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# Towards a Formal Foundation for DeMarco Data Flow Diagrams*

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**ABSTRACT**

In thie paper, we describe a proposal for formalizing data flow diagrams through extended Petri nets. We illustrate the usefulness of the approach by describing how it can be used to analyse the consistency of requirements specifications.

## 1. INTRODUCTION

Quite a number of tools have been proposed under the name of structured analysis and design. Examples are data flow diagrams [7, 8, 35], Jackson structure diagrams, Jackson structure text [9], system specification diagrams, system implementation diagrams [10], Warnier/Orr diagrams [17] and structure charts [36]. They are widely accepted by software engineering professionals because of the top down nature of the methodologies and the graphical nature of the tools [3]. They enable practitioners to visualize the target systems and to communicate with users much more easily than traditional methods. Unfortunately, structured systems development has still remained a manual method, due to the fact that there is no theoretical foundation behind the tools. A series of studies [27, 29, 28, 30, 31] are being made to define such theoretical foundations.

Amongst the structured analysis tools, data flow diagrams have become the most popular [2]. They have a graphical representation with only a few primitives and concepts. A complex system specification can be decomposed into a modular and hierarchical structure which is easily comprehensible. Because of the lack of a formal framework, however, only a couple of automated aids [6, 11] have been developed to support its use. In this paper, we shall describe a proposal for formalizing data flow diagrams through extended Petri nets. We shall illustrate the usefulness of the approach by describing how it can be used to analyse the consistency of requirements specifications.

## 2. REASONS FOR CHOOSING PETRI NETS

In order to remedy the defects of informality in the structured analysis tools, an attempt is made to add a mathematical structure to data flow diagrams. Petri net is found to be an appropriate model in this respect because of the following reasons:

(a) Petri nets can be represented both graphically and algebraically. The graphical representation closely resembles data flow diagrams. Transitions and places of Petri nets correspond, respectively, to processes and data flows of DFD's. A subnet concept is also supported, so that a

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hierarchical representation of a system at various levels of abstraction can be created in a manner similar to that of DFD's. Parallelism is supported and irrelevant processing sequence can be ignored to allow freedom in design and implementation.

(b) The algebraic representation of Petri nets, on the other hand, provides a theoretical basis for the analysis of a specification. The concepts of tokens and markings, not found in any other model, provide an excellent means of analysing the behavioural properties of target systems.

(c) Surveys in [5, 4, 12, 13] reveal that Petri nets serve as an excellent tool for systems design and testing because of their rich formalism. They are, however, not acceptable as a systems analysis tool because users find them difficult to understand. If the user-friendliness of data flow diagrams is added to Petri nets, the resulting specification language will have assets in both aspects.

The concept of Petri nets has been applied in other projects on the design of system specification tools. Examples are IML-inscribed predicate/transition nets [22], abstract process nets [14, 15] and EDDA [25]. The present project differs from the others in the use of DeMarco data flow diagrams as the user interface, and the application of special consistency analyses on the resulting language to safeguard the correctness of a specification.

A brief description of Petri nets will be given in the appendix. More details can be found in [1, 18, 19, 21].

## 3. FORMAL DATA FLOW DIAGRAMS (FDFD)

Our specification language — Formal Data Flow Diagrams (FDFD) — provide data flow diagrams with a theoretical framework through extended Petri nets. As pointed out in [32], a requirements specification language should be graphics based and augmented by a symbolic description which is in one-to-one correspondence with the graphics. Moreover, a symbolic description is more easily input into an automated system for analysis and maintenance. Hence we define FDFD in two equivalent forms — graphic and symbolic. The graphical representation retains the user-friendly advantages of the original data flow diagrams. The symbolic representation makes use of the algebraic foundation of Petri nets. It also has a formal syntax so that it can be processed easily by a computer. The one-to-one correspondence between the graphics and symbolic representations enables consistency and traceability between the two. It also enhances the maintainability of the specification.

An FDFD consists of two types of primitive elements — data flows and tasks. They correspond to data flows and processes, respectively, of an ordinary data flow diagram.

To avoid ambiguities in a specification, we require that the relationships among input/output data flows for any given task must be defined explicitly. They are described by the operators "**and**" and "**or**" (or "$*$" and "$\oplus$" in the graphical representation). The "**and**" connector of data flow diagrams fits well with Petri nets, because the latter assumes an "**and**" operation on places connected to a transition. The "**or**" problem can be solved by extending the Petri net model to include input and output logic functions, as discussed below.

Let $D = \{d_1, d_2, ..., d_m\}$, where $m \geq 1$, be a finite set of data flows. Suppose $E$ denotes the set of all data flow expressions over the operators "**and**" and "**or**" (such as "$d_1$ **and** $d_2$ **or** $d_3$"). An *FDFD* is defined as a 4-tuple $G = (D, T, \mathbf{I}, \mathbf{O})$ such that:

- $D$ is the set of data flows.
- $T = \{t_1, t_2, ..., t_n\}$, where $n \geq 1$, is a finite set of tasks.
- $D$ and $T$ are disjoint.

- **I**: $T \rightarrow E$ and **O**: $T \rightarrow E$ are functions which map tasks to data flow expressions. **I** is called the input logic function and **O** the output logic function.

The graphical representation of a sample FDFD and its symbolic equivalent are shown in Figure 1.

To model the behaviour of a system over time, we have also incorporated the notions of token and firing from Petri nets into FDFD. Tokens can be placed in the data flows of an FDFD. The presence of a token means that input through a given data flow is ready for a task. A *marking* of an FDFD is defined as a set of tokens assigned to its data flows. It indicates the state of a system represented by the FDFD at a certain point in time. Mathematically, it is a function $u: D \rightarrow N$ from the set of data flows $D$ of an FDFD to the set of non-negative integers $N$. Given an FDFD $G$ and a marking $u$, we shall call the ordered couple $M = (G, u)$ a *marked FDFD*.

The marking can be changed by the execution of one or more tasks. A task is said to be *executable* if a combination of data flows satisfying its input logic function contains at least one token each, or in other words, a combination of data satisfying the input logic is available. After the execution of a task, one token is removed from each of the input data flows, and new tokens are placed in a combination of data flows satisfying the output logic function. A marking $v$ is said to be *reachable* from another marking $u$ if there exists a sequence of executions that changes $u$ into $v$.

These dynamic elements will provide the basis for analysing the dynamic behaviour of the system. The analysis will help to detect problems which may not otherwise be apparent in the static model, such as deadlocks or tasks that will never be activated.

## 4. CONSISTENCY ANALYSES

To demonstrate the feasibility of the language, a specification system based on FDFD has been implemented. Details of the specification system can be found in [20].

One important area in the analysis of a requirements specification is consistency. Consistency analysis will provide information on the completeness and correctness of a requirements specification. In addition, if consistency between different decomposition levels in a hierarchical specification is maintained, it will also reduce the amount of effort required in systems design and maintenance. Following the line of [5, 4, 16], we shall discuss in this section three types of consistency analyses useful for requirements specifications. They are global consistency, structural consistency and behavioural consistency. We shall illustrate how they can be achieved through FDFD.

### 4.1 Global Consistency Analysis

Before we concentrate on the consistency analysis at each abstraction level, we must make sure that the specification as a whole is defined as a hierarchical structure. Decomposition should not be done recursively, as illustrated in Figure 2. Otherwise, not only is the resulting specification confusing to users, but some non-primitive elements may in fact remain undefined. We shall refer to this type of consistency checking as *global consistency analysis*.

The decomposition of a task into a hierarchy of subtasks can be regarded as the creation of directed graphs whose vertices represent data/tasks and whose edges represent parent-child relationships between data/tasks. A partial ordering would result and the corresponding directed graph should contain no cycle. The presence of any cycle would imply recursive decomposition. This is illustrated in the last part of Figure 2.

Our algorithm for the detection of cycles is as follows: The directed graph can be represented by an adjacency matrix $A$. The entry $A(i, j)$ has a value of 1 if there is an edge from vertex $i$ to vertex $j$, and 0 otherwise. From this adjacency matrix, we can compute its corresponding path matrix $P$, where the entry $P(i, j)$ has a value of 1 if there is a path from vertex $i$ to vertex $j$, and 0 otherwise. Hence, $P(i, i) = 1$ will indicate the presence of one or more cycles passing through $i$.

The path matrix $P$ can be computed from the adjacency matrix $A$ by using the Warshall algorithm [26]:

```
begin
P := A
for i = 1 to n do
    for j = 1 to n do
        for k = 1 to n do
            P(j, k) := P(j, k) or (P(j, i) and P(i, k))
end.
```

We can then determine whether or not the hierarchy contains cycles by simply inspecting the diagonal of $P$. Any such cycle, if exists, can be located systematically from the original adjacency matrix.

In addition, the precedence analyser developed in [33, 34] can be applied to the adjacency matrix to generate useful reports for further analyses of the specification.

### 4.2 Structural Consistency Analysis

In order to spell out the details of a task in a data flow diagram, it can be redrawn as subtasks in another data flow diagram. One important principle to bear in mind is the balancing rule in structured analysis [7]: any data flow entering or leaving a parent bubble must be be represented on the lower level diagram by the same data flow into or out of some child bubble(s). We shall refer to this rule as *structural consistency*.

Before we spell out the conditions for structural consistency, we must define the concepts of external input and output data flows. Given an FDFD $G$, an *external input data flow* is defined as a data flow which is an input to some task in $G$ but not an output to any task in $G$. The set of all external input data flows of $G$ will be denoted by *ext_input*($G$). The set *ext_output*($G$) of *external output data flows* is similarly defined.

A necessary and sufficient condition for structural consistency can now be stated. Given a task $t_0$, let $G_0 = (D_0, \{t_0\}, \mathbf{I}_0, \mathbf{O}_0)$ be the FDFD formed by $t_0$ and its associated input and output flows. Let $G_1 = (D_1, T_1, \mathbf{I}_1, \mathbf{O}_1)$ be the new FDFD formed by decomposing $t_0$ into subtasks. The decomposition of $t_0$ into subtasks in $G_1$ will be structurally consistent if and only if:

(a) *ext_input*($G_1$) = $\mathbf{I}_0(t_0)$, and
(b) *ext_output*($G_1$) = $\mathbf{O}_0(t_0)$.

The example in Figure 3 shows a violation of structural consistency.

An algorithm has been developed for finding *ext_input*($G_1$) and *ext_output*($G_1$). It is summarized as follows:

(1) Let $t_1, t_2, ..., t_n$ be the tasks of $G_1$. Relate each $t_i$ with a data transformation of the form

$$L(t_i) \rightarrow R(t_i),$$

where $L(t_i) = \mathbf{I}(t_i)$ and $R(t_i) = \mathbf{O}(t_i)$. Hence represent $G_1$ by a combined transformation

$$L(t_1) \rightarrow R(t_1) \textbf{ and } L(t_2) \rightarrow R(t_2) \textbf{ and } \ldots \textbf{ and } L(t_n) \rightarrow R(t_n).$$

(2) Using the distributive property of the logical operator "**and**" over the operator "**or**", expand the input/output expressions *within* each transformation. For example,

$$(L_1 \textbf{ and } (L_2 \textbf{ or } L_3)) \rightarrow ((R_1 \textbf{ or } R_2) \textbf{ and } R_3)$$

becomes

$$(L_1 \textbf{ and } L_2 \textbf{ or } L_1 \textbf{ and } L_3) \rightarrow (R_1 \textbf{ and } R_2 \textbf{ or } R_1 \textbf{ and } R_3).$$

(3) For every transformation of the form

$$(L_1 \textbf{ or } L_2 \textbf{ or } \ldots \textbf{ or } L_p) \rightarrow R,$$

expand it to read

$$L_1 \rightarrow R \textbf{ or } L_2 \rightarrow R \textbf{ or } \ldots \textbf{ or } L_p \rightarrow R.$$

(4) For every transformation of the form

$$L \rightarrow (R_1 \textbf{ or } R_2 \textbf{ or } \ldots \textbf{ or } R_q),$$

expand it to read

$$L \rightarrow R_1 \textbf{ or } L \rightarrow R_2 \textbf{ or } \ldots \textbf{ or } L \rightarrow R_q.$$

(5) For every transformation of the form

$$L \rightarrow (R_1 \textbf{ and } R_2 \textbf{ and } \ldots \textbf{ and } R_s),$$

expand it to read

$$L \rightarrow R_1 \textbf{ and } L \rightarrow R_2 \textbf{ and } \ldots \textbf{ and } L \rightarrow R_s.$$

(6) Using the distributive property of the logical operator "**and**" over the operator "**or**", expand the expressions of transformations. For example,

$$L_1 \rightarrow R_1 \textbf{ and } (L_2 \rightarrow R_2 \textbf{ or } L_3 \rightarrow R_3)$$

becomes

$$L_1 \rightarrow R_1 \textbf{ and } L_2 \rightarrow R_2 \textbf{ or } L_1 \rightarrow R_1 \textbf{ and } L_3 \rightarrow R_3.$$

(7) We define a *term* as a combination of transformations joined together only by "**and**"s but not "**or**"s. Do the following for each term:

(7.1) For any $L' \rightarrow R'$ such that $R'$ is a subexpression of $L$ for some $L \rightarrow R$ in the same term:

(a) Substitute $L'$ into $R'$ in the transformation $L \rightarrow R$;
(b) Remove $L' \rightarrow R'$ from the term.

(7.2) Remove all transformations $L \rightarrow R$ such that

(a) $R$ is not in *ext_output* $(G_1)$, or
(b) there is some $d$ in $L$ but not in *ext_input* $(G_1)$.

(8) Combine transformations *within* a term into a single transformation by converting

$$L_1 \rightarrow R_1 \textbf{ and } L_2 \rightarrow R_2 \textbf{ and } \ldots \textbf{ and } L_p \rightarrow R_p$$

into

$$(L_1 \textbf{ and } L_2 \textbf{ and } \ldots \textbf{ and } L_p) \rightarrow (R_1 \textbf{ and } R_2 \textbf{ and } \ldots \textbf{ and } R_p).$$

(9) Combine all the transformations into a single transformation by converting

$$L_1 \rightarrow R_1 \textbf{ or } L_2 \rightarrow R_2 \textbf{ or } ... \textbf{ or } L_q \rightarrow R_q$$

into

$$(L_1 \textbf{ or } L_2 \textbf{ or } ... \textbf{ or } L_q) \rightarrow (R_1 \textbf{ or } R_2 \textbf{ or } ... \textbf{ or } R_q).$$

(10) Let $L \rightarrow R$ be the resulting transformation. Then

(a) $ext\_input(G_1) = L$
(b) $ext\_output(G_1) = R$.

It is assumed in the above algorithm that $G_1$ is a connected net. If this were not the case, the parent FDFD should be redefined so that all the tasks are connected by data flows.

### 4.3 Behavioural Consistency Analysis

Besides checking consistencies among the static properties of a system, we must also ensure that the dynamic properties are preserved during the decomposition of an FDFD. This will be known as *behavioural consistency analysis*.

As discussed in Section 3, the dynamic behaviour of a system over time is modelled by the notion of token and firing. A marking of an FDFD represents a state of the system. It reflects the data available for transformation and the tasks to be executed next. Changes in markings via the execution of tasks portray the changes in the states of the system over time and hence the dynamic behaviour.

Let $G$ be an FDFD and let $U(G)$ be the set of all its markings. We define the *external input markings* as the set of all markings such that only external input data flows contain tokens. More formally,

$$ext\_input\_mark(G) = \{u \in U(G) \mid u(d_i) > 0 \text{ ☞ } d_i \in ext\_input(G)\}.$$

Similarly, we define the *external output markings* as the set of all markings such that only output data flows contain tokens, or

$$ext\_output\_mark(G) = \{u \in U(G) \mid u(d_i) > 0 \text{ ☞ } d_i \in ext\_output(G)\}.$$

Given a marking $u$ in $ext\_input\_mark(G)$, we define the *final markings*, or $final\_mark(G, u)$, as the set of markings which are reachable from $u$ but have no potential for further execution.

Let $t_0$ be a task in $G$ and let $G'$ be the new FDFD formed from $G$ by decomposing $t_0$ into subtasks. Let $G_0$ be the FDFD formed by the task $t_0$ and its associated data flows. To preserve behavioural characteristics, it is necessary that, for every external input marking of $G_0$, both $G$ and $G'$ reach the same set of final markings.

We note also that, whereas the final marking of the original $G$ will only have tokens in the external input/output data flows, this may not be the case for $G'$. To complete the conditions for behaviourally consistency, therefore, we require that all the final markings of $G'$ must be external output markings only.

More formally, the decomposition of $G$ into $G'$ will be behaviourally consistent if and only if:

(a) The decomposition is structurally consistent.
(b) For each marking $u$ in $ext\_input\_mark(G_0)$,
$final\_mark(G, u) = final\_mark(G', u)$ and $final\_mark(G', u) \subseteq ext\_output\_mark(G')$.

The example in Figure 4 shows a violation of behavioural consistency.

In order to detect any violation of these conditions, we need to introduce the concepts of reachability sets and reachability trees. Given a marked FDFD $(G, u)$, we define the *reachability set* $R(G, u)$ as the set of all markings which are reachable from the present marking $u$. Then we have:

$$final\_mark(G, u) = \{v \in R(G, u) \mid R(G, v) = \{v\}\}.$$

Since there may be infinitely many paths leading from the marking $u$, we must find a systematic way of finding the final markings. We shall follow [19] and construct a *reachability tree*, which is a finite representation of the relationships in a reachability set. The nodes of the tree represent markings reachable from $u$. The branches represent the paths leading from one marking to another through the execution of tasks. The leaves of the tree can be one of the following:

- Terminal nodes, which represent markings with no potential for further execution.
- Duplicated nodes, which also appear elsewhere in the tree and hence their successors need not be shown again.

An example of the reachability tree of a marked formal data flow diagram is given in Figure 5.

To identify any violation of behavioural consistency, we should construct the reachability trees of $G$ and $G'$ for every marking $u$ in $ext\_input\_mark(G_0)$. We can then locate the terminal nodes, which correspond to the final markings. Any discrepancy from condition (b) above can therefore be detected.

## 5. COMPARISON WITH RELATED WORKS

In most of the other system specification tools based on Petri nets, such as in IML-inscribed predicate/transition nets and abstract process nets, the existing formalisms in Petri nets are extended to incorporate new concepts that are necessary in describing a system. As pointed out in [4, 12, 13], however, practitioners are rather hesitant to use such tools which involve an unfamiliar formal language. EDDA and our FDFD, on the other hand, are attempts to incorporate the concepts in Petri nets into existing systems analysis tools such as SADT [23, 24] and DeMarco data flow diagrams. They are therefore more acceptable by practising systems analysts.

Although the approaches taken by EDDA and FDFD are similar, the choice of coupling EDDA with SADT may complicate the issue, because not every one of the 40 features of SADT has a Petri net counterpart. Even if we can extend the Petri net notions to incorporate all the SADT features, the number of concepts involved will be a definite hindrance to user understanding.

## 6. CONCLUSION

We have provided data flow diagrams with a formal foundation through extended Petri nets. We have developed a specification language which is both comprehensible to users and analysable by computers. The resulting language, known as formal data flow diagrams (FDFD), has two equivalent representations — a graphical form similar to standard data flow diagrams, as well as a symbolic form. Based on the formal foundation, analyses of the requirements specification can be made on such areas as global consistency, structural consistency and behavioural consistency.

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## REFERENCES

[1] T. Agerwala, ''Putting Petri nets to work'', *IEEE Computer* **12** (12): 85−94 (1979).

[2] L.L. Beck and T.E. Perkins, ''A survey of software engineering practice: tools, methods, and results'', *IEEE Transactions on Software Engineering* **9** (5): 541−561 (1983).

[3] M.A. Colter, ''Evolution of the structured methodologies'', in *Advanced System Development / Feasibility Techniques*, J.D. Couger, M.A. Colter, and R.W. Knapp (eds.), John Wiley, New York, NY, pp. 73−96 (1982).

[4] A.M. Davis, ''The design of a family of application-oriented requirements languages'', *IEEE Computer* **15** (5): 21−28 (1982).

[5] A.M. Davis and T.G. Rauscher, ''Formal techniques and automatic processing to ensure correctness in requirements specifications'', in *Proceedings of the IEEE Conference on Specifications of Reliable Software*, M.V. Zelkowitz (ed.), IEEE Computer Society, New York, NY, pp. 15−35 (1979).

[6] N.M. Delisle, D.E. Menicosy, and N.L. Kerth, ''Tools for supporting structured analysis'', in *Automated Tools for Information Systems Design*, H.-J. Schneider and A.I. Wasserman (eds.), Elsevier, Amsterdam, The Netherlands, pp. 11−20 (1982).

[7] T. DeMarco, *Structured Analysis and System Specification*, Yourdon Press Computing Series, Prentice Hall, Englewood Cliffs, NJ (1979).

[8] C.P. Gane and T. Sarson, *Structured Systems Analysis: Tools and Techniques*, Prentice Hall, Englewood Cliffs, NJ (1979).

[9] M.A. Jackson, *Principles of Program Design*, Academic Press, London, UK (1975).

[10] M.A. Jackson, *System Development*, Prentice Hall International Series in Computer Science, Prentice Hall, London, UK (1983).

[11] G.R. Kampen, ''SWIFT: A requirements specification system for software'', in *Requirements Engineering Environments: Proceedings of the International Symposium on Current Issues of Requirements Engineering Environments*, Y. Ohno (ed.), Elsevier, Amsterdam, The Netherlands, pp. 77−84 (1982).

[12] J. Martin, *Program Design which is Provably Correct*, Savant Institute, Carnforth, Lancashire, UK (1983).

[13] J. Martin, *An Information Systems Manifesto*, Prentice Hall, Englewood Cliffs, NJ (1984).

[14] L.J. Mekly, *A Systems Approach to Software Design Representation*, PhD Thesis, Northwestern University, Evanston, IL (1979).

[15] L.J. Mekly and S.S. Yau, ''Software design representation using abstract process networks'', *IEEE Transactions on Software Engineering* **6** (5): 420−435 (1980).

[16] T.J. Miller and B.J. Taylor, ''A system requirements methodology'', in *Proceedings of ELECTRO '81*, IEEE Computer Society, New York, NY, pp. 18.5.1−18.5.5 (1981).

[17] K.T. Orr, *Structured Systems Development*, Yourdon Press Computing Series, Prentice Hall, Englewood Cliffs, NJ (1977).

[18] J.L. Peterson, ''Petri nets'', *ACM Computing Surveys* **9** (3): 223−252 (1977).

[19] J.L. Peterson, *Petri Net Theory and the Modeling of Systems*, Prentice Hall, Englewood Cliffs, NJ (1981).

[20] L. Pong, *Formal Data Flow Diagrams (FDFD): A Petri-Net Based Requirements Specification Language*, MPhil Thesis, The University of Hong Kong, Pokfulam, Hong Kong (1986).

[21] W. Reisig, *Petri Nets: An Introduction*, EATCS Monographs on Theoretical Computer Science, vol. 4, Springer, Berlin, Germany (1985).

[22] G. Richter and R. Durchholz, ''IML-inscribed high-level Petri nets'', in *Information Systems Design Methodologies, a Comparative Review: Proceedings of the IFIP WG 8.1 Working Conference (CRIS '82)*, T.W. Olle, H.G. Sol, and A.A. Verrijn-Stuart (eds.), Elsevier, Amsterdam, The Netherlands, pp. 335−368 (1982).

[23] D.T. Ross, ''Structured analysis (SA): A language for communicating ideas'', in *Advanced System Development / Feasibility Techniques*, J.D. Couger, M.A. Colter, and R.W. Knapp (eds.), John Wiley, New York, NY, pp. 135−163 (1982).

[24] D.T. Ross and K.E. Schoman, ''Structured analysis for requirements definition'', in *Classics in Software Engineering*, E. Yourdon (ed.), Yourdon Press Computing Series, Prentice Hall, Englewood Cliffs, NJ, pp. 365−385 (1979).

[25] W. Trattnig and H. Kerner, ''EDDA: A very-high-level programming and specification language in the style of SADT'', in *Proceedings of the 4th Annual International Computer Software and Applications Conference (COMPSAC '80)*, IEEE Computer Society, New York, NY, pp. 436−443 (1980).

[26] J.P. Tremblay and P.G. Sorenson, *An Introduction to Data Structures with Applications*, McGraw-Hill, New York, NY (1976).

[27] T.H. Tse, ''A unified algebraic view of structured systems development models'', Doctoral Consortium, 6th International Conference on Information Systems, Indianapolis, IN (1985).

[28] T.H. Tse, ''An algebraic formulation for structured systems development tools'', *Hong Kong Computer Journal* **2** (4): 44−49 (1986).

[29] T.H. Tse, ''Integrating the structured analysis and design models: An initial algebra approach'', *Journal of Research and Practice in Information Technology (formerly the Australian Computer Journal)* **18** (3): 121−127 (1986).

[30] T.H. Tse, ''Towards a unified algebraic view of structured systems development models'', *ACM SIGMIS Database* **17** (4) (1986).

[31] T.H. Tse, ''Integrating the structured analysis and design models: A category-theoretic approach'', *Journal of Research and Practice in Information Technology (formerly the Australian Computer Journal)* **19** (1): 25−31 (1987).

[32] T.H. Tse and L. Pong, ''An examination of requirements specification languages'', *The Computer Journal* **34** (2): 143−152 (1991).

[33] S.J. Waters, ''CAM 01: A precedence analyser'', *The Computer Journal* **19** (2): 122−126 (1976).

[34] S.J. Waters, ''CAM 02: A structured precedence analyser'', *The Computer Journal* **20** (1): 2−5 (1977).

[35] V. Weinberg, *Structured Analysis*, Yourdon Press Computing Series, Prentice Hall, Englewood Cliffs, NJ (1980).

[36] E. Yourdon and L.L. Constantine, *Structured Design: Fundamentals of a Discipline of Computer Program and Systems Design*, Yourdon Press Computing Series, Prentice Hall, Englewood Cliffs, NJ (1979).

## APPENDIX

### A Brief Summary of Petri Nets

A *Petri net* is a directed bipartite graph consisting of two types of nodes — places and transitions. Algebraically, it is defined as a 4-tuple $C = (P, T, I, O)$ such that:

- $P = \{p_1, p_2, ..., p_n\}$, where $n \geq 1$, is a finite set of places.
- $T = \{t_1, t_2, ..., t_m\}$, where $m \geq 1$, is a finite set of transitions.
- $P$ and $T$ are disjoint.
- $I: T \rightarrow 2^P$ and $O: T \rightarrow 2^P$, where $2^P$ denote the power set of $P$, are functions which map transitions to sets of places. $I$ is called the input function and $O$ the output function.

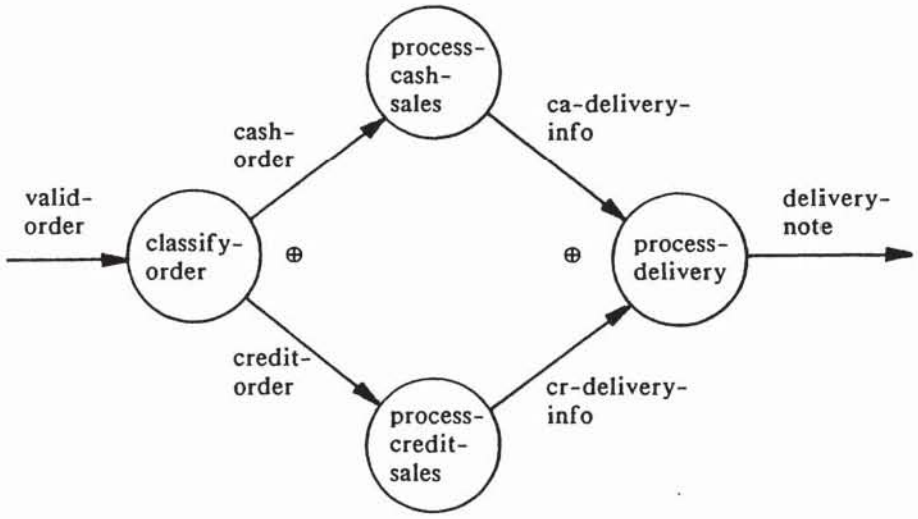
In the graphical representation, places are represented by circles and transitions by bars, as illustrated in Figure 6. This graphical model describes the static properties of a system.

The notions of tokens and firing are used to model system dynamics. *Tokens*, each represented by a dot, can be defined inside the places of a Petri net. A place can contain any number of tokens. A transition is said to be *enabled* if each of its input places contains at least one token. A transition can be *fired* if and only if it has been enabled. During the firing of a transition, one token is removed from each of its input places and one token is deposited into each of its output places. An example is shown in Figure 7.

The assignment of a set of tokens to the places of a Petri net is known as a *marking* of the net. Formally, it is defined as a function $u: P \rightarrow N$ from the set of places $P$ to the set of non-negative integers $N$. A marking $v$ is said to be *reachable* from another marking $u$ if there exists a sequence of executions that changes $u$ into $v$. Given a Petri net $C$ and a marking $u$, we call the ordered couple $M = (C, u)$ a *marked Petri net*.

A Petri net is an uninterpreted mathematical model, in the sense that we can assign any meaning to the states, transitions, tokens or markings. We can, for example, assign meanings to these concepts through formal data flow diagrams as shown in the main paper.

**(a) Graphical Representation:**




**(b) Algebraic Representation:**

*D* = {valid-order, cash-order, credit-order, ca-delivery-info, cr-delivery-info, delivery-note}

*T* = {classify-order, process-cash-sales, process-credit-order, process-delivery}

**I** = {(classify-order, valid-order),
(process-cash-sales, cash-order),
(process-credit-sales, credit-order),
(process-delivery, ca-delivery-info **or** cr-delivery-info)}

**O** = {(classify-order, cash-order **or** credit-order),
(process-cash-sales, ca-delivery-info),
(process-credit-sales, cr-delivery-info),
(process-delivery, delivery-note)}

**Figure 1. Graphical and algebraic representations of FDFD.**

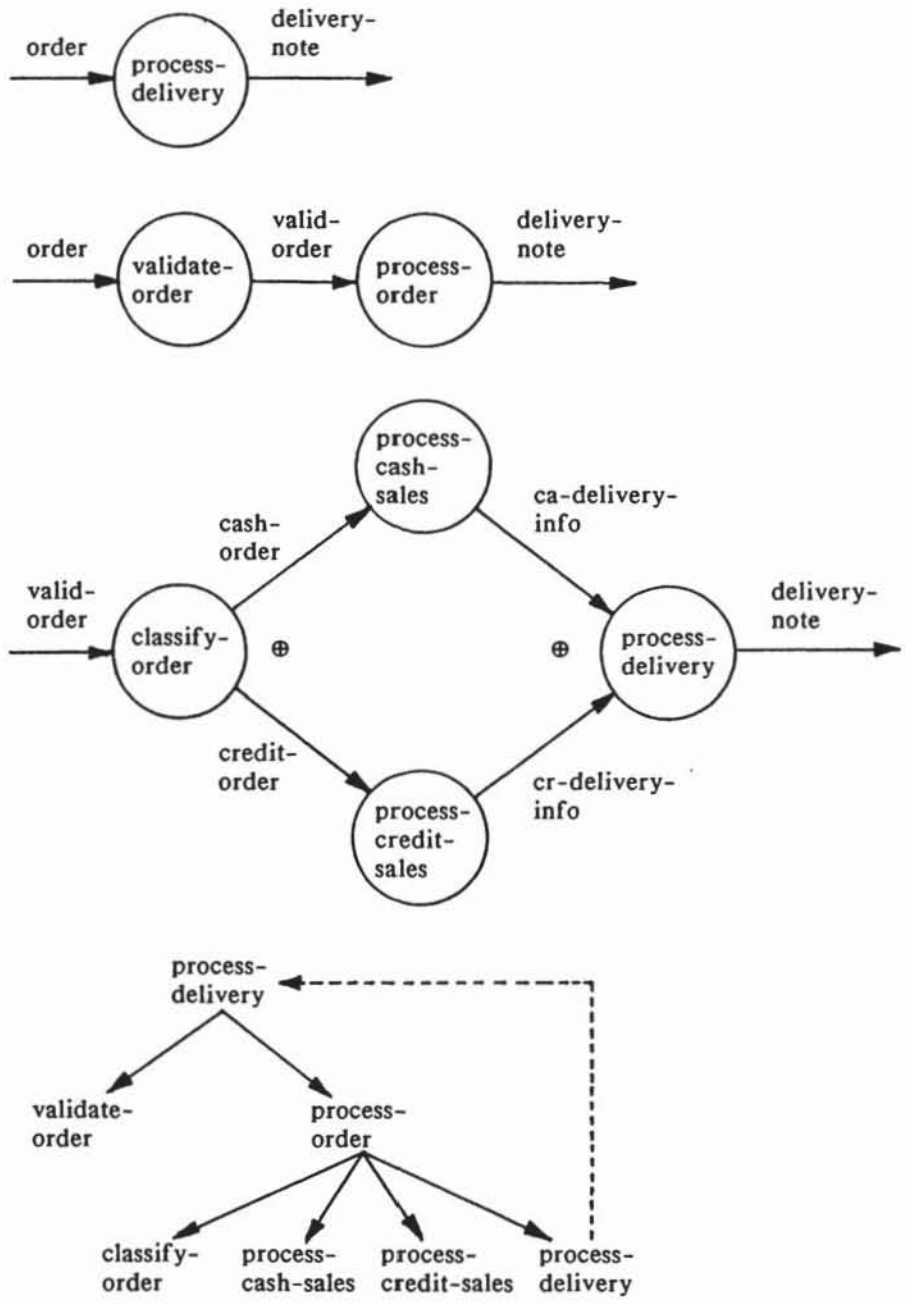
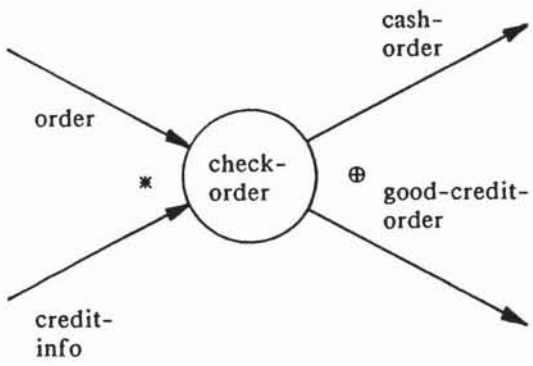
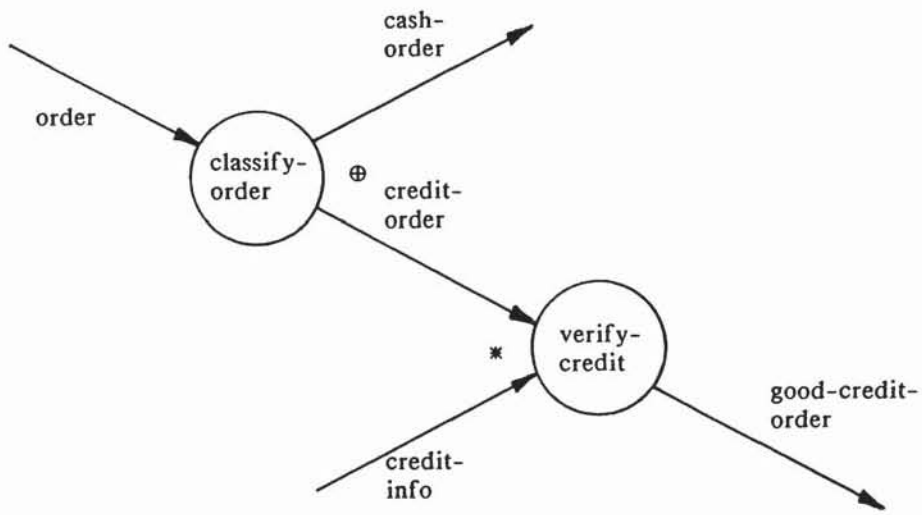



**Figure 2. Violation of global consistency.**

(a) FDFD $G_0$ (before decomposition):




(b) FDFD $G_1$ (after decomposition):




**Algebraically:**

$\mathbf{I}_0(t_0)$ = order **and** credit-info

$\mathbf{O}_0(t_0)$ = cash-order **or** good-credit-order

$exp\_inp(G_1)$ = order **or** order **and** credit-info

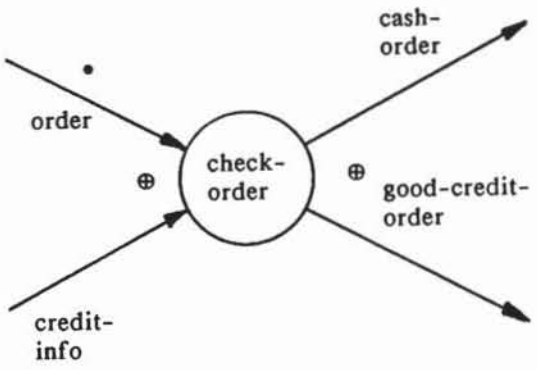
$exp\_outp(G_1)$ = cash-order **or** good-credit-order

Hence $exp\_inp(G_1) \neq \mathbf{I}_0(t_0)$

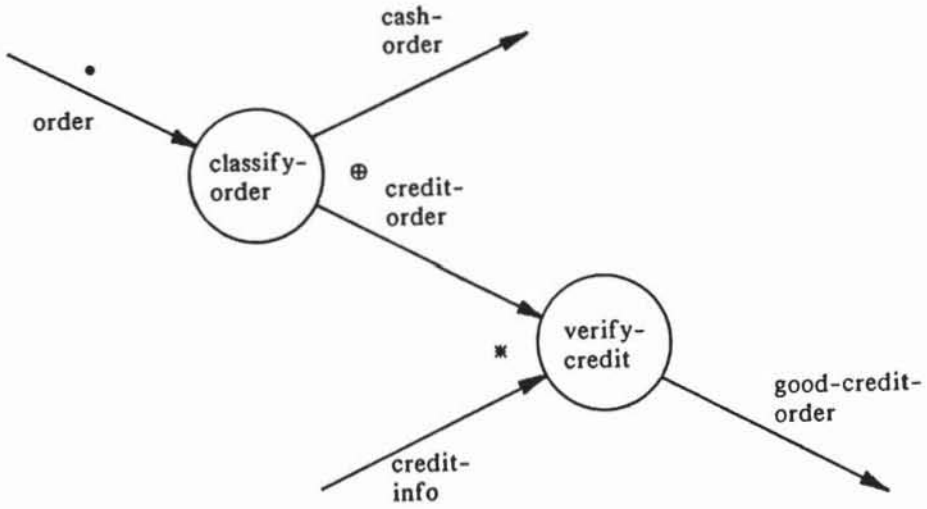
**Figure 3. Violation of structural consistency.**

**Marking $u$ (order ready):**

**(a) FDFD $G$ (before decomposition):**

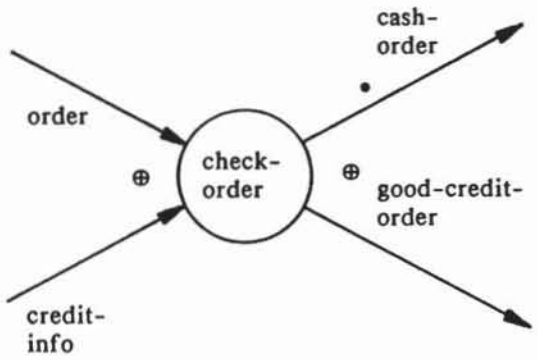



**(b) FDFD $G'$ (after decomposition):**

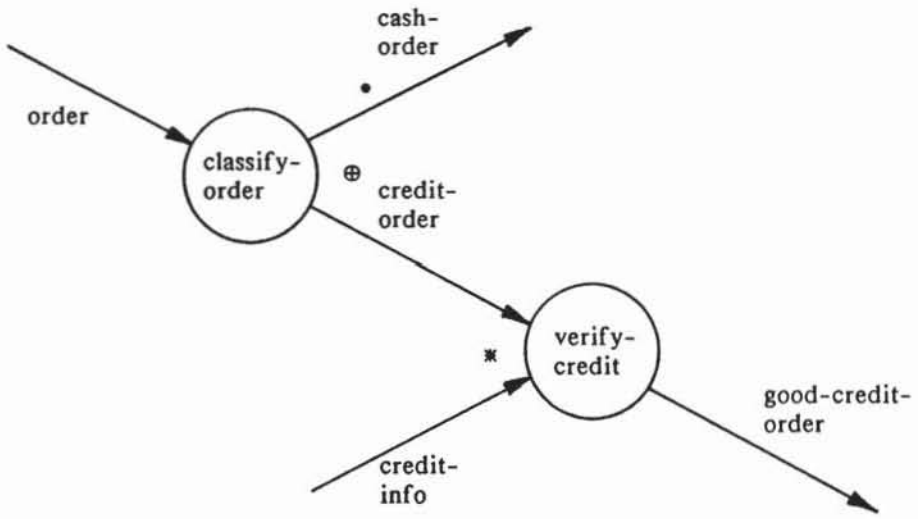



**Marking $u_1$ (cash-order ready):**

**(a) FDFD $G$ (before decomposition):**

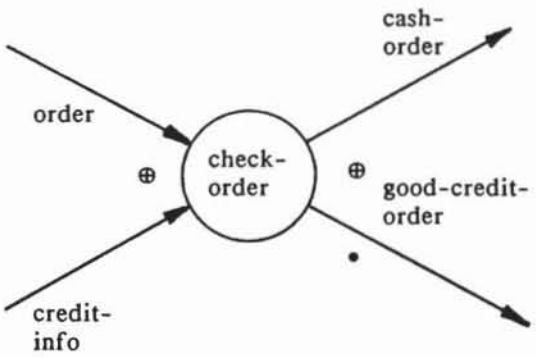



**(b) FDFD $G'$ (after decomposition):**

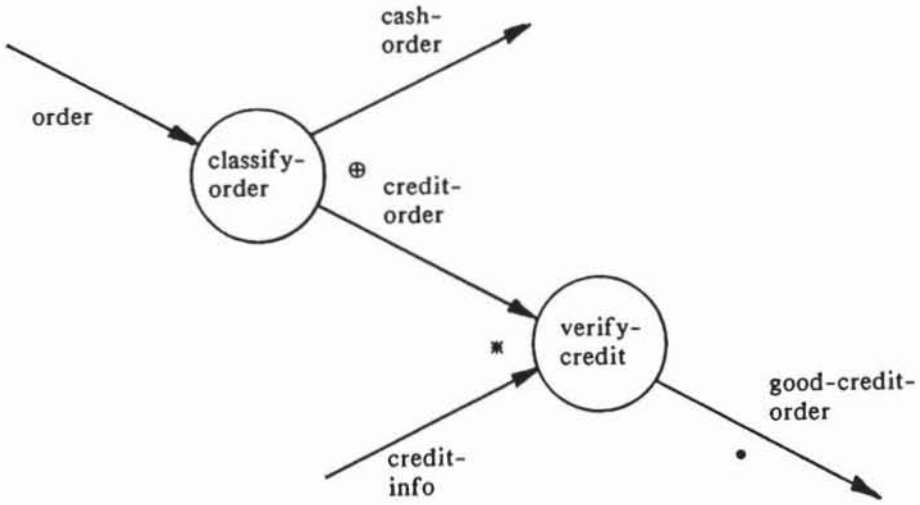


**Marking $u_2$ (good-credit-order ready):**

**(a) FDFD $G$ (before decomposition):**




**(b) FDFD $G'$ (after decomposition):**

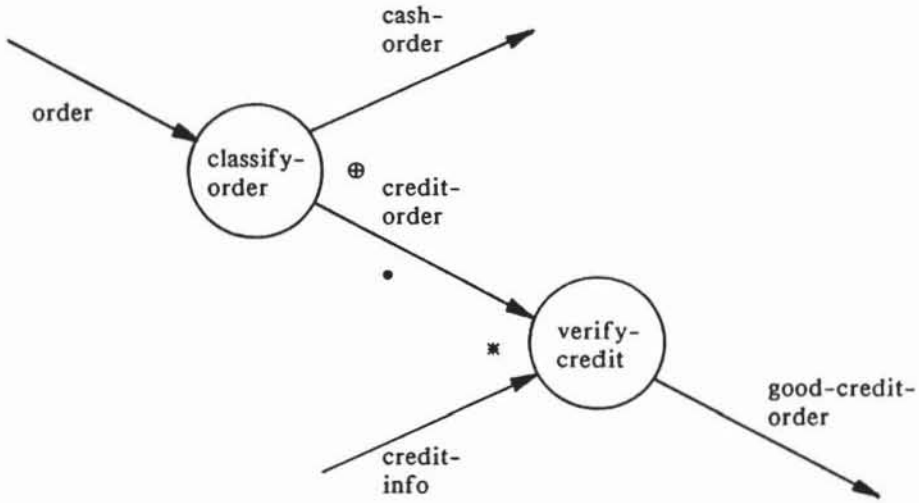



**Marking $u_3$ (only credit-order ready):**

**(a) FDFD $G$ (before decomposition):**

Such a marking does not exist.

**(b) FDFD $G'$ (after decomposition):**




**Algebraically:**

$final\_mark(G, u) = \{u_1, u_2\}$

$final\_mark(G', u) = \{u_1, u_3\}$

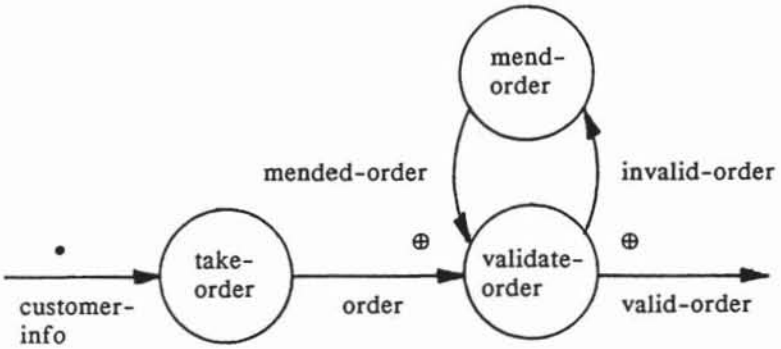
$ext\_outp\_mark(G') = \{u_1, u_2\}$

Hence $final\_mark(G, u) \neq final\_mark(G', u)$

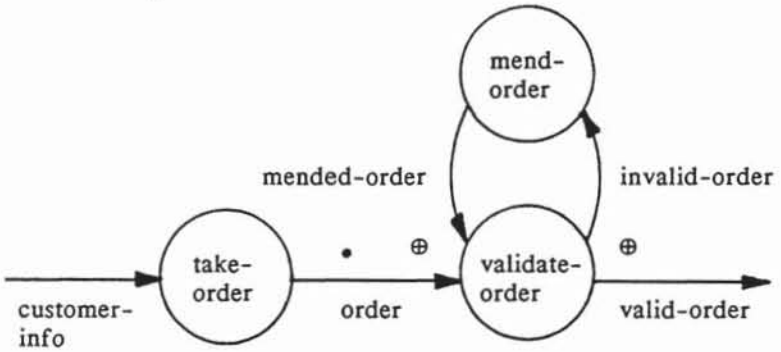
and $final\_mark(G', u) \not\subset ext\_outp\_mark(G')$

**Figure 4. Violation of behavioural consistency.**

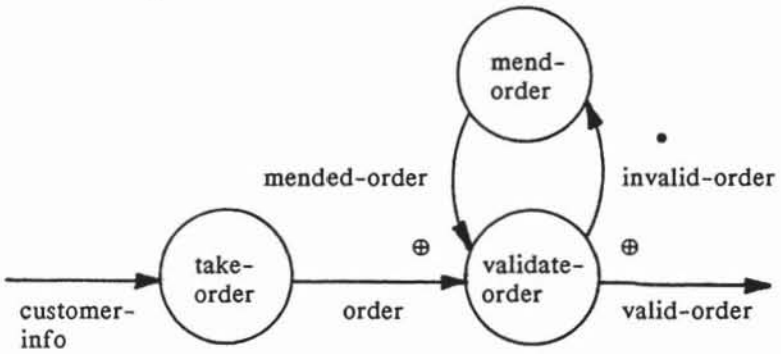
**Marking $u_1$ (customer-info ready):**



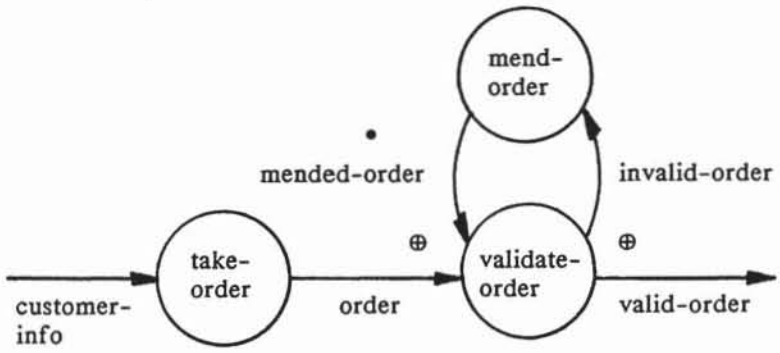

**Marking $u_2$ (order ready):**



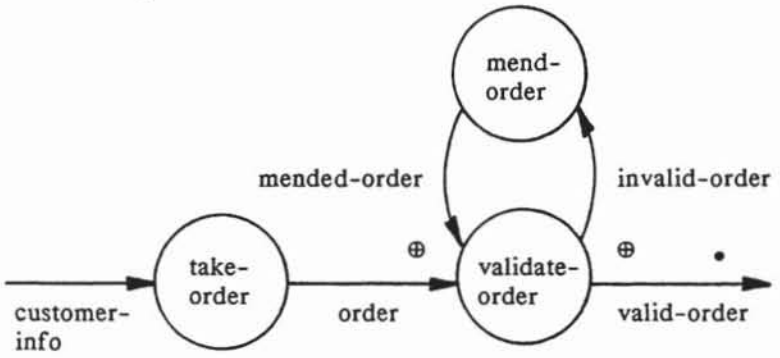

**Marking $u_3$ (invalid-order case):**



**Marking $u_4$ (mended-order ready):**




**Marking $u_5$ (valid-order ready):**




**Reachability Tree:**

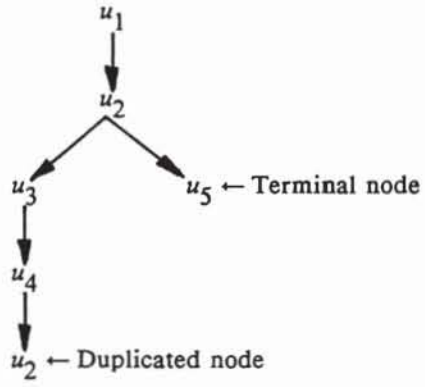



**Figure 5. Reachability tree.**