Time-Domain Signal Detection Based on Second-Order Statistics for MIMO-OFDM Systems

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Abstract—In this paper, a time-domain signal detection algorithm based on second-order statistics (SOS) is proposed for multiple input-multiple output (MIMO)-orthogonal frequency division multiplexing (OFDM) systems over frequency selective fading channels. A new system model in which the $j$th received OFDM block is left shifted by $J$ samples is introduced. Based on some structural properties of the new system model, an equalizer is designed using SOS of the received signals to cancel most of the intersymbol interference. The transmitted signals are then detected from the equalizer output. In the proposed algorithm, only $2P/P$ ($P$ is the number of transmit antennas/users in MIMO-OFDM systems) columns of the channel matrix need to be estimated, and channel length estimation is unnecessary, which is an advantage over existing algorithms. In addition, the proposed algorithm is applicable irrespective of whether the channel length is shorter than, equal to or longer than the cyclic prefix (CP) length. Simulation results verify the effectiveness of the proposed algorithm, and show that it outperforms the existing ones in all cases.

Index Terms—Multiple input-multiple output (MIMO), orthogonal frequency division multiplexing (OFDM), second-order statistics (SOS), signal detection.

I. INTRODUCTION

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ultiple input-multiple output (MIMO) using multiple antennas at both the transmit and the receive ends becomes an important system for future wireless communications [1]–[4] as it has the potential to greatly increase the system capacity without extra bandwidth. Generally, MIMO is applied in two situations. One is in space-time coding systems where the transmission quality [bit-error rate (BER)] is improved due to spatial diversity. The other is in spatial multiplexing or spatial multiple access systems where independent data streams are transmitted over different antennas, thus increasing the transmission rate or improving the system capacity. In this paper, the latter situation is considered. On the other hand, orthogonal frequency division multiplexing (OFDM) technique has been widely utilized in digital audio broadcasting (DAB), digital video broadcasting (DVB), and broadband wireless local area networks (IEEE 802.11a) [5]–[8], due to its ability to resist frequency selective fading. It is, therefore, desirable to combine OFDM with MIMO for high system capacity, as well as better performance. A lot of research interest has thus been attracted to MIMO-OFDM systems in recent years [9]–[17].

In MIMO-OFDM systems over frequency selective fading channels, signal detection can be easily implemented by a set of parallel per-subcarrier signal detectors applicable to flat fading channels [16] when the channel length is shorter than or equal to the cyclic prefix (CP) length. However, when the channel length is longer than the CP length, interblock interference (IBI) occurs and the orthogonal property of subcarriers will be destroyed, resulting in substantial performance degradation of the signal detection algorithm. To the best knowledge of the authors, two indirect algorithms (which require estimating the channel matrix before signal detection) have been proposed for detecting the signals in this case. The first one is a frequency-domain algorithm [17], which applies the conventional detection algorithm for MIMO systems to each subcarrier after modeling the smoothed per-subcarrier received signal similarly to the smoothed received signal of a MIMO system. The second one is a time-domain algorithm [18], in which an equalizer is first inserted to reduce the MIMO channels to ones with channel length shorter than or equal to the CP length. The general signal detection algorithm for MIMO-OFDM systems [16] is then applied. Unfortunately, both algorithms involve estimation of the channel matrix (refers to $\mathbf{H}$ in [17] and $\mathbf{H}$ in [18]) which requires channel length estimation followed by channel coefficient estimation. In general, channel length is estimated using information theoretic criteria such as Akaike’s Information Criterion (AIC) [21] or Minimum Description Length (MDL) [22], which are highly complex and computationally intensive. In addition, accurate channel length estimation is difficult to achieve in practice and estimation error usually occurs, which will degrade the system performance. As for channel coefficient estimation, it is obvious that at least $(1 + L)P/P$ ($P$ is the number of transmit antennas/users in MIMO-OFDM systems, and $L$ is the maximum channel length) pilot symbols are required. The number of pilot symbols required increases linearly with the channel length $L$, thereby reducing the transmission efficiency when the channel length is large.

In this paper, a time-domain signal detection algorithm based on second-order statistics (SOS) is proposed for general MIMO-OFDM systems over frequency selective channels. A new system model is first introduced in which the $j$th received OFDM block is left shifted by $J$ samples. The new system model has certain structural properties that enable an equalizer to be designed to cancel most of the intersymbol interference (ISI) using SOS of the received signals. At the output of the equalizer, only two paths of the transmitted signals are retained and the signals can readily be detected. Due to the special
structure of the new system model, it turns out that only $2P$ columns of the channel matrix need to be estimated. It follows that the minimum number of pilot symbols required to estimate the $2P$ columns of the channel matrix is $4P$ (compared with $(1 + L)P$ needed generally to estimate the channel matrix) and is independent of the channel length. It also means that channel length estimation is unnecessary, which implies that the proposed algorithm has substantial advantage, computational as well as avoiding performance degradation due to channel length estimation error, over existing algorithms [17], [18]. Furthermore, the proposed algorithm is applicable irrespective of whether the channel length is shorter than, equal to or longer than the CP length. Simulation results confirm the effectiveness of the proposed algorithm, and show that it outperforms the existing ones in all cases.

The rest of the paper is organized as follows. In Section II, the new MIMO-OFDM system model is introduced. The time-domain signal detection algorithm is presented in Section III and in Section IV, the performance of the proposed algorithm is demonstrated by simulation. Finally, Section V draws the conclusion.

II. SYSTEM MODEL

Consider a MIMO-OFDM system with $P$ transmit antennas/users and $M$ receive antennas. The signal corresponding to the $n$th user and the $m$th receive antenna is $\beta_{n,m}[n]$, $n \in \{0,1,\ldots,N-1\}$ (where $N$ is the number of subcarriers in OFDM systems), $p \in \{1,2,\ldots,P\}$. This is the so-called “frequency-domain” signal. Denote the frequency-domain signal vector of the $i$th OFDM block from the $p$th transmit antenna/user as

$$\mathbf{b}_{i,p} = \mathbf{F}_N \mathbf{\beta}_{i,p}$$

where $(\mathbf{\cdot})^T$ represents matrix transpose. Performing an $N$-point IFFT on it, the so-called “time-domain” signal vector from the $p$th transmit antenna/user can be generated as

$$\mathbf{s}_{i,p} = \mathbf{F}_N^* \mathbf{b}_{i,p}$$

and $\mathbf{F}_N$ is the $N \times N$ IFFT matrix with the $(n+1,k+1)$th entry as $e^{-j2\pi nk/N} \sqrt{N}$, $n,k \in \{0,1,\ldots,N-1\}$. It is obvious that the FFT matrix is $\mathbf{F}_N^* \mathbf{F}_N = \mathbf{I}_N$, where $(\mathbf{\cdot})^*$ represents conjugate transpose and $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. A CP of length $D$ is inserted into $\mathbf{s}_{i,p}$ to generate the $i$th transmitted signal block from the $p$th transmit antenna/user, denoted by $\mathbf{s}_{i,p}$

$$\mathbf{s}_{i,p} = \left[ \begin{array}{c} \mathbf{s}_{i,p}[0] \\ \mathbf{s}_{i,p}[1] \\ \vdots \\ \mathbf{s}_{i,p}[N'-1] \end{array} \right]$$

in which $N' = N + D$ and

$$\mathbf{s}_{i,p}[n] = \left\{ \begin{array}{ll} \mathbf{s}_{i,p}[n-D+N] & 0 \leq n \leq D-1 \\ \mathbf{s}_{i,p}[n-D] & D \leq n \leq N'-1 \end{array} \right.$$  

Denote the frequency selective channel between the $p$th transmit antenna/user and the $m$th receive antenna as $h_{pm}(l)$, which is modeled as an $L_{pm}$-th-order FIR filter. Here, the maximum channel length is defined as $L = \max_{1 \leq p \leq P, 1 \leq m \leq M} (L_{pm})$. Without loss of generality, it is assumed to satisfy $L < N - D$ which implies that the number of subcarriers is larger than the channel length plus the CP length. The $i$th received block at the $m$th receive antenna is, therefore

$$y_{i,m}[n] = \sum_{p=0}^{P} \sum_{l=1}^{L} h_{pm}(l) s_{i,p}[n-l] + w_{i,m}[n], \quad n = 0,1,\ldots,N'-1, \quad m = 1,2,\ldots,M$$

where $h_{pm}(l)$ is zero-padded for $L_{pm} < l \leq \min_{1 \leq m \leq M}(L_{pm})$ without channel noise, and $s_{i,p}[n] = s_{i,(p-1)+l}[N' + n]$ for $n < 0$.

By defining

$$\mathbf{y}_i[n] = [y_{i,1}[n] \quad y_{i,2}[n] \quad \ldots \quad y_{i,M}[n]]^T$$

$$\mathbf{h}_p(l) = [h_{p,1}(l) \quad h_{p,2}(l) \quad \ldots \quad h_{p,M}(l)]^T$$

$$\mathbf{w}_i[n] = [w_{i,1}[n] \quad w_{i,2}[n] \quad \ldots \quad w_{i,M}[n]]^T$$

equation (6) can be expressed in vector form as

$$\mathbf{y}_i[n] = \sum_{p=0}^{P} \sum_{l=0}^{L} \mathbf{h}_p(l) s_{i,p}[n-l] + \mathbf{w}_i[n], \quad n = 0,1,\ldots,N'-1.$$

Traditionally, signal detection is performed based on the $i$th received OFDM block as

$$\mathbf{y}_i^{(0)} = [\mathbf{y}_i[0]^T \quad \mathbf{y}_i[1]^T \quad \ldots \quad \mathbf{y}_i[N'-1]^T]^T$$

A new system model is now introduced in which the $i$th received OFDM block is left shifted by $J$ samples as

$$\mathbf{y}_i^{(j)} = [\mathbf{y}_i[-J]^T \quad \mathbf{y}_i[-J+1]^T \quad \ldots \quad \mathbf{y}_i[N'-1-J]^T]^T$$

$$J = 0, \pm 1, \pm 2, \ldots$$

where $\mathbf{y}_i[n] = \mathbf{y}_{i-[n+N]}$ for $n < 0$ and $\mathbf{y}_i[n] = \mathbf{y}_{i+[n-N]}$ for $N' \leq n$. It is apparent that $\mathbf{y}_i^{(j)}$ contains the information from two consecutive received OFDM blocks. It follows that

$$\mathbf{y}_i^{(j)} = \sum_{p=1}^{P} \mathbf{H}_p \mathbf{x}_i^{(j)} + \mathbf{w}_i^{(j)}$$

in which $[\mathbf{H}_1 \quad \mathbf{H}_2 \quad \ldots \quad \mathbf{H}_P]$ (17)

$$\mathbf{x}_i^{(j)} = \left[ \begin{array}{c} \mathbf{x}_{i+1}^{(j)} \\ \mathbf{x}_{i+2}^{(j)} \\ \vdots \\ \mathbf{x}_{i+P}^{(j)} \end{array} \right]^T$$

the received signal vector $\mathbf{y}_i^{(j)}$ can be written as

$$\mathbf{y}_i^{(j)} = \mathbf{H}_i \mathbf{x}_i^{(j)} + \mathbf{w}_i^{(j)}$$

$$J = 0, \pm 1, \pm 2, \ldots$$
When $0 \leq J \leq N-L$, we observe that the signal vector $x_{i,p}^{(j)}$ in (15) can be expressed as

$$x_{i,p}^{(j)} = F_{2N}^{(j)} c_{i,p}, \quad p \in \{1, 2, \ldots, P\}, \ 0 \leq J \leq N-L \quad (20)$$

where

$$c_{i,p} = \begin{bmatrix} \beta_{i(-1),p}^T & \beta_{i,p}^T \end{bmatrix}^T \quad (21)$$

and

$$F_{2N}^{(j)} = \begin{bmatrix} F_{N}(N-L-J+1: N) & 0 \\ 0 & F_{N}(N-D+1: N) \\ 0 & F_{N}(1: N-J) \end{bmatrix} \quad (22)$$

In (22), $F_{2N}^{(j)}$ is a $(N' + L) \times 2N$ matrix and $F_{N}(a: b)$ denotes a submatrix of $F_{N}$, composed by the rows between the $a$th row and the $b$th row of $F_{N}$. It follows that the transmit signal vector $x_{i}^{(j)}$ in (18) can be written as

$$x_{i}^{(j)} = F^{(j)} c_{i}, \quad 0 \leq J \leq N-L \quad (23)$$

in which

$$F^{(j)} = \begin{bmatrix} F_{2N}^{(j)} & 0 & 0 & 0 \\ 0 & F_{2N}^{(j)} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & F_{2N}^{(j)} \end{bmatrix} \quad (24)$$

$$c_{i} = \begin{bmatrix} c_{i,1}^T \\ c_{i,2}^T \\ \vdots \\ c_{i,p}^T \end{bmatrix} \quad (25)$$

Applying (23) into (19), the received signal vector $y_{i}^{(j)}$ can be remodeled as

$$y_{i}^{(j)} = H x_{i}^{(j)} + w_{i}^{(j)} = H F^{(j)} c_{i} + w_{i}^{(j)}, \quad 0 \leq J \leq N-L \quad (26)$$

Based on the new system model (26), an equalizer is designed in the next section to cancel most of the ISI before signal detection.

### III. TIME-DOMAIN SIGNAL DETECTION

In this section, a time-domain signal detection algorithm based on SOS is proposed for MIMO-OFDM systems over frequency selective fading channels. Before discussing the algorithm, the following general assumptions are made.

#### A1) Signals from different transmit antennas/users are statistically independent, and signals from each transmit antenna/user at different subcarriers are independent with zero mean and unit variance. It implies that each transmit antenna/user modulates all subcarriers with equal power. This situation arises in spatial multiplexing and spatial multiple access systems as independent data streams are transmitted over different antennas and different subcarriers. Note that this system is different from orthogonal frequency division multiple access (OFDMA) systems where each transmit antenna/user modulates only a portion of the subcarriers. The autocorrelation matrix of the signals before IFFT $c_{i}$ is, therefore

$$R_{c} = E\{c_{i} c_{i}^*\} = I_{2NP} \quad (27)$$

where $E\{\cdot\}$ is the expectation operator.

#### A2) The noise components are independently identically distributed (i.i.d.) and independent of the signals from all transmit antennas/users.

#### A3) The $MN' \times (L + N')P$ matrix $H$ is of full column rank after removing all-zero columns, which means the nonzero columns are independent. This is a sufficient condition for detecting the signals based on SOS [23]. In order to meet this condition, the number of receive antennas, $M$, must be chosen to satisfy $M \geq (L + N')P/N'$, so that there are more rows than columns. In most cases, when the number of receive antennas ($M$) is chosen equal to the number of transmit antennas/users ($P$) plus one, the above inequality will be satisfied. Under this assumption, the matrix $H$ has the property

$$H^* (HH^*)^{-1} H = A_{(N'+L)P} \quad (28)$$

in which $(\cdot)^* \#$ represents pseudoinverse and $A_{(N'+L)P}$ is an $(N' + L)P \times (N' + L)P$ matrix with one along the major diagonal except the rows corresponding to the all-zero columns of $H$. which is equal to zero. In other words, $A_{(N'+L)P}$ is an $(N' + L)P \times (N' + L)P$ identity matrix with all-zero rows corresponding to the all-zero columns of $H$.

#### A. Zero-Noise Case

To simplify the derivation of the algorithm, zero noise is first assumed. The effect of noise on the algorithm is then examined.
In the absence of noise, \( y_i^{(J)} \) can be expressed as

\[
y_i^{(J)} = H x_i^{(J)}, \quad J = 0, \pm 1, \pm 2, \cdots \quad (29)
\]

When \( 0 \leq J \leq N - L \), \( y_i^{(J)} \) can also be modeled as

\[
y_i^{(J)} = H F^{(J)} c_i, \quad 0 \leq J \leq N - L. \quad (30)
\]

1) Equalization and Signal Detection: It is apparent from (15) and (29) that the received signal vector \( y_i^{(J)} \) includes \((N' + L) \) path signals [each path signal refers to one sample signal, see (15)] from each transmit antenna/user. Before deriving the equalizer designed to cancel most of the ISI, some useful properties are described first in the following.

**Property 1:** When \( 0 \leq J \leq N - L \), the matrix \( F^{(J)}_2 \) is given by

\[
F^{(J)}_2 = \begin{bmatrix}
I_{L+J} & 0 & 0 & 0 \\
0 & I_D & 0 & 0 \\
0 & 0 & I_{D-J} & 0 \\
0 & 0 & 0 & I_{N-J}
\end{bmatrix}
\]

in which, \( I_{D-J} = 0 \) when \( J \geq D \).

**Proof:** Using the orthogonal property of the IFFT matrix \( F_N F_N^* = I_N \), each column (row) of the matrix \( F_N \) is orthogonal to the other columns (rows). It follows that

\[
F_N(N - L - J + 1 : N) F_N(N - L - J + 1 : N)^* = I_{L+J} \quad (32)
\]

\[
F_N(N - D + 1 : N) F_N(N - D + 1 : N)^* = I_{D-J} \quad (33)
\]

\[
F_N(N - D + 1 : N) F_N(N - D + 1 : N)^* = I_D \quad (34)
\]

\[
F_N(1 : N - J) F_N(1 : N - J)^* = I_{N-J} \quad (35)
\]

The property follows from (22) when (32)–(36) are applied.

**Property 2:** The matrices satisfy

\[
F^{(J)}_2 F^{(J)}_2^* = \begin{bmatrix}
I_{L+D} & 0 & 0 & 0 \\
0 & I_D & 0 & 0 \\
0 & 0 & I_{D-J} & 0 \\
0 & 0 & 0 & I_{N-D}
\end{bmatrix}
\]

and

\[
F^{(0)}_2 F^{(0)}_2^* - F^{(1)}_2 F^{(1)}_2^* = U \quad (38)
\]

where \( U \) is an \((N' + L) \times (N' + L) \) matrix with only two entries having values of one at the positions \((L + 1, N + L + 1)\) and \((N + L + 1, L + 1)\), while all remaining entries are zeros.

**Proof:** The result follows from Property 1.

Now consider the autocorrelation of the received signal vector \( y_i^{(J)} \). Under Assumption A1), when \( 0 \leq J \leq N - L \)

\[
R_y^{(J)} = E \left\{ y_i^{(J)} y_i^{(J)^*} \right\} = H F^{(J)} E \left\{ c_i c_i^* \right\} F^{(J)^*} H^* \\
= H F^{(J)} F^{(J)^*} H^* \\
= H \left( I_P \otimes F^{(J)}_2 F^{(J)^*}_2 \right) H^*, \quad 0 \leq J \leq N - L \quad (39)
\]

where \( \otimes \) denotes Kronecker product. It follows that

\[
R_y^{(0)} - R_y^{(1)} = H \left( I_P \otimes \left( F^{(0)}_2 F^{(0)^*}_2 - F^{(1)}_2 F^{(1)^*}_2 \right) \right) H^*. \quad (40)
\]

Applying Property 2 into (39) and (40), we have

\[
R_y^{(D)} = H \begin{bmatrix}
I_{N' + L} & 0 & 0 & 0 \\
0 & I_{N' + L} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & I_{N' + L}
\end{bmatrix} H^*
\]

\[
= H \tilde{U} H^* \quad (41)
\]

\[
R_y^{(0)} - R_y^{(1)} = H \tilde{U} H^* \quad (42)
\]

where

\[
\tilde{U} = \begin{bmatrix}
U & 0 & 0 & 0 \\
0 & U & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & U
\end{bmatrix}. \quad (43)
\]

An equalizer is now constructed based on SOS \( \left( R_y^{(0)}, R_y^{(1)} \right) \) and \( R_y^{(D)} \) as

\[
G = \left( R_y^{(0)} - R_y^{(1)} \right) R_y^{(D)^*}. \quad (44)
\]

It follows that

\[
G = \left( R_y^{(0)} - R_y^{(1)} \right) R_y^{(D)^*} = H \tilde{U} H^* \tilde{H}^* \quad (45)
\]

In (45), the matrix \( H_p U, p \in \{1, 2, \cdots, P\} \), has the following property.

**Property 3:** The matrix \( H_p U, p \in \{1, 2, \cdots, P\} \), has all-zero columns except the \((L + 1)\)th and the \((N + L + 1)\)th columns given by \( H_p(N + L + 1) \) and \( H_p(L + 1) \), respectively, where \( H_p(a) \) denotes the \( a \)th column of \( H_p \).

**Proof:** As the matrix \( U \) is a special \((N' + L) \times (N' + L) \) matrix with only two nonzero entries at the positions \((L + 1, N + L + 1)\) and \((N + L + 1, L + 1)\), it follows:

\[
H_p U = \begin{bmatrix}
0 & \cdots & H_p(N + L + 1) & \cdots & H_p(L + 1) & \cdots & 0
\end{bmatrix}, \quad p \in \{1, 2, \cdots, P\}. \quad (46)
\]
From the definition of $H_p$ in (14), it is apparent that $H_p(N + L + 1)$ and $H_p(L + 1)$ are not all-zero columns and the proof follows.

To proceed with signal detection, the equalizer is applied to the received signal vector $y^{(j)}_k$ (29) to yield
\[
\mathbf{o}^{(j)}_k = \mathbf{G}_y^{(j)} = \left( \mathbf{R}_y^{(0)} - \mathbf{R}_y^{(1)} \right)^{\#} y^{(j)}_k
\]
\[
= \mathbf{H}_u \mathbf{H}^* (\mathbf{H} \mathbf{H}^*)^{\#} \mathbf{H}_x^{(j)}, \quad J = 0, \pm 1, \pm 2, \cdots (47)
\]

Using (15), (18), (28), (45), and Property 3, the output of the equalizer $\mathbf{o}^{(j)}_k$ is given by
\[
\mathbf{o}^{(j)}_k = \left[ \begin{array}{cc}
H_1 & H_2 \\
\vdots & \vdots \\
H_{p} & \end{array} \right] \mathbf{x}^{(j)}_{\text{part}}
\]
\[
= \sum_{j=1}^{P} \left[ H_p(N + L + 1) s_{i_p}[N - J] \right.
\]
\[
+ H_p(L + 1) s_{i_p}[N - J], \quad J = 0, \pm 1, \pm 2, \cdots
\]
\[
= \mathbf{H}_{\text{part}} \mathbf{x}^{(j)}_{\text{part}} \quad (48)
\]

where
\[
\mathbf{H}_{\text{part}} = \left[ \begin{array}{cccc}
H_1(N + L + 1) & H_1(L + 1) & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
H_p(N + L + 1) & H_p(L + 1) & \end{array} \right]
\]
\[
\mathbf{x}^{(j)}_{\text{part}} = \begin{bmatrix} s_{i_1}[N - J] & s_{i_1}[N - J] & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
s_{i_p}[N - J] & s_{i_p}[N - J] & \end{bmatrix}^T. \quad (49)
\]

It is apparent that most of the ISI are cancelled by the equalizer $\mathbf{G}$. The equalizer output $\mathbf{o}^{(j)}_k$ only contains two paths of the transmitted signals from each transmit antenna/user, i.e., $s_{i_p}[N - J]$ and $s_{i_p}[N - J], \quad p \in \{1, 2, \cdots, P\}$. In other words, only $2P$ columns of the channel matrix $\mathbf{H}$ are retained. Note that channel length information is not needed in this step.

When the matrix $\mathbf{H}_{\text{part}}$ which contains only $2P$ columns of the channel matrix $\mathbf{H}$ is known, $\mathbf{x}^{(j)}_{\text{part}}$ in (50) can be easily detected from the equalizer output based on the least-squares criteria [19]
\[
\mathbf{x}^{(j)}_{\text{part}} = \left( \mathbf{H}_{\text{part}}^* \mathbf{H}_{\text{part}} \right)^{\#} \mathbf{H}_{\text{part}}^* \mathbf{o}^{(j)}_k, \quad J = 0, \pm 1, \pm 2, \cdots, (51)
\]

As $\mathbf{x}^{(j)}_{\text{part}}$ contains two paths of the estimated transmitted signals from each transmit antenna/user, either path can be used for signal detection. For example, by setting the parameter $J = -D + 1, -D + 2, \cdots, N - D$, the estimation of the transmitted signal $s_{i_p}[n], \quad p \in \{1, 2, \cdots, P\}, \quad n \in \{N + D - 1, N + D - 2, \cdots, D\}$, are obtained from the path $s_{i_p}[N - J]$ of the vector $\mathbf{x}^{(j)}_{\text{part}}$. The frequency-domain signal vector $\mathbf{f}_{\text{eq}}$ is then detected by the FFT of $s_{i_p}[n], \quad p \in \{1, 2, \cdots, P\}, \quad n \in \{D, D + 1, \cdots, N + D - 1\}$, based on (2) and (5).

2) $\mathbf{H}_{\text{part}}$ Estimation: In order to perform signal detection, knowledge of the $MN' \times 2P$ matrix $\mathbf{H}_{\text{part}}$ is necessary. For better performance, pilot symbols will be used. Note that the channel length information is not needed for the selection of pilot symbols as it is now embedded in the matrix $\mathbf{H}_{\text{part}}$ after equalization. Suppose the pilot symbols are inserted into each transmit antenna/user’s signal and $\mathbf{X}^{(j)}_{\text{pilot}}$ consists of the pilot symbols, i.e.
\[
\mathbf{X}^{(j)}_{\text{pilot}} = \left[ \begin{array}{cccc}
\mathbf{x}^{(j)}_{\text{pilot}, 1} & \mathbf{x}^{(j)}_{\text{pilot}, 2} & \cdots & \mathbf{x}^{(j)}_{\text{pilot}, K} \\
\end{array} \right], \quad J_1, J_2, \cdots, J_K = 0, \pm 1, \pm 2, \cdots (52)
\]

The matrix $\mathbf{H}_{\text{part}}$ can be straightforwardly estimated from the equalizer output [19] as
\[
\hat{\mathbf{H}}_{\text{part}} = \mathbf{O}_{\text{pilot}} \mathbf{X}^{(j)}_{\text{pilot}} \mathbf{X}^{(j)}_{\text{pilot}} \mathbf{X}^{(j)}_{\text{pilot}} \mathbf{X}^{(j)}_{\text{pilot}}^{-1} \quad (53)
\]

where
\[
\mathbf{O}_{\text{pilot}} = \left[ \begin{array}{cccc}
o_{1} & o_{2} & \cdots & o_{K} \\
\end{array} \right], \quad (54)
\]

In order to achieve a unique estimate of $\mathbf{H}_{\text{part}}$ based on (53), some conditions on pilot symbols for identifiability need to be satisfied. These are discussed in the following.

(i) The $MN' \times 2P$ matrix $\mathbf{H}_{\text{part}}$ requires at least $2P$ sets of (48) to be estimated, which means at least $2P$ pilot vectors are required. In other words, the parameter $K$ in (52) must be greater than or equal to $2P$. Based on the definition in (50), each pilot vector $\mathbf{x}^{(j)}_{\text{pilot}, k}, \quad k \in \{1, 2, \cdots, K\}$, in $\mathbf{X}^{(j)}_{\text{pilot}}$ contains two paths of the transmitted signals from each transmit antenna/user. Therefore, the minimum number of pilot symbols required by each transmit antenna/user is $2P \times 2$.

(ii) In order to yield a unique estimation of $\mathbf{H}_{\text{part}}$, the $2P \times 2P$ matrix $\mathbf{X}^{(j)}_{\text{pilot}}$ in (53) must be nonsingular. It follows that the rank of the $2P \times 2$ matrix $\mathbf{X}^{(j)}_{\text{pilot}}$ must be greater than or equal to $2P$, which implies $K \geq 2P$ and $\mathbf{X}^{(j)}_{\text{pilot}}$ is of full row rank.

(iii) Each pilot vector $\mathbf{x}^{(j)}_{\text{pilot}, k}, \quad k \in \{1, 2, \cdots, K\}$, in $\mathbf{X}^{(j)}_{\text{pilot}}$ has the structure as shown in (50). It contains two symbols $s_{i_p}[N - J]$ and $s_{i_p}[N - J]$, which is the symbol in the cyclic prefix and is equal to the second symbol $s_{i_p}[N - J]$. Consequently, the pilot matrix $\mathbf{X}^{(j)}_{\text{pilot}}$ will not be of full row rank as the $(2a + 1)$th row is the same as the $(2a + 2)$th row, $a \in \{0, 1, \cdots, P - 1\}$. Condition (ii) above is, therefore, not satisfied and it follows that the pilot symbols need to be distributed in more than one OFDM block. Note that when the pilot symbols are distributed in more than one OFDM block, generally condition (ii) can be satisfied. However, there is no guarantee and the chosen pilot symbols need to be checked to ensure that condition (ii) is satisfied.

3) Remark: In the algorithms [17], [18], knowledge of the channel matrix $\mathbf{H}$ is necessary. Due to the structure of $\mathbf{H}$ [see (14) and (17)], only the channel coefficients $(h_p(0), h_p(1), \cdots, h_p(L), p \in \{1, 2, \cdots, P\})$ are required to be estimated. Since they are directly estimated from the received signal which contains $(L + 1)$ paths of the transmitted signals from each transmit antenna/user, the minimum number of pilot symbols required is, therefore, $(1 + L)P$ and linearly increases with the channel length. Also, channel length estimation is necessary before selecting the pilot symbols. On the
other hand, in the proposed algorithm, only knowledge of the matrix \( \mathbf{H}_{\text{part}} \), which includes \( 2P \) columns of the channel matrix \( \mathbf{H} \), is required. Each pair of columns of the matrix \( \mathbf{H}_{\text{part}} \), corresponding to the \( p \)th transmit antenna/user (\( p \in \{1,2,\ldots,P\} \)), contains all the channel coefficients \( \{\mathbf{h}_p(0),\mathbf{h}_p(1),\ldots,\mathbf{h}_p(L)\} \) [see (14), (17), and (49)]. When \( \mathbf{H}_{\text{part}} \) is estimated, it follows that the channel matrix \( \mathbf{H} \) is effectively estimated. In this case, only \( 4P \) pilot symbols are required to estimate \( \mathbf{H}_{\text{part}} \). As aforementioned, computationally intensive channel length estimation is not needed and the transmitted signals are detected straightforwardly (51) without the need to reconstruct the estimated channel matrix.

### B. Channel Noise Consideration

In the presence of noise, \( \mathbf{y}_i^{(J)} \) is given by (19) and (26). Let the variance of noise be \( \sigma^2 \). When \( 0 \leq J \leq N-L \), the autocorrelation matrix of \( \mathbf{y}_i^{(J)} \) is

\[
\mathbf{R}_{\mathbf{y}}^{(J)} = \mathbf{E}\left\{\mathbf{y}_i^{(J)}\mathbf{y}_i^{(J)*}\right\} = \mathbf{H}\mathbf{F}^{(J)}\mathbf{F}^{(J)*}\mathbf{H}^* + \sigma^2 \mathbf{I}_{MN}, \tag{55}
\]

If \( \sigma^2 \) is known, the noise contribution can be subtracted from \( \mathbf{R}_{\mathbf{y}}^{(0)}, \mathbf{R}_{\mathbf{y}}^{(1)}, \mathbf{R}_{\mathbf{y}}^{(D)} \), and, therefore, it has no impact on the equalizer which can be constructed as

\[
\mathbf{G} = \left( \left( \mathbf{R}_{\mathbf{y}}^{(0)} - \sigma^2 \mathbf{I}_{MN} \right) - \left( \mathbf{R}_{\mathbf{y}}^{(1)} - \sigma^2 \mathbf{I}_{MN} \right) \right)^{\#} \times \left( \left( \mathbf{R}_{\mathbf{y}}^{(D)} - \sigma^2 \mathbf{I}_{MN} \right)^{\#} \right) \tag{56}
\]

The output of the equalizer \( \mathbf{o}_i^{(J)} \) becomes

\[
\mathbf{o}_i^{(J)} = \mathbf{G}\mathbf{y}_i^{(J)} = \mathbf{H}_{\text{part}}\mathbf{x}_i^{(J)} + \mathbf{G}\mathbf{w}_i^{(J)}. \tag{57}
\]

Taking into account the noise contribution \( \mathbf{G}\mathbf{w}_i^{(J)} \) in \( \mathbf{o}_i^{(J)} \), the matrix \( \mathbf{H}_{\text{part}} \) and the signal vector \( \mathbf{x}_i^{(J)} \) can be detected based on the minimum mean square error (MMSE) criterion [19].

If \( \sigma^2 \) is unknown, it can be estimated from the singular value decomposition of \( \mathbf{R}_{\mathbf{y}}^{(D)} \) [20] where \( \mathbf{R}_{\mathbf{y}}^{(D)} = \mathbf{H}\mathbf{H}^* + \sigma^2 \mathbf{I}_{MN} \). Since some error generally exists in the estimation of \( \sigma^2 \) and this error will degrade the performance, it is generally preferred not to subtract the noise contribution from \( \mathbf{R}_{\mathbf{y}}^{(0)}, \mathbf{R}_{\mathbf{y}}^{(1)}, \mathbf{R}_{\mathbf{y}}^{(D)} \). Instead, the equalizer \( \mathbf{G} \) is constructed based on \( \mathbf{G} = \left( \mathbf{R}_{\mathbf{y}}^{(0)} - \mathbf{R}_{\mathbf{y}}^{(1)} \right)^{\#} \mathbf{R}_{\mathbf{y}}^{(D)} \) as if it were noiseless. It follows that the equalizer \( \mathbf{G} \) includes two parts: the effective equalizer as \( \mathbf{G}_{\text{effective}} = \mathbf{H}\mathbf{H}^* \) and the noise contribution to the equalizer as \( \mathbf{G}_{\text{noise}} = \left( \mathbf{R}_{\mathbf{y}}^{(0)} - \mathbf{R}_{\mathbf{y}}^{(1)} \right) \mathbf{R}_{\mathbf{y}}^{(D)}^{\#} \). The output of the equalizer \( \mathbf{o}_i^{(J)} \) is, therefore

\[
\mathbf{o}_i^{(J)} = (\mathbf{G}_{\text{effective}} + \mathbf{G}_{\text{noise}})\mathbf{y}_i^{(J)} = \mathbf{H}_{\text{part}}\mathbf{x}_i^{(J)} + (\mathbf{G}_{\text{effective}}\mathbf{w}_i^{(J)} + \mathbf{G}_{\text{noise}}\mathbf{y}_i^{(J)}). \tag{58}
\]

Here \( (\mathbf{G}_{\text{effective}}\mathbf{w}_i^{(J)} + \mathbf{G}_{\text{noise}}\mathbf{y}_i^{(J)} \) is considered as the noise contribution to \( \mathbf{o}_i^{(J)} \). As the noise contribution is not known, \( \mathbf{H}_{\text{part}} \) estimation and signal detection will be performed based on the least-squares criteria in the simulation in Section IV. Results will show that the proposed algorithm performs well in the noisy case.

### IV. Simulation Results

Computer simulations have been conducted to investigate the performance of the proposed algorithm. In the following examples, a MIMO-OFDM system with \( P = 2 \) transmit antennas/users and \( M = 3 \) receive antennas \((2 \times 3)\) system is considered. The OFDM parameters are selected as: \( N = 64 \) and \( D = 16 \). All transmitted signals are modulated with QPSK scheme. The channel parameters are assumed constant over 1500 blocks and two consecutive OFDM block pilots are inserted into each transmit antenna/user’s signal at the beginning of every 1500 blocks for reliable estimation. The frequency selective fading channel responses are randomly generated with a Rayleigh probability distribution. The autocorrelation of the received signal vector is computed from a finite number of received signal vectors as

\[
\mathbf{R}_{\mathbf{y}}^{(J)} \approx \frac{1}{N_s} \sum_{i=1}^{N_s} \mathbf{y}_i^{(J)}\mathbf{y}_i^{(J)*} \tag{59}
\]

where \( N_s \) is the number of OFDM block used and is selected as 1500 unless otherwise indicated. The signal-to-noise ratio (SNR) is defined as

\[
\text{SNR} = \frac{\sum_{m=1}^{M} E \left\{ \sum_{l=0}^{P} \sum_{m=0}^{L} h_{pm}(l)s_{lm}[n-l] \right\}^2 }{\sum_{m=1}^{M} E \left\{ |w_{lm}[n]|^2 \right\} }. \tag{60}
\]

The noise variance is unknown in the proposed algorithm. The performance measure, BER, is computed by averaging over 300 000 OFDM blocks of the Monte Carlo test results among all transmit antennas/users.

#### A. The Case Where The Channel Length is Shorter than or Equal to the CP Length: \( L \leq D \)

In this case, all subcarriers are orthogonal to each other and there is no IBI. The conventional signal detection algorithm [16] and the MMSE signal detection algorithm [19] are also implemented for comparison. Note that the conventional algorithm [16] does the FFT in the receiver to transform the frequency selective channels into flat fading channels and then performs parallel signal detection on each subcarrier with \( P \) OFDM block pilots. In the MMSE algorithm [19], the channel length is overestimated by one as \( L = L + 1 \) and the channel coefficients are estimated using Maximum Likelihood method with two consecutive OFDM block pilots. The BER performance of various algorithms under consideration for \( L = 14 (L < D) \) and \( L = 16 (L = D) \) are shown in Fig. 1 and Fig. 2, respectively. It is obvious that the proposed algorithm performs substantially better than the conventional algorithm [16] and the MMSE algorithm [19] over the range of SNR considered.

#### B. The Case Where The Channel Length is Longer Than the CP Length: \( L > D \)

In this case, the orthogonality between all subcarriers is destroyed and IBI occurs. Two indirect signal detection algorithms
[17], [18] with exact knowledge of the channel length and coefficients, and the MMSE algorithm [19] aforementioned are implemented for comparison. Figs. 3 and 4 show the performance of various algorithms for $L = 18$ and $L = 20$, respectively. In these figures, per-tone equalization (PTEQ) represents the frequency-domain algorithm in [17] and time-domain equalization (TEQ) represents the time-domain channel shortening algorithm in [18]. These results indicate that the proposed algorithm outperforms the existing ones [17]–[19]. Note that there is no need to estimate the channel length in the proposed algorithm while it is assumed precisely estimated in [17], [18] and is overestimated by one in [19]. In practice, channel length estimation is performed using computationally intensive methods such as AIC [21] or MDL [22] and estimation error usually occurs, which will degrade the performance of the existing algorithms [17]–[19].

C. Comparison
To illustrate the impact of the channel length and the CP length on the proposed algorithm, the performance for $L = 14, 16, 18, 20$ cases are shown in Fig. 5. It is obvious that the performance is only slightly degraded when the channel length increases from 14 ($L < D$) to 20 ($L > D$). It demonstrates that the channel and CP lengths have insignificant effect on the proposed algorithm. It also verifies that the proposed algorithm is applicable irrespective of whether the channel length is shorter than, equal to or longer than the CP length.

D. Data Length Effect
In the proposed algorithm, SOS of the received signal vectors is utilized to design the equalizer. In practice, it is computed from finite number of received signal vectors (59) and data length may affect the performance of the algorithm. This is different from the existing algorithms in which the statistics of the received signals is not used.
A time-domain signal detection algorithm for MIMO-OFDM systems has been proposed in this paper. A new system model has been introduced in which the jth received OFDM block is left shifted by J samples. Based on some structural properties of the new system model, an equalizer has been designed to cancel most of the ISI using the SOS of the received signals before signal detection. It has been shown that the channel length information is not needed and only 2P columns of the channel matrix need to be estimated with a minimum of 4P pilot symbols for identifiability. In addition, it has been demonstrated that the proposed algorithm is applicable to general MIMO-OFDM systems irrespective of whether the CP length is longer than, equal to or shorter than the channel length. Simulation results have shown that the proposed algorithm outperforms the existing ones in all cases.

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