TABLE II
PERFORMANCE COMPARISON BETWEEN INDEPENDENT TRAINING, NCCD (ON
THE ENSEMBLES OF 1) FOUR MLPS, 2) FOUR RBFS, AND 3) TWO RBFS PLUS
TWO MLPS AND 4) FOUR SVRS ON AUSTRALIAN CREDIT CARD ASSESSMENT.
n_{ex} REPRESENTS THE AMOUNT OF REQUIRED NETWORK COMMUNICATIONS.
NOTE THAT CELS IS ONLY APPLICABLE TO ENSEMBLES OF MLPS

Ensemble	Indep.learning	NCCD		CELS	
	Mean Err.	Mean Err.	n_{ex}	Mean Err.	n_{ex}
4 MLPs	12.4%	12.2%	100	12.0%	12,950
2 RBFs+2 MLPs	12.9%	12.6%	100	/	/
4 RBFs	13.9%	12.9%	100	/	/
4 SVRs	14.0%	13.4%	100	/	/

10^{-4} and $\varepsilon = 0.1$, and they are trained using the quadratic programming method. Note that neither the RBFs nor SVRs use backpropagation-type learning algorithms.

Results are shown in Table II. Although each ensemble performs differently from each other, NCCD improves the generalization performance of all by 0.2%–1.0%. Comparing with CELS on the ensemble of four MLPs, NCCD scores slightly higher error (12.2% c.f. 12.0%), yet requiring 130 time lesser network communications $((M \times g_{\mathrm{tot}}/g_{\mathrm{update}}) = (4 \times 25/1) = 100$ c.f. $(N \times g_{\mathrm{tot}}) = (518 \times 25) = 12\,950)$. NCCD is clearly the more cost-effective method.

V. CONCLUSION

The theory and experiments have demonstrated the proposed NC learning method.

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Exponential ε-Regulation for Multi-Input Nonlinear Systems Using Neural Networks

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Abstract—This paper considers the problem of robust exponential ε-regulation for a class of multi-input nonlinear systems with uncertainties. The uncertainties appear not only in the feedback channel but also in the control channel. Under some mild assumptions, an adaptive neural network control scheme is developed such that all the signals of the closed-loop system are semiglobally uniformly ultimately bounded and, under the control scheme with initial data starting in some compact set, the states of the closed-loop system is guaranteed to exponentially converge to an arbitrarily specified ϵ-neighborhood about the origin. The important contributions of the present work are that a new exponential uniformly ultimately bounded performance is proposed and that the design parameters and initial condition set can be determined easily. The development generalizes and improves earlier results for the single-input case.

Index Terms—Adaptive control, multi-input system, neural network, nonlinear system, uncertain system, uniform ultimate boundedness.

I. INTRODUCTION

The study of uncertain nonlinear systems has been one of the most active research topics in recent years (see, for example, [3]–[5], [7]–[9] and references therein). Neural network techniques have been found to be particularly useful for controlling nonlinear systems with uncertainties [5], [8]. Neural network approximators can be used to parameterize an unknown nonlinear function over a compact set to any degree of accuracy [2]. Representative work can be found in, to just name a few, [5], [6], [8]–[11], and [13]. By using neural network, Polycarpou and Mears [8] developed control laws that guarantee semiglobal uniform ultimate boundedness. Zhang *et al.* [11], [12] developed control laws to solve the control problem with uncertainty in control channel. All the plants considered in [8] and [10]–[12] are single-input nonlinear systems with uncertainties. Recently, Kwan and Lewis [5] developed control laws to control a more general class of multi-input plants without uncertainty in the control channel. It is noted that the developments in [5], [8], and [10]–[12] all guarantee uniform ultimate boundedness of the closed-loop signals. That is, all the closed-loop signals converge not to a point but to a ball-type residual set (asymptotic bounding). As the magnitude of the uncertain nonlinearities increases, the size of the residual set may also increase. Therefore, these control schemes may not be applicable if the uncertain nonlinearities dominate the system dynamics to the extent that the achievable residual set is beyond the application range.

Motivated by [5], [8], and [10]–[12], this paper develops a dynamic feedback control scheme based on neural networks for a new class of multi-input plants with dominating uncertain nonlinearity in the feedback channel and time-invariant uncertainty in the control channel. The

control problem with multi-input nonlinear systems of a similar structure with control channel uncertainty has been seldom addressed in the past. Moreover, an improved uniformly ultimately bounded (UUB) performance is employed in this paper. One major contribution of this paper is to formally establish that, for any initial data starting in a compact set, the states of the closed-loop system exponentially converge to an arbitrarily specified neighborhood about the origin. In particular, the control scheme developed guarantees that all signals of the closed-loop system are semiglobally UUB. An advantage of the approach is that the design parameters of the controller can be easily determined.

The organization of this paper is as follows. Section II presents a class of multi-input nonlinear systems with uncertainties. Properties of general three-layer neural networks used in the controller will be described. The controller design is given in Section III, with its performance analyzed in Section IV. The paper ends with concluding remarks in Section V.

Notation: Throughout this paper, the superscript τ represents the transpose. All vectors and matrices are assumed to be real. $\|.\|_F$ denotes the Frobenius norm of a matrix. $\|.\|$ denotes the Euclidean norm of a vector or the induced matrix norm (spectral norm) of a matrix. A real matrix $P > (\geq)0$ denotes P being a real symmetric positive definite (or positive semidefinite) matrix, and $A > (\geq)B$ means $A - B > (\geq)0$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations. Throughout this paper, we may also simplify the notations by dropping their arguments whenever no ambiguity arises.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following multi-input nonlinear system with uncertainties:

$$\dot{x} = \mathcal{F}(x) + \mathcal{G}u \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state and $u(t) \in \mathbb{R}^n$ is the control input. The vector-valued continuous function $\mathcal{F}$ and constant matrix $\mathcal{G}$ are unknown.

In order to formulate the problem considered in this paper, we give the following definition.

Definition 1: Given a nonlinear system

$$\dot{x} = f(x, t), \quad x \in \mathbb{D}, \quad t \geq t_0$$

where $\mathbb{D}$ is a domain contained in $\mathbb{R}^n$. If

1) there exists an open sphere $\tilde{\Omega}_0 \subset \mathbb{D}$ containing the origin and at least one point $p \neq 0$;
2) there exist a $\gamma > 0$ and a number $T(x(t_0), \gamma)$ such that $\|x(t)\| < \gamma$ for all $t \geq t_0 + T$ and any open sphere $\Omega_0 \subset \tilde{\Omega}_0$ with Ω_0 containing the origin and at least one point $\hat{p} \neq 0$ such that for all $x(t_0) \in \Omega_0$;

then a solution of the system is said to be *uniformly ultimately bounded* (UUB) with open sphere $\tilde{\Omega}_0$.

Remark 1: The UUB property is a local property on a set (open sphere $\tilde{\Omega}_0$) of initial data. If $\mathbb{D} = \mathbb{R}^n$, $\tilde{\Omega}_0 = \mathbb{R}^n$, the solution of the system is then said to be *globally uniformly ultimately bounded* (GUUB). If $\mathbb{D} = \mathbb{R}^n$, $\tilde{\Omega}_0 = \{x : \|x\| < \rho\}$ where ρ can be chosen arbitrarily large, the solution of the system is said to be *semiglobally uniformly ultimately bounded* (SGUUB). It should be pointed out that the definitions of UUB and GUUB are the same as those given in [4]. The initial data set $\tilde{\Omega}_0$ is given here mainly for providing the definition of SGUUB.

Motivated by the fact that the UUB performance of the closed-loop signals, especially those of the system states, may be beyond the allowable domain of application for the control scheme, we propose a performance called *exponential ε-regulation* (EX ε-REG) for uncertain nonlinear control systems. In particular, the problem we considered is exponential ε-regulation of system (1) by dynamic feedback.

EX ε-REG Performance Control: For any given scalar constant $\varepsilon > 0$, find a dynamic feedback law of the form

$$\dot{\Pi} = \pi(\Pi, x, \varepsilon) \tag{2}$$

$$u = \alpha(\Pi, x, \varepsilon) \tag{3}$$

such that there exist

1) two open spheres $\tilde{\Omega}_0$ and $\tilde{\Pi}_0$, for any closed sphere $\Omega_0 \subset \tilde{\Omega}_0$ and $\Pi_0 \subset \tilde{\Pi}_0$, the solutions of the closed-loop system (1)–(3) are UUB with open spheres $\tilde{\Omega}_0$ and $\tilde{\Pi}_0$ for all initial conditions $(x(t_0), \Pi(t_0)) \in \Omega_0 \times \Pi_0$;
2) two compact sets $\bar{\Omega}_0 \subset \tilde{\Omega}_0$ and $\bar{\Pi}_0 \subset \tilde{\Pi}_0$, $\omega[x(t_0), \Pi(t_0), \varepsilon] > 0$ and $\beta[x(t_0), \Pi(t_0), \varepsilon] > 0$ such that for all $t \geq t_0$

$$\|x(t)\| \leq \omega e^{-\beta(t - t_0)} + \varepsilon$$

for any initial conditions $(x(t_0), \Pi(t_0)) \in \bar{\Omega}_0 \times \bar{\Pi}_0$.

Remark 2: EX ε-REG is a local property defined on a compact of initial data. As in Definition 1, it is required that all the relevant sets contain the origin and at least one point not being the origin.

Remark 3: Compared with UUB, exponential ε-regulation performance needs more information of the plant and results in a more complex controller. Due to its more superior characteristics in general, EX ε-REG performance may be more practical and useful than UUB performance.

Under certain mild assumptions, the EX ε-REG performance control of (1) can be solved by using adaptive neural network techniques. The following assumptions for (1) will be used as a basis in the development of this paper.

Assumption 1: For any given admissible control u and initial condition $x(t_0) \in \mathbb{R}^n$, there exists a unique trajectory $x(t)$ of (1) starting at $x(t_0)$.

Remark 4: In general, finite escape time [4] may occur for (1), as one has no constraints on $\mathcal{F}(x)$ except continuity.

Assumption 2: The uncertain matrix $\mathcal{G}$ in the control channel satisfies

$$0 < G \leq \mathcal{G} \leq H \tag{4}$$

where G and H are known constant positive definite matrices.

Remark 5: Assumption 2 is a natural multi-input extension of the single-input case where the control channel with uncertainty has known control direction (for example, [10] and [11]). The knowledge of $\mathcal{G}$ is crucial for the construction of the controller such that the system (1) has the EX ε-REG performance.

In the sequel, we will make use of the universal approximation property of a feedforward network to estimate the unknown parts of the given system. For a vector $z = (z_1, \ldots, z_m)^\tau \in \mathbb{R}^m$, the sigmoidal activation Φ is defined as

$$\Phi[z] = (\phi(z_1), \ldots, \phi(z_m))^\tau$$

where

$$\phi(z_i) = \frac{1}{1 + e^{-\alpha z_i}}, \quad i = 1, \ldots, m$$

with $\alpha > 0$. A three-layer feedforward network is used

$$y = W^\tau \Phi[V^\tau \bar{x}]$$

where x is the input, y is the output, $\bar{x} = [x^\tau \ 1]^\tau$, and $W \in \mathbb{R}^{m \times n}$ and $V \in \mathbb{R}^{(n+1) \times m}$ are weighting matrices.

Select an appropriately large number $0 < R < +\infty$ and define a compact set

$$\Omega = \left\{ x \in \mathbb{R}^n : \frac{1}{2} x^\tau H^{-1} x \leq R \right\}.$$

For the compact set $\Omega \subset \mathbb{R}^n$, one can define a function $F(x)$ as follows:

$$F(x) = \mathcal{G}^{-1} \mathcal{F}(x), \quad x \in \Omega. \tag{5}$$

According to the universal approximation theorem [2] for the function $F(x)$ defined on compact set Ω, and for any given scalar constant $\mu > 0$, there exist ideal weighting matrices W^* and V^* such that the unknown nonlinear function $F(x)$ can be approximated as follows:

$$F(x) = F^*(x) + \delta F(x) = W^{*\tau} \Phi[V^{*\tau} \bar{x}] + \delta F(x) \tag{6}$$

with $\| \delta F(x) \| < \mu$. In general, the ideal weights W^* and V^* are unknown and need to be estimated as part of the controller design process. Let $\hat{W}$ and $\hat{V}$ be the estimates of W^* and V^*. Define the corresponding estimation errors as

$$\tilde{W} = \hat{W} - W^*, \quad \tilde{V} = \hat{V} - V^*. \tag{7}$$

Let

$$\hat{F} = \hat{W}^\tau \Phi[\hat{V}^\tau \bar{x}]. \tag{8}$$

The approximation error $\delta \hat{F}$ can be written as

$$\begin{aligned}
\delta \hat{F} &= \hat{F} - F^* \\
&= \tilde{W}^\tau \left[\Phi[\hat{V}^\tau \bar{x}] - \Phi'[\hat{V}^\tau \bar{x}](\hat{V}^\tau \bar{x}) \right] \\
&\quad + \hat{W}^\tau \Phi'[\hat{V}^\tau \bar{x}] \tilde{V}^\tau \bar{x} + \eta
\end{aligned} \tag{9}$$

where $\Phi'[z] = \partial \Phi[z]/\partial z^\tau$. Then the following lemma can be established using the same line as in [6], [10], and [11].

Lemma 1: For sigmoidal activation Φ, one has

$$\| \eta \| \leq a_1 \| V^* \|_F \| \bar{x} \| \| \hat{W} \| + a_2 \| W^* \|_F \| \bar{x} \| \| \hat{V} \| + a_3 \| W^* \|_F \tag{10}$$

where $a_i > 0$, $i = 1, 2, 3$, are constants that can be computed by using Φ and the dimensions of W^* and V^*.

III. CONTROL DESIGN

Define a storage function

$$E(x, \hat{V}, \hat{W}) = \frac{1}{2} x^\tau \mathcal{G}^{-1} x + \frac{1}{2} \text{tr}[\tilde{W}^\tau P \tilde{W}] + \frac{1}{2} \text{tr}[\tilde{V}^\tau Q \tilde{V}]. \tag{11}$$

After some algebraic manipulation, the time derivative of $E(x, \hat{V}, \hat{W})$ along the solution of the system (1) is given by

$$\dot{E}(x, \hat{V}, \hat{W}) = x^\tau \left[\mathcal{G}^{-1} \mathcal{F}(x) + u \right] + \text{tr}[\tilde{W}^\tau P \dot{\tilde{W}}] + \text{tr}[\tilde{V}^\tau Q \dot{\tilde{V}}].$$

It follows from (5), (6), (8), and (9) that

$$\begin{aligned}
\dot{E}(x, \hat{V}, \hat{W}) &= x^\tau \left\{ u + \delta F + \hat{F} - \hat{W}^\tau \Phi'[\hat{V}^\tau \bar{x}] \tilde{V}^\tau \bar{x} \right. \\
&\quad \left. - \eta - \tilde{W}^\tau \left[\Phi[\hat{V}^\tau \bar{x}] - \Phi'[\hat{V}^\tau \bar{x}](\hat{V}^\tau \bar{x}) \right] \right\} \\
&\quad + \text{tr}[\tilde{W}^\tau P \dot{\tilde{W}}] + \text{tr}[\tilde{V}^\tau Q \dot{\tilde{V}}] \\
&= \text{tr}[\tilde{W}^\tau P \dot{\tilde{W}}] + \text{tr}[\tilde{V}^\tau Q \dot{\tilde{V}}] + x^\tau \delta F - x^\tau \eta \\
&\quad + x^\tau \left\{ u + \hat{F} - \hat{W}^\tau \Phi'[\hat{V}^\tau \bar{x}] \tilde{V}^\tau \bar{x} - \tilde{W}^\tau \right. \\
&\quad \left. \times \left[\Phi[\hat{V}^\tau \bar{x}] - \Phi'[\hat{V}^\tau \bar{x}](\hat{V}^\tau \bar{x}) \right] \right\}. \tag{12}
\end{aligned}$$

Given any $k > 0$, noting that

$$k \left(x - \frac{\delta F}{2k} \right)^\tau \left(x - \frac{\delta F}{2k} \right) \geq 0$$

one has

$$x^\tau \delta F \leq k x^\tau x + \frac{\| \delta F(x) \|^2}{4k}. \tag{13}$$

It follows from (13) and $\| \delta F(x) \| \leq \mu$ that

$$x^\tau \delta F \leq k x^\tau x + \frac{\mu^2}{4k}. \tag{14}$$

Similarly, one obtains

$$\begin{aligned}
-x^\tau \eta &\leq k x^\tau x \left(\| \bar{x} \| \| \hat{W} \| \right)^2 + \frac{a_1^2 \| V^* \|_F^2}{4k} + k x^\tau x \\
&\quad + \frac{a_2^2 \| W^* \|_F^2}{4k} + k x^\tau x \left(\| \bar{x} \| \| \hat{V} \| \right)^2 + \frac{a_3^2 \| W^* \|_F^2}{4k}.
\end{aligned}$$

Then, we have

$$\begin{aligned}
x^\tau (\delta F - \eta) &\leq x^\tau \left[k x \left(\| \bar{x} \| \| \hat{W} \| \right)^2 + 2 k x + k x \left(\| \bar{x} \| \| \hat{V} \| \right)^2 \right] \\
&\quad + \frac{a_1^2 \| V^* \|_F^2 + \left(a_2^2 + a_3^2 \right) \| W^* \|_F^2 + \mu^2}{4k} \tag{15}
\end{aligned}$$

where k is a scalar constant to be chosen. It follows from (12) and (15) that

$$\begin{aligned}
\dot{E}(x, \hat{V}, \hat{W}) &\leq x^\tau (u + \hat{F} + 2kx) - \text{tr} \left[\tilde{V}^\tau \bar{x} x^\tau \hat{W}^\tau \Phi'(\hat{V}^\tau \bar{x}) \right] \\
&\quad + x^\tau \left[\left(\| \bar{x} \| \| \hat{W} \| \right)^2 + \left(\| \bar{x} \| \| \hat{V} \| \right)^2 \right] kx \\
&\quad - \text{tr} \left\{ \tilde{W}^\tau \left[\Phi[\hat{V}^\tau \bar{x}] - \Phi'[\hat{V}^\tau \bar{x}](\hat{V}^\tau \bar{x}) \right] x^\tau \right\} \\
&\quad + \frac{a_1^2 \| V^* \|_F^2 + \left(a_2^2 + a_3^2 \right) \| W^* \|_F^2 + \mu^2}{4k} \\
&\quad + \text{tr}[\tilde{W}^\tau P \dot{\hat{W}}] + \text{tr}[\tilde{V}^\tau Q \dot{\hat{V}}]. \tag{16}
\end{aligned}$$

One can choose the control law and weight updating rule as follows:

$$\begin{aligned}
u &= - \left[\left(\| \bar{x} \| \| \hat{W} \| \right)^2 + \left(\| \bar{x} \| \| \hat{V} \| \right)^2 + 2 \right] kx \\
&\quad - \hat{F} - \sigma G^{-1} x \tag{17}
\end{aligned}$$

$$\dot{\hat{W}} = -2\sigma \hat{W} + P^{-1} \left[\Phi[\hat{V}^\tau \bar{x}] - \Phi'[\hat{V}^\tau \bar{x}](\hat{V}^\tau \bar{x}) \right] x^\tau \tag{18}$$

$$\dot{\hat{V}} = -2\sigma \hat{V} + Q^{-1} \bar{x} x^\tau \hat{W}^\tau \Phi'[\hat{V}^\tau \bar{x}]. \tag{19}$$

By using (16)–(19), one has

$$\begin{aligned}
\dot{E}(x, \hat{V}, \hat{W}) &\leq -\sigma x^\tau \mathcal{G}^{-1} x - 2\sigma \text{tr}[\tilde{W}^\tau P \hat{W}] \\
&\quad + \frac{a_1^2 \| V^* \|_F^2 + \left(a_2^2 + a_3^2 \right) \| W^* \|_F^2 + \mu^2}{4k} \\
&\quad - 2\sigma \text{tr}[\tilde{V}^\tau Q \hat{V}] \tag{20}
\end{aligned}$$

where $x \in \Omega$ and $\sigma > 0$ is a scalar constant to be chosen.

IV. PERFORMANCE ANALYSIS

In this section, we will show, by imposing a further mild assumption, that all the signals in the closed-loop system are SUUB and the states have the EX ε-REG performance. The assumption is very common in the neural control setting [5].

Assumption 3: For the vector-valued function $F(x)$ defined on $x \in \Omega$ in (5), given $\mu > 0$, the ideal weights (6) are bounded by a scalar $C_\mu > 0$. That is

$$\max \left\{ \| W^* \|_F^2, \| V^* \|_F^2 \right\} \leq C_\mu. \tag{21}$$

Theorem 1: For nonlinear system (1) with Assumptions 1–3, the adaptive neural controller (17) and weight updating rule (18) and (19), if the design parameters $\{P, Q, k, \sigma\}$ are chosen such that $P > 0$ and $Q > 0$

$$\|P\| + \|Q\| < \frac{R}{C_\mu} \qquad (22)$$

$$k\sigma > \frac{C_\mu \sum_{i=1}^{3}\left(a_i^2\right) + \mu^2}{4R} \qquad (23)$$

then there exists scalar $R_I > 0$ such that the compact set Ω_I defined by

$$\Omega_I = \left\{(x, \hat{V}, \hat{W}) : E \leq R_I\right\}$$

satisfies that any trajectory starting in Ω_I at t_0 stays in Ω_I for all $t \geq t_0$.

Proof: Consider

$$\frac{1}{2}\mathrm{tr}[W^{*\tau}PW^*] + \frac{1}{2}\mathrm{tr}[V^{*\tau}QV^*]$$
$$\leq \frac{1}{2}\|P\|\,\|W^*\|_F^2 + \frac{1}{2}\|Q\|\,\|V^*\|_F^2$$
$$\leq \frac{1}{2}\left(\|P\| + \|Q\|\right)C_\mu.$$

Recalling (22) leads to

$$\frac{1}{2}\mathrm{tr}[W^{*\tau}PW^*] + \frac{1}{2}\mathrm{tr}[V^{*\tau}QV^*] \leq \frac{R}{2}.$$

It is easy to see that

$$\frac{a_1^2\|V^*\|_F^2 + \left(a_2^2 + a_3^2\right)\|W^*\|_F^2 + \mu^2}{8k\sigma} \leq \frac{C_\mu \sum_{i=1}^{3}\left(a_i^2\right) + \mu^2}{8k\sigma} < \frac{R}{2}.$$

Let

$$M = \frac{a_1^2\|V^*\|_F^2 + \left(a_2^2 + a_3^2\right)\|W^*\|_F^2 + \mu^2}{4k}$$
$$+ \sigma\mathrm{tr}[V^{*\tau}QV^*] + \sigma\mathrm{tr}[W^{*\tau}PW^*]. \qquad (24)$$

Then, we have

$$\frac{M}{2\sigma} \leq \frac{C_\mu \sum_{i=1}^{3}\left(a_i^2\right) + \mu^2}{8k\sigma} + \frac{1}{2}\left(\|P\| + \|Q\|\right)C_\mu < R.$$

Let

$$\frac{C_\mu \sum_{i=1}^{3}\left(a_i^2\right) + \mu^2}{8k\sigma} + \frac{1}{2}\left(\|P\| + \|Q\|\right)C_\mu < R_I \leq R.$$

Then, one has $M/2\sigma < R_I$. It is shown that

$$(x, \hat{V}, \hat{W}) \in \Omega_I \Rightarrow \frac{1}{2}x^\tau H^{-1}x \leq \frac{1}{2}x^\tau \mathcal{G}^{-1}x \leq E \leq R_I \leq R$$
$$\Rightarrow x \in \Omega. \qquad (25)$$

Now, it is easy to verify that

$$\frac{1}{2}\mathrm{tr}[\tilde{W}^\tau P\tilde{W}] - \frac{1}{2}\mathrm{tr}\left[(\tilde{W} - \hat{W})^\tau P(\tilde{W} - \hat{W})\right] \leq \mathrm{tr}[\tilde{W}^\tau P\hat{W}].$$

Observing (7), one has

$$\frac{1}{2}\mathrm{tr}[\tilde{W}^\tau P\tilde{W}] - \frac{1}{2}\mathrm{tr}[W^{*\tau}PW^*] \leq \mathrm{tr}[\tilde{W}^\tau P\hat{W}] \qquad (26)$$

$$\frac{1}{2}\mathrm{tr}[\tilde{V}^\tau Q\tilde{V}] - \frac{1}{2}\mathrm{tr}[V^{*\tau}QV^*] \leq \mathrm{tr}[\tilde{V}^\tau P\hat{V}]. \qquad (27)$$

It follows from (20), (26), and (27) that

$$\dot{E}(x, \hat{V}, \hat{W}) \leq -\sigma x^\tau \mathcal{G}^{-1}x - \sigma\mathrm{tr}[\tilde{W}^\tau P\tilde{W}] - \sigma\mathrm{tr}[\tilde{V}^\tau Q\tilde{V}]$$
$$+ \frac{a_1^2\|V^*\|_F^2 + \left(a_2^2 + a_3^2\right)\|W^*\|_F^2 + \mu^2}{4k}$$
$$+ \sigma\mathrm{tr}[W^{*\tau}PW^*] + \sigma\mathrm{tr}[V^{*\tau}QV^*]. \qquad (28)$$

Then, using (11), one has

$$\dot{E}(x, \hat{V}, \hat{W}) \leq -2\sigma E + \sigma\mathrm{tr}[W^{*\tau}PW^*] + \sigma\mathrm{tr}[V^{*\tau}QV^*]$$
$$+ \frac{a_1^2\|V^*\|_F^2 + \left(a_2^2 + a_3^2\right)\|W^*\|_F^2 + \mu^2}{4k}. \qquad (29)$$

Therefore, we have

$$\dot{E} \leq -2\sigma E + M = -2\sigma\left(E - \frac{M}{2\sigma}\right) \qquad (30)$$

with $x \in \Omega$. This implies that (30) holds for $(x, \hat{V}, \hat{W}) \in \Omega_I$. It is shown that Ω_I is a positively invariant set of the closed-loop system since $\dot{E}$ is negative on the boundary of Ω_I. The result follows. $\qquad \square$

Remark 6: Theorem 1 shows that all the signals of the closed-loop system are UUB. The semiuniform ultimate boundedness can be concluded by considering that the parameter R can be chosen appropriately large. Since Ω_I is a positively invariant set of the closed-loop system, one has that any trajectory starting in Ω_I at t_0 stays in Ω_I for all $t \geq t_0$, that is, $(x(t), \hat{W}(t), \hat{V}(t)) \in \Omega_I$. Then, it follows by (25) that $x(t) \in \Omega$, which is the key condition to guarantee the satisfaction of (20).

In what follows, we will show that the signal $x(t)$ in the closed loop has the EX ε-REG performance. The establishment of the conditions for EX ε-REG performance requires the following lemma.

Lemma 2: [1] Let $E : [0, \infty) \to R$ satisfy the inequality

$$\dot{E} \leq -2\sigma E + M$$

where $\sigma > 0$ and $M > 0$ are scalar constants. Then for any $t \geq t_0$

$$E(t) \leq E(t_0)e^{-2\sigma(t-t_0)} + \frac{M}{2\sigma}.$$

Theorem 2: Consider nonlinear system (1) under Assumptions 1–3. For any scalar constant $\varepsilon > 0$, if the design parameters $\{P, Q, k, \sigma\}$ of the adaptive neural controller (17) and weight updating rule (18) and (19) are chosen such that $P > 0$, $Q > 0$

$$\|P\| + \|Q\| < \min\left\{\frac{2\varepsilon^2}{3C_\mu\|H\|}, \frac{R}{C_\mu}\right\} \qquad (31)$$

$$k\sigma > \max\left\{\frac{3\|H\|\left[C_\mu \sum_{i=1}^{3}\left(a_i^2\right) + \mu^2\right]}{4\varepsilon^2}\right.$$
$$\left.\frac{C_\mu \sum_{i=1}^{3}\left(a_i^2\right) + \mu^2}{4R}\right\} \qquad (32)$$

then, for any $(x(t_0), \hat{W}(t_0), \hat{V}(t_0)) \in \Omega_I$, the state vector $x(t)$ of the closed-loop system satisfies

$$\|x(t)\| \le \sqrt{2\|H\| E\left(x(t_0), \hat{V}(t_0), \hat{W}(t_0)\right)} \exp(-\sigma(t - t_0)) + \varepsilon.$$

Proof: It is easy to verify that the conditions of Theorem 1 hold. For any initial data $(x(t_0), \hat{W}(t_0), \hat{V}(t_0)) \in \Omega_I$, by using Theorem 1, we know that trajectory starting in Ω_I at t_0 stays in Ω_I for all $t \ge t_0$, that is, $(x(t), \hat{W}(t), \hat{V}(t)) \in \Omega_I$. By employing the arguments in the proof of Theorem 1, we know that

$$\dot{E}\left(x(t), \hat{W}(t), \hat{V}(t)\right) \le -2\sigma E\left(x(t), \hat{W}(t), \hat{V}(t)\right) + M.$$

By Lemma 1, we have

$$E(t) \le E(t_0)e^{-2\sigma(t-t_0)} + \frac{M}{2\sigma}. \tag{33}$$

Since $(1/2)\mathrm{tr}[\tilde{W}^\tau P\tilde{W}] + (1/2)\mathrm{tr}[\tilde{V}^\tau Q\tilde{V}] \ge 0$, it is easy to see from (11) and (33) that

$$\frac{1}{2}x^\tau \mathcal{G}^{-1}x \le E(t) \le E(t_0)e^{-2\sigma(t-t_0)} + \frac{M}{2\sigma}.$$

Recalling (4), one has

$$\frac{1}{2}x^\tau H^{-1}x \le E(t) \le E(t_0)e^{-2\sigma(t-t_0)} + \frac{M}{2\sigma}$$

which implies

$$\|x\| \le \sqrt{2\|H\| E(t_0)}e^{-\sigma(t-t_0)} + \sqrt{\frac{\|H\| M}{\sigma}}. \tag{34}$$

It is clear that

$$\|P\| + \|Q\| < \frac{2\varepsilon^2}{3C_\mu \lambda_{\max}(H)}$$

$$\Rightarrow \|H\| \left(\mathrm{tr}[V^{*\tau}PV^*] + \mathrm{tr}[W^{*\tau}QW^*]\right) < \frac{2\varepsilon^2}{3}$$

$$k\sigma > \frac{3\|H\| \left[C_\mu \sum_{i=1}^3 \left(a_i^2\right) + \mu^2\right]}{4\varepsilon^2}$$

$$\Rightarrow \frac{\|H\| \left[a_1^2\|V^*\|_F^2 + \left(a_2^2 + a_3^2\right)\|W^*\|_F^2 + \mu^2\right]}{4k\sigma} < \frac{\varepsilon^2}{3}.$$

Recalling (24), we have

$$\sqrt{\frac{\|H\| M}{\sigma}} < \varepsilon. \tag{35}$$

The proof can be concluded by using (34) and (35). $\qquad\square$

Remark 7: It is noted that the bounds of $\mathcal{G}$ in (4) by G and H are very useful. The matrix G is used for constructing the controller (17) while H affects the selection of P and Q in (31). It is easy to see from (32) that there is a tradeoff between the gain-damping product $k\sigma$ and

ε. When ε is required to be small, a large $k\sigma$ is needed. Notice that a large damping of $x(t)$ (that is, σ is large) does not require a large k, as seen in (32). However, a compromise should be reached between k and σ since they both affect the control magnitude [see (17)].

Remark 8: If inequality (4) in Assumption 2 is replaced by $G \le \mathcal{G} \le H < 0$, where G and H are known constant negative definite matrices, the EX ε-REG performance control problem can be solved by using the same technique.

Remark 9: In practice, the bound C_μ of the ideal weights may not be obtained easily. In this case, for some given $\varepsilon > 0$, $R > 0$ and $\mu > 0$, one can obtain appropriate values of $k\sigma$ and b by increasing the values of $k\sigma$ and decreasing the values of $\|P\| + \|Q\|$ step by step to achieve the desired ε-regulation. Note that large $k\sigma$ may generate large control signal. Thus, there is a tradeoff between control energy and regulation performance.

V. Conclusion

In this paper, the EX ε-REG performance control problem is formulated for a class of multi-input systems with dominating uncertain nonlinearity in the feedback channel and time-invariant uncertainty in the control channel. A dynamic feedback control scheme based on neural networks is developed for solving this problem. Under certain mild assumptions, it is shown that the semiglobal uniform ultimate boundedness of the state of the closed-loop system can be achieved. Moreover, for any initial data starting in a compact set, the states of the closed-loop system converge exponentially to an arbitrarily specified ε-neighborhood about the origin. The selection of the controller parameters can be carried out using some explicit inequalities.

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