbf{C}_k / \widetilde{\mathbf{C}}_k \in \Re^{N\times L}$ include the right/left shift of the *kth* user's code sequence, whose *lth* ($l = 1,\ldots,L$) column is

$$\mathbf{C}_k(:,l) = \mathbf{0}_{N\times 1};\ \mathbf{C}_k(l + p_k : l + L_c - 1, l) = \mathbf{c}_k(1 : L_c - p_k)$$
$$\widetilde{\mathbf{C}}_k(:,l) = \mathbf{0}_{N\times 1};\ \widetilde{\mathbf{C}}_k(l : l + p_k - 1, l) = \mathbf{c}_k(L_c - p_k + 1 : L_c)\,;$$

$\mathbf{s}_k(i) = [\alpha_{k,1}(i) b_k(i), \ldots, \alpha_{k,L}(i) b_k(i)]^T$, $\mathbf{s}_k(i-1) = [\alpha_{k,L}(i) \cdot b_k(i-1), \ldots, \alpha_{k,L}(i) b_k(i-1)]^T$; and $\mathbf{v}(i)$ is the noise vector.

Let User 1 be the desired user, and its synchronization to the first multipath component has been obtained, i.e., $\tau_{1,1} = 0$. The vector $\mathbf{y}(i)$ can be rewritten as

$$\mathbf{y}(i) = \sqrt{E_1}\,\mathbf{C}_1 \mathbf{s}_1(i) + \sum_{k=2}^{K} \sqrt{E_k}\,(\mathbf{C}_k \mathbf{s}_k(i) + \widetilde{\mathbf{C}}_k \mathbf{s}_k(i-1)) + \mathbf{v}(i)\,. \quad (5)$$

### *B. MOE Detection*

The basic idea of the MOE detector is to minimize the overall variance of the receiver output, while preserving the components of the desired signal. The MOE detector can be formulated as the following constrained optimization problem

$$\min_{w} \sum_{i=0}^{n} \lambda^{n-i} \left|\mathbf{w}^H(n)\mathbf{y}(i)\right|^2, \quad s.t. \quad \mathbf{D}^H \mathbf{w}(n) = \mathbf{f}\,, \quad (6)$$

where $0 < \lambda < 1$ is the forgetting factor, $\mathbf{w}$ is the multiuser detector to be determined, $\mathbf{D}$ is a ($N \times d$) matrix whose $d$ columns specify the constraints, and $\mathbf{f}$ is a ($d \times 1$) vector of constraint values. Denoting $\mathbf{R}_y = E\{\mathbf{y}\mathbf{y}^H\}$, the optimal solution $\mathbf{w}$ to the constrained optimization problem is then given by

$$\mathbf{w}_o = \mathbf{R}_y^{-1}\mathbf{D}(\mathbf{D}^H \mathbf{R}_y^{-1}\mathbf{D})^{-1}\mathbf{f}\,. \quad (7)$$

This MOE detector can also be implemented as a GSC [5] shown in Fig. 1. Here, $\mathbf{w}_o$ is decomposed into two orthogonal components as follows

$$\mathbf{w} = \mathbf{w}_c - \mathbf{B}\mathbf{w}_a\,, \quad (8)$$

where $\mathbf{w}_c = \mathbf{D}(\mathbf{D}^H\mathbf{D})^{-1}\mathbf{f}$ is the fixed constraint weight vector, while $\mathbf{w}_a = (\mathbf{B}^H \mathbf{R}_y \mathbf{B})^{-1}\mathbf{B}^H \mathbf{R}_y \mathbf{w}_c$ is an adaptive filter orthogonal to $\mathbf{w}_c$ in order to mitigate the interference. The blocking matrix $\mathbf{B} \in \Re^{N\times(N-d)}$ satisfies $\mathbf{B}^H\mathbf{D} = \mathbf{0}$, and blocks the desired user's signal from entering the adaptive filter. Otherwise, signal cancellation will occur. There are different

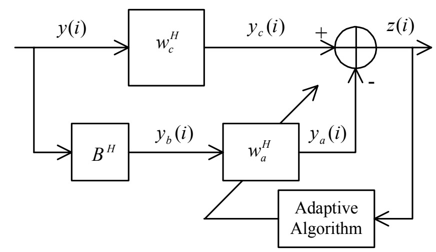


Fig. 1. The generalized sidelobe canceller.

ways of choosing the linear constrains to handle multipath channels. If we choose $\mathbf{D} = \mathbf{C}_1$, the optimal $\mathbf{f}$ is given as $\mathbf{f} = \mathbf{C}_1^H \mathbf{R}_y^{-1}\mathbf{C}_1\mathbf{g}_1$, where $\mathbf{g}_1$ is the channel state information. In this paper, we use the algorithm proposed in [9] to develop the MOE detector. In [9], the first multipath component is preserved while the $d-1$ delayed copies of the signal of interest are forced to zero, i.e., $\mathbf{D} = \mathbf{C}_1$ and $\mathbf{f} = [1\,0\cdots0]^T$. Hence, the output of the MOE receiver is expressed as

$$z(n) = \mathbf{w}^H(n)\mathbf{y}(n) = \alpha_{1,1} b_1(n) + e(n)\,, \quad (9)$$

where $\alpha_{1,1}$ denotes the desired user's channel gain of the first path, and $e(n)$ is the overall interference and noise after filtering. Assume the channel information is available. Using the equal gain combiner (EGC), the symbol estimation is given by

$$\hat{b}_1(n) = (\alpha_{1,1}^* / \left|\alpha_{1,1}\right|^2) z(n)\,. \quad (10)$$

Next, we shall consider the recursive adaptation of the adaptive part $\mathbf{w}_a$.

## III. THE *p*-A-QR-LS ALGORITHM

Consider a least-squares adaptive filter design problem. A set of desired signal $z(i)$ and the input signals $\mathbf{y}(i) \in \mathbf{C}^{N\times 1}$ have been taken for $0 < i < n$. In least-squares estimation, the optimal linear filter $\mathbf{w}_o$ is chosen to minimize the cost function

$$\xi_N(n) = \sum_{i=0}^{n} \lambda^{n-i} \left|e(i)\right|^2 = \sum_{i=0}^{n} \lambda^{n-i} \left|z(i) - \mathbf{w}^H(n)\mathbf{y}(i)\right|^2. \quad (11)$$

Eq. (11) can be written more compactly in matrix form as

$$\xi_N(n) = \mathbf{e}^H(n)\mathbf{\Lambda}^2(n)\mathbf{e}(n) = \left\|\mathbf{\Lambda}(n)\mathbf{e}(n)\right\|^2\,, \quad (12)$$

where $\mathbf{e}(n) = [e(0), e(1), \cdots, e(n)]^H$, and $\mathbf{\Lambda}^2(n) = diag\{\lambda^n, \lambda^{n-1}, \cdots, 1\}$ is a diagonal matrix.. Defining $\mathbf{z}(n) = [z(0), z(1), \cdots, z(n)]^H$ and $\mathbf{Y}^H(n) = [\mathbf{y}(0), \mathbf{y}(1), \cdots, \mathbf{y}(n)]$, we have $\mathbf{e}(n) = \mathbf{z}(n) - \mathbf{Y}(n)\mathbf{w}(n)$. The QR-RLS method [5] is summarized below in Table 1.

1. Given the augmented data matrix

$$\mathbf{D}(n-1) = \mathbf{\Lambda}(n-1)[\mathbf{Y}(n-1) \quad \mathbf{z}(n-1)]$$

and its QRD at time (n-1):

$$\mathbf{D}^*(n-1) = \mathbf{Q}(n-1)\mathbf{D}(n-1) = \begin{bmatrix} \Re(n-1) & \widetilde{\mathbf{k}}(n-1) \\ \mathbf{0}_{1\times N} & \widetilde{e}^*(n-1) \end{bmatrix}$$

where $\mathbf{Q}(n-1)$ and $\Re(n-1)$ are unitary and upper triangular matrices, respectively. $(\cdot)^*$ denotes the complex conjugate.

2. (**QRD**) Form the new augmented data matrix

$$\mathbf{D}(n) = \mathbf{A}(n)[\mathbf{Y}(n) \quad \mathbf{z}(n)] = \begin{bmatrix} \sqrt{\lambda}\mathbf{D}(n-1) \\ \psi(n) \end{bmatrix}$$

where $\psi(n) = [\mathbf{y}^H(n) \quad z^*(n)]$. Get the new QRD by Givens rotations or Householder reflections as

$$\mathbf{Q}^{(N)}(n)\cdots\mathbf{Q}^{(1)}(n)\mathbf{Q}'(n)\mathbf{D}(n) = \begin{bmatrix} \Re(n) & \widetilde{\mathbf{k}}(n) \\ \mathbf{0}_{1\times N} & \widetilde{e}^*(n) \end{bmatrix}.$$

3. (**Back-solving**) Solve the triangular system $\Re_N(n)\mathbf{w}(n) = \widetilde{\mathbf{k}}(n)$ for the *LS* estimate $\mathbf{w}(n)$ at time *n* by back-substitution:

$$w_N(n) = r_{N,N+1}(n)/r_{N,N}(n),$$

$$w_i(n) = [r_{i,N+1}(n) - \sum_{j=i+1}^{N} r_{i,j}(n)w_j(n)]/r_{i,i}(n), i = N-1,\cdots,1$$

where $r_{i,j}$ and $r_{i,N+1}$ are the corresponding elements in $\Re(n)$ and $\widetilde{\mathbf{k}}(n)$. $w_i(n)$ is the i-*th* element of $\mathbf{w}(n)$.

Table 1. QR-RLS algorithm.

In [6], Liu proposed an approximate QR-LS (A-QR-LS) algorithm with a complexity of $O(N)$ by approximating the upper triangular matrix as a diagonal matrix, which simplifies the QRD and the back substitution. More precisely, the quantities inside the square bracket in Step 3 of Table 1 are computed from $w_i(n-1)$ and are denoted by

$$s_N(n-1) = r_{N,N+1}(n-1),$$

$$s_i(n-1) = [r_{i,N+1}(n-1) - \sum_{j=i+1}^{N} r_{i,j}(n-1)w_j(n-1)],$$

$$i = N-1, N-2, \cdots, 1, \tag{13}$$

or $$r_{i,i}(n-1)w_i(n-1) = s_i(n-1), \quad i = 1,\cdots,N. \tag{14}$$

Given the values of $s_i(n-1)$ and $r_{i,i}(n-1)$, the combination of (14) and the approximation $z(n) = \mathbf{w}^H(n)\mathbf{y}(n)$ forms linear equations in the variable $\mathbf{w}(n)$

$$\begin{aligned} &\sqrt{\lambda}\, r_{i,i}(n-1)w_i(n-1) = \sqrt{\lambda}\, s_i(n-1), \quad i = 1,\cdots,N, \\ &\mathbf{w}^H(n)\mathbf{y}(n) = z(n). \end{aligned} \tag{15}$$

Equation (15) can also be written in matrix form as the following

$$\boldsymbol{\Phi}(n)\mathbf{w}(n) = \mathbf{b}(n), \tag{16}$$

where $\boldsymbol{\Phi}(n) = \begin{bmatrix} \sqrt{\lambda}\widetilde{\mathbf{R}}(n-1) \\ \mathbf{y}^H(n) \end{bmatrix}$, $\mathbf{b}(n) = \begin{bmatrix} \sqrt{\lambda}\widetilde{\mathbf{R}}(n-1)\mathbf{w}(n-1) \\ z^*(n) \end{bmatrix}$, and

$\widetilde{\mathbf{R}}(n-1) = diag\{r_{1,1}(n-1),\cdots,r_{N,N}(n-1)\}$. Therefore, (16) can be solved by computing the QRD of $\boldsymbol{\Phi}(n)$, which works with the following appended matrix:

$$\widetilde{\mathbf{D}}(n) = \begin{bmatrix} \sqrt{\lambda}r_{1,1}(n-1) & & & \sqrt{\lambda}s_1(n-1) \\ & \ddots & \mathbf{0} & \sqrt{\lambda}s_2(n-1) \\ \mathbf{0} & & \ddots & \vdots \\ & & \sqrt{\lambda}r_{N,N}(n-1) & \sqrt{\lambda}s_N(n-1) \\ y_1^*(n) & y_2^*(n) \cdots & y_N^*(n) & z^*(n) \end{bmatrix}. \tag{17}$$

From (17), we can see that the A-QR-LS algorithm retains an approximate triangular factor of the augmented data matrix so as to reduce the arithmetic complexity to $O(N)$. It is therefore natural to expect that better performance can be achieved by retaining more off diagonal elements of this factor. Obviously, when the whole upper triangular matrix is retained, we obtain the QR-RLS algorithm but the back-solving step has a complexity of $O(N^2)$. In this paper, we proposed to retain the main diagonal and $p-1$ nearby off-diagonals of the triangular factor, hence the name *p*-A-QR-LS algorithms. We shall show later that the *p*-A-QR-LS algorithm with a given positive integer $p$ has a complexity of order $O(Np)$ and a performance that generally improves as *p* increases. Therefore, the family not only provides a link between the QR-LMS-type and QR-RLS-type algorithms, but also a practical tradeoff between performance and complexity when $p-1$ is varied from 1 to $N$.

In the proposed *p*-A-QR-LS algorithm, the diagonal as well as nearby (*p*-1) off-diagonals are retained. It yields,

$$\widetilde{\mathbf{R}}_p(n-1) = \begin{bmatrix} r_{1,1}(n-1) & r_{1,2}(n-1 & \cdots & r_{1,p}(n-1) & & \cdots & 0 & 0 \\ & r_{2,2}(n-1) & \cdots & r_{2,p}(n-1) & r_{2,p+1}(n-1) & 0 & \mathbf{0} & 0 \\ & & \ddots & & & \ddots & 0 & \vdots \\ & \mathbf{0} & & \ddots & & & \ddots & \\ & & & & \ddots & & \vdots & r_{N\text{-}p+1,N}(n-1) \\ & & & & & r_{N\text{-}2,N-2}(n-1) & r_{N-2,N-1}(n-1) & \vdots \\ 0 & 0 & \cdots & & & 0 & r_{N-1,N-1}(n-1) & r_{N-1,N}(n-1) \\ 0 & 0 & \cdots & & & & 0 & r_{N,N}(n-1) \end{bmatrix} \tag{18}$$

The items of (16) are now modified as

$$\begin{aligned} &\boldsymbol{\Phi}(n) = \begin{bmatrix} \sqrt{\lambda}\widetilde{\mathbf{R}}_p(n-1) \\ \mathbf{y}^H(n) \end{bmatrix}, \\ &\mathbf{b}(n) = \begin{bmatrix} \sqrt{\lambda}\widetilde{\mathbf{R}}_p(n-1)\mathbf{w}(n-1) \\ z^*(n) \end{bmatrix} = \begin{bmatrix} \sqrt{\lambda}\mathbf{s}(n-1) \\ z^*(n) \end{bmatrix}. \end{aligned} \tag{19}$$

The QRD is then applied to solve for (19) by eliminating $\mathbf{y}^H(n)$ sequentially using Givens rotations. Since the elements of both the diagonal and the ( $p-1$ ) off-diagonals are retained, the back substitution consists of two parts. The first part, for the nonzero diagonal and off-diagonal entries, is similar to the conventional back-substitution of the QR-RLS algorithm. It has a complexity of $O(Np)$. The second part consists of the approximation resulting from the zero off-diagonal entries, and it can be updated at a complexity of $O(N)$. The *p*-TA-QR-LS algorithm is summarized in Table 2. It can be seen that when $p=1$ and $p=N$, the proposed algorithm reduces respectively to the A-QR-LS and the QR-RLS algorithms. With this additional flexibility, the new algorithm can be tailored for various applications with different performance and complexity requirements. It is also possible to select a $p$ that yields an algorithm with low complexity while the performance remains comparable to that of the QR-RLS algorithm. Due to page limitation, the mean convergence analysis of the *p*-A-QR-LS algorithm is omitted here. Interested readers are referred to [7] for more details.

## IV. NUMERICAL RESULTS

In this section, numerical results are presented to illustrate the performance of the blind MOE receiver, where the *p*-A-QR-LS algorithms are used to recursively update the adaptive weight vector $\mathbf{w}_a$. Consider an asynchronous DS-CDMA system in the frequency-selective fading channel. Assume that the multipath channel has $L=4$ paths, and the multipath intensity profile decays exponentially. There are $K=10$ users with $L_c=31$ Gold code spreading sequences. The interference-to-signal ratio of the interferers is 5-dB, i.e., $10\cdot\log 10(E_k/E_1)=5$. A set of $p$ is tested with the same forgetting factor $\lambda=0.995$. The results are averaged over 600 Monte-Carlo trials. Fig. 2 depicts the output power of the MOE detector, $\left|\mathbf{w}^H(n)\mathbf{y}(n)\right|^2$, versus the data samples. The MSE performance, $E\{\left|b_1(n)-\hat{b}_1(n)\right|^2\}$, is illustrated in Fig. 3. It shows that the algorithm with larger value of $p$ has faster initial convergence, which complies well with our previous analyses.

1. *Initialization*

$\mathbf{w}=[0,0,\cdots,0]^T$, $\boldsymbol{\delta}=[1,1,\cdots,1]^T$, $\alpha_0=1$

2. *Recursive Operations*

$\mathbf{b}=\widetilde{\mathbf{R}}_p\mathbf{w}, \boldsymbol{\delta}'=\sqrt{\lambda}\boldsymbol{\delta}$,

*Upper triangular Algorithm*

*For* $i=1,2,\cdots,N$ *Loop*

$\alpha'=\alpha_{i-1}[\widetilde{b}_{N+1,i}^{(i)}]^*$, $\delta_i=\delta_i'+\alpha'\widetilde{b}_{N+1,i}^{(i)}$,

$\sigma_i=\delta_i'/\delta_i$, $\rho_i=\alpha'/\delta_i$,

$\alpha_i=\alpha_{i-1}\sigma_i$, $\tau=b_{i,N+1}$,

$b_{i,N+1}''=\sigma_i b_{i,N+1}+\rho_i b_{N+1,N+1}^{(i)}$,

$b_{N+1,N+1}^{(i+1)}=b_{N+1,N+1}^{(i)}-\tau b_{N+1,i}$

} $8N$ Multiplications, $3N$ Additions

*If* $i>N-p$ *or* $j<i+p$

$\widetilde{b}_{N+1,j}^{(i+1)}=\widetilde{b}_{N+1,j}^{(i)}-\widetilde{b}_{i,j}\widetilde{b}_{N+1,i}^{(i)}$,

$\widetilde{b}_{i,j}''=\sigma_i\widetilde{b}_{i,j}+\rho_i\widetilde{b}_{N+1,j}^{(i)}$

} (3/2)*(2$N$-$p$)($p$-1) Multiplications, (2$N$-$p$)($p$-1) Additions

*End*

*End of Loop*

*Back Substitution*

$\gamma_{1\sim N-1}=0, \widetilde{w}(N)=b_{N,N+1}''$

*For* $i=N-1,N-2,\cdots,N-p+1$ *Loop*

*For* $j=N,N-1,\cdots,i+1$ *Loop*

$\gamma_i=\gamma_i+\widetilde{b}_{i,j}''\widetilde{w}(j)$

*End of Loop*

$\widetilde{w}(i)=b_{i,N+1}''-\gamma_i$

} $p$($p$-1)/2 Multiplications, $p$($p$-1) /2 Additions

*End of Loop*

$\hat{\gamma}_{N-p+1}=0$

*For* $i=N-p,N-p-1,\cdots,2,1$ *Loop*

$\hat{\gamma}_i=\hat{\gamma}_{i+1}+b_{N+1,j+1}\widetilde{w}(i+p)$

$\hat{b}_{i,N+1}''=b_{i,N+1}''-\rho_i\hat{\gamma}_i$

*For* $j=i+p,\cdots,i+1$ *Loop*

$\gamma_i=\gamma_i+\widetilde{b}_{i,j}''\widetilde{w}(j)$

*End of Loop*

$\widetilde{w}(i)=\hat{b}_{i,N+1}''-\gamma_i$

} ($N$-$p$)($p$+1) Multiplications, ($N$-$p$)($p$+1) Additions

*End of Loop*

Table 2. Square root free Givens rotation-based $p$-A-QR-LS algorithm.

## V. CONCLUSIONS

A new family of approximate QR-based LS adaptive algorithms, called $p$-A-QR-LS algorithms, is presented for MOE detection in DS-CDMA communications systems. It retains a different number of diagonal plus off-diagonals in the triangular factor of the augmented data matrix and generates a series of new QR-based algorithms with different complexity-performance tradeoffs. Simulation results were presented to illustrate the performance of the algorithm.

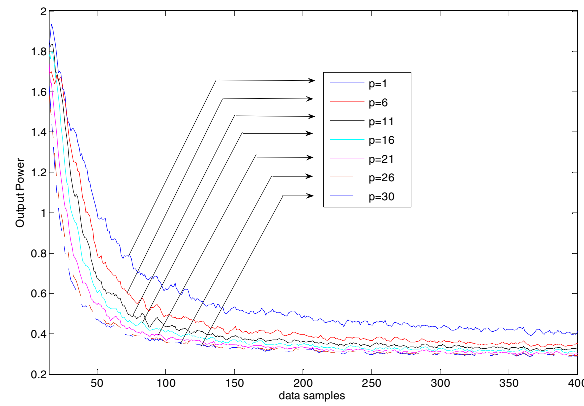


Fig. 2. The averaged output power of the MOE detection.

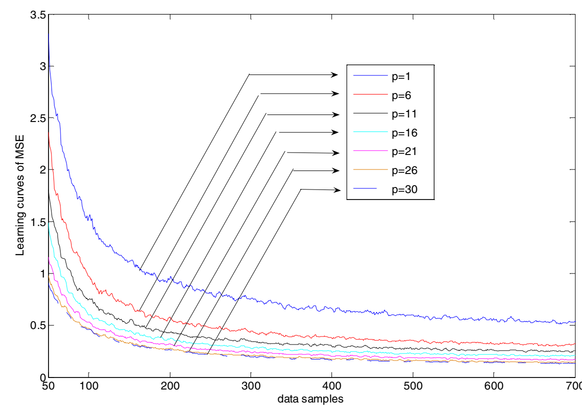


Fig. 3. Steady-state MSE after equal gain combining.

## REFERENCES


[1] U. Madhow and M. Honig, "MMSE interference suppression for direct sequence spread spectrum CDMA", *IEEE Trans. Commun.*, vol. 42, pp. 3178-88, 1994.

[2] M. Honing, U. Madhow, and S. Verdu, "Blind adaptive multiuser dectection", *IEEE Trans. Inform. Theory*, pp. 944-960, 1995.

[3] M. K. Tsatsanis and Z. Y. Xu, "Performance analysis of minimum variance CDMA receiver", *IEEE Trans. Commun.*, vol. 46, pp. 3014-3022, 1998.

[4] O. L. Frost, "An algorithm for linearly constrained adaptive array processing", *Proc. IEEE*, vol. 60, pp. 926-935, 1972.

[5] D. G. Manolakis, V. K. Ingle and S. M. Kogon, *Statistical and Adaptive Signal Processing*, McGraw-Hill, 2000.

[6] Z. S. Liu. "QR methods of $O(N)$ complexity in adaptive parameter estimation," *IEEE Trans. Signal Proc.*, vol. 43, pp. 720-729, 1995.

[7] S. C. Chan, "A new family of approximate QR-based algorithms for adaptive filtering," Internal report, Department of EEE, The University of Hong Kong, 2003.

[8] S. C. Chan and X. X. Yang, "Improved Approximate QR-LS Algorithms for Adaptive Filtering", *IEEE Trans. on Circuits and Systems II: Express Briefs*, vol. 51, no. 1, pp. 29-39, 2004.

[9] M. K. Tsatsanis, "Inverse filtering criteria for CDMA systems", *IEEE Trans. Signal Processing*, vol. 45, pp. 102-112, 1997.