Spin-resolved Hall effect driven by spin-orbit coupling

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Spin and electric Hall currents are calculated numerically in a two-dimensional mesoscopic system with Rashba and Dresselhaus spin-orbit coupling by means of the Landauer-Büttiker formalism. It is found that both electric and spin Hall currents circulate when two spin-orbit couplings coexist, while the electric Hall conductance vanishes if either one is absent. The electric and spin Hall conductances are suppressed in strong disorder, but survive in weak disorder. Physically it can be understood that the spin transverse "force" generated by spin-orbit coupling is responsible for the formation of the spin Hall current and the lack of transverse reflection symmetry is the origin of the electric Hall current.

When a metallic sample is subjected to a perpendicular external magnetic field, the Lorentz force acting on the charge carriers gives rise to a transverse voltage between two edges of the sample; this is well known as the ordinary Hall effect. Actually, the Hall effect family has a number of important members. The anomalous Hall effect may occur even in the absence of an external magnetic field in a ferromagnetic metal with spin-orbit coupling.[1–8] In the past few years it has been recognized that the spin-orbit coupling may provide an efficient way to manipulate a spin-resolved current in metals and semiconductors.[9–15] In a two-dimensional electron gas (2DEG) lacking bulk and structure inversion symmetries, the effective Hamiltonian is given by

$$H = \frac{p^2}{2m^*} + \lambda(\sigma^x p_y - \sigma^y p_x) + \beta(\sigma^x p_x - \sigma^y p_y), \quad (1)$$

where the second term is the Rashba spin-orbit coupling and the third one is the Dresselhaus spin-orbit coupling. σ^μ ($\mu = x, y, z$) are the Pauli matrices, and the coupling parameters λ and β have the dimension of velocity. Using the Heisenberg equation of motion the second derivative of the position operator **r** gives

$$m^* \frac{\partial^2 \mathbf{r}}{\partial t^2} = + \frac{2m^*(\lambda^2 - \beta^2)\sigma^z}{\hbar} \mathbf{p} \times \hat{z}. \quad (2)$$

Compared with the Lorentz force brought by the magnetic field upon a charged particle, the spin-orbit coupling produces a spin transverse "force" on a moving electron. It has no classical counterpart as the coefficient is divided by $\hbar$, but it reflects the tendency of spin asymmetric scattering of a moving electron subject to the spin-orbit coupling. When charge carriers are driven by an external electric field, this force tends to form a transverse spin current.

In this paper we calculate the spin and electric Hall conductances in a 2DEG mesoscopic system with Rashba and Dresselhaus coupling by using the Landauer-Büttiker formula and the Green's function technique. It is found that both electric and spin Hall currents circulate while these two types of spin-orbit coupling coexist, but the electric Hall current disappears when either one is absent. The spin and elec-

tric Hall conductances are suppressed in strong disorder, but survive in weak disorder. The numerical results are in good agreement with the symmetry analysis of the system.

We consider a cross-shape device with four semi-infinite metallic leads as shown in Fig. 1. The scattering region (the shadowed part in Fig. 1) is described by the effective Hamiltonian in Eq. (1), and when it is treated as an $L \times L$ lattice with the tight-binding approximation, the model Hamiltonian reads,

$$H = -t \sum_{ij,\sigma=\uparrow,\downarrow} c_{i,\sigma}^\dagger c_{j,\sigma} + t_{so}^R \sum_i (J_{i,y}^x - J_{i,x}^y) + t_{so}^D \sum_i (J_{i,x}^x - J_{i,y}^y),$$
$$(3)$$

where t_{so}^R and t_{so}^D are the dimensionless parameters for Rashba and Dresselhaus coupling strength in the unit of t, respectively, and the local spin current operator $J_{i,\alpha}^\mu$ is defined as[16]

$$J_{i,\alpha}^\mu = -it \sum_{\sigma,\sigma'} (c_{i,\sigma}^\dagger \sigma_{\sigma\sigma'}^\mu c_{i+\alpha,\sigma'} - \text{H.c.}), \quad (4)$$

where α stands for the unit vector along the axes of the lattice and μ stands for the direction of spin polarization.[17]
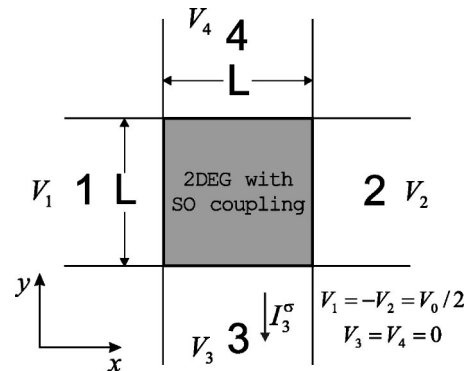

FIG. 1. Cross-shape device with four semi-infinite metallic leads. The spin-orbit coupling is supposed to exist in the shadowed area only, and the effect of the semi-infinite leads is treated exactly through self-energy terms.

The calculation of electric and spin currents is based on the Landauer-Büttiker formalism.[18,19] Assume $T_{q,p}^{\nu,\mu}$ to be the spin-resolved transmission probability of electrons transmitted from spin channel μ of lead p to spin channel ν of lead q, and V_p to be the electric voltage in lead p, then, respectively, the outgoing electric current and spin current polarized along μ direction in lead p are

$$I_p^c = \frac{e^2}{h} \sum_{q,\mu,\nu} (T_{p,q}^{\mu,\nu} V_q - T_{q,p}^{\nu,\mu} V_p), \tag{5}$$

$$I_p^\mu = -\frac{e}{4\pi} \sum_{q,\nu} [(T_{p,q}^{\mu,\nu} - T_{p,q}^{-\mu,\nu})V_q - (T_{q,p}^{\nu,\mu} - T_{q,p}^{\nu,-\mu})V_p]. \tag{6}$$

The transmission probability coefficients can be calculated by using the Green's function technique, $T_{q,p}^{\nu,\mu} = \text{Tr}[\Gamma_q^\nu G^R \Gamma_p^\mu G^A]$. And the retarded and advanced Green functions are given by $G^{R,A}(E) = 1/(E - H_c - \Sigma_{p=1}^4 \Pi_p^{R,A})$, where E is the electron energy and H_c is the model Hamiltonian in the shadowed region in Fig. 1. The retarded and advanced self-energy terms introduced by the semi-infinite lead p, $(\Pi_p^R)_{p_i\sigma,p_j\sigma'} = -t\Sigma_m \chi_m(p_i)e^{ik_m a}\chi_m(p_j)\delta_{\sigma\sigma'}$ and $\Pi_p^A = (\Pi_p^R)^\dagger$, where $\chi_m(p_i)$ is the transverse mode wave function at site p_i in lead p connected to the scattering region. It should be noted that in Eq. (6) μ may stand for an arbitrary direction of spin polarization, and this is done by incorporating a transformation in the definition of Γ_p, that is, $\Gamma_p^\mu(p_i,\sigma,p_j,\sigma') = 2tR_{\mu\sigma}R_{\sigma'\mu}^{-1}\Sigma_m\chi_m(p_i)\sin(k_m a)\chi_m(p_j)$ and R is the rotation matrix to transform the eigenstates of σ^z to those of $\hat{r}\cdot\boldsymbol{\sigma}$ ($\hat{r}$ is a unit vector).[20] The Landauer-Büttiker formalism has been applied extensively to study the spin transport in mesoscopic systems numerically.[21–24]

In this paper we consider an initial electric current driven through leads 1 and 2, $I_1^c = -I_2^c$, by setting the bias voltage $V_1 = -V_2 = V_0/2$ and $V_3 = V_4 = 0$. The currents in leads 3 and 4 are perpendicular to the current through leads 1 and 2. Thus the electric and spin Hall conductances are defined as

$$G_{cH} = I_3^c/(V_1 - V_2); \tag{7}$$

$$G_{sH}^\mu = I_3^\mu/(V_1 - V_2), \tag{8}$$

respectively, where the spin current has three components, $\mu = x, y, z$. Electric and spin Hall conductances are evaluated as functions of the Fermi energy for different ratios of Rashba and Dresselhaus coupling strength in Fig. 2. Generally speaking, the electric Hall conductance is symmetric about the Fermi energy E_f while the spin Hall conductance is antisymmetric such that the spin Hall conductance vanishes at the band center, $E_f = 0$. This is consistent with the symmetry analysis for the tight-binding Hamiltonian.[23] In the case of the pure Rashba or Dresselhaus coupling, the electric Hall conductance disappears, but the spin Hall conductance still exists. In the two cases of $t_{SO}^R = 1$ and $t_{SO}^D = 1/2$ and of $t_{SO}^R = 1/2$ and $t_{SO}^D = 1$, the electric Hall conductances are equal. However the spin Hall conductances G_{sH}^z differ by a minus sign, with G_{sH}^x and G_{sH}^y swapped, and the former is consistent with the work of Shen[25] and Sinitsyn et al.[26] for free 2DEG systems. A special case is at the symmetric point of t_{SO}^D
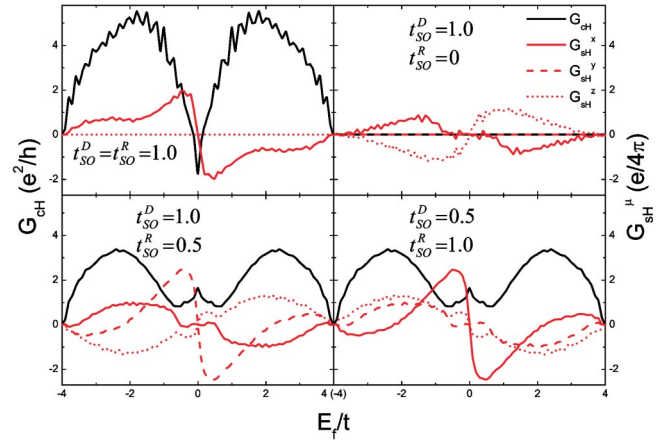

FIG. 2. (Color online) Electric and spin Hall conductances as functions of the electron Fermi energy for different ratios of the Rashba and Dresselhaus coupling strengths. $L=40$ for all the results in this figure.

$= t_{SO}^R$. The spin Hall conductance $G_{sH}^z = 0$, while G_{sH}^x and G_{sH}^y are equal and nonzero, which means the current is polarized within the x-y plane. In this case the operator $\sigma^x + \sigma^y$ commutes with the total Hamiltonian, and actually there is no spin flip in the scattering region.[27] On the other hand the longitudinal electric and spin conductances are also nonzero. The longitudinal conductances are about one order larger than the Hall conductances in magnitude, i.e., $I_3^c/I_2^c \sim 10$. And the electric conductance is also symmetric with respect to the Fermi energy, just like the electric Hall conductance, while the longitudinal spin current is antisymmetric. According to the symmetry properties of such a system,[25] under the transformation: $\sigma^x \rightarrow \sigma^y$, $\sigma^y \rightarrow \sigma^x$, and $\sigma^z \rightarrow -\sigma^z$, $t_{SO}^D \rightarrow t_{SO}^R$, and $t_{SO}^R \rightarrow t_{SO}^D$, correspondingly $G_{sH}^x \rightarrow G_{sH}^y$, $G_{sH}^y \rightarrow G_{sH}^x$, and $G_{sH}^z \rightarrow -G_{sH}^z$ while the electric Hall conductance remains unchanged. At the symmetric point, $t_{SO}^R = t_{SO}^D$, it yields that $G_{sH}^x = G_{sH}^y$ and $G_{sH}^z = 0$. Our numerical results obviously agree with this symmetry analysis.

The Hall conductances as functions of the Rashba coupling strength are also evaluated, with the specific Dresselhaus coupling strength $t_{so}^D = 1.0$ and electron Fermi energy $E_f = -2.0t$ as shown in Fig. 3. It indicates clearly that the electric Hall conductance increases with increasing the Rashba couple strength, and reaches its maximal value at $t_{so}^R = t_{so}^D$. Then it turns to decrease when $t_{so}^R > t_{so}^D$, and approaches to zero for a large Rashba coupling strength. The figure shows that $G_{sH}^z = 0$ and $G_{sH}^x = G_{sH}^y$ at $t_{so}^R = t_{so}^D$ as expected by the symmetry analysis. For a large spin-orbit coupling both electric and spin Hall conductance approaches to zero because the spin-orbit coupling in the scattering region forms a large potential barrier and the incident electrons will be completely reflected. Unlike bulk systems[25,26] where the spin Hall conductance in the clean limit has a universal value $\pm e/8\pi$ and the sign is given by the relative ratio of two coupling strength in Eq. (1), Fig. 3 shows that the value of spin Hall conductance varies with the relative ratio as well as the sign, but the change of sign is compatible with the bulk systems case. And this result is also compatible with the previous numerical work in the case of pure Rashba coupling.[23,24,28]
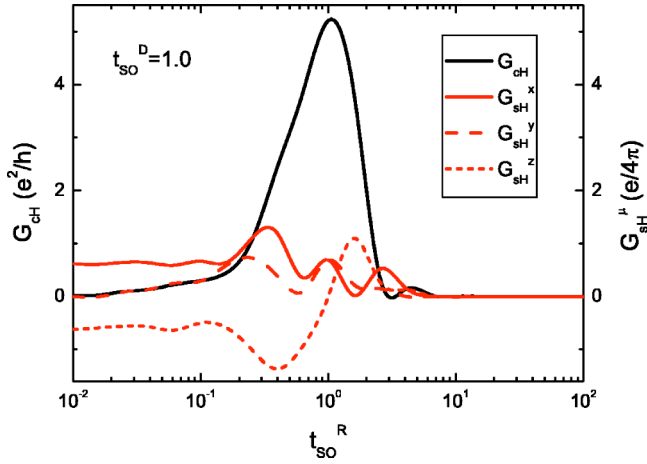
FIG. 3. (Color online) Electric and spin Hall conductances as functions of the Rashba coupling strength with a fixed Dresselhaus coupling strength $t_{so}^D = 1.0$ at $E_f = -2.0t$ and $L = 40$. Similar results are obtained for the Hall conductances as functions of the Dresselhaus coupling strength with a fixed Rashba coupling strength, which are consistent with symmetry analyses.

To see the finite-size effect we calculate the electric and spin Hall conductances for $L = 20$, 30, 40, and 50. G_{cH}/L and G_{sH}/L as functions of E_f are plotted in Fig. 4. We notice that these curves for different sizes fit a single one very well. Thus we conclude that both electric and spin Hall conductance are proportional to the size L of the sample. In other words, in our calculation the electric and spin Hall currents are determined by both the number of the incident channels and that of the outgoing channels. Thus the Hall currents induced by a specified longitudinal electric field are not proportional to the size L linearly, but to $L \times L$.

The disorder effect is an interesting issue in the spin Hall effect in 2DEG. It is still greatly controversial whether the spin Hall effect may survive when the impurity scattering is taken into account.[29–31] We consider the disorder effect by
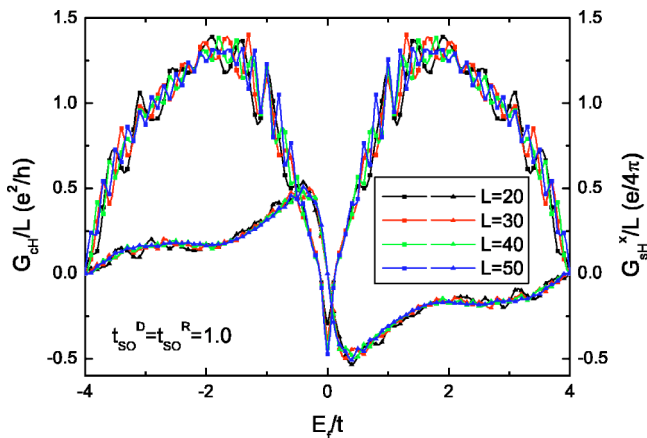

FIG. 4. (Color online) Electric and spin Hall conductances divided by sample size L as functions of the electron Fermi energy. Here the Rashaba and Dresselhaus coupling strengths are equal and the results for $L = 20$, 30, 40, and 50 are shown simultaneously. It implies a size effect that both Hall conductances are proportional to L in this calculation.
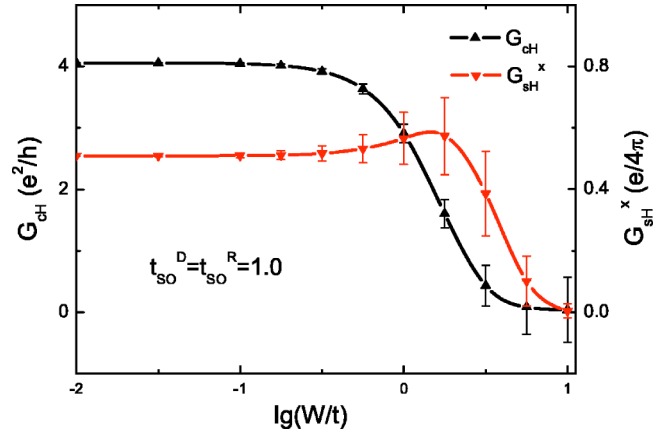

FIG. 5. (Color online) Electric and spin Hall conductances as functions of the logarithm of the disorder strength W/t. Results are obtained with equal Rashba and Dresselhaus coupling strengths at $E_f = -2.0t$ and $L = 30$. Standard deviations in the calculation are shown through the error bars.

including the disorder potential term $V_{disorder} = \Sigma_{i,\sigma=\uparrow,\downarrow} \epsilon_i c_{i,\sigma}^\dagger c_{i,\sigma}$ in Eq. (3), where ϵ_i are randomly distributed between $[-W/2, +W/2]$. Selectively the electric and spin Hall conductances, G_{cH} and G_{sH}^x, for two couplings with equal strength are plotted in Fig. 5. G_{sH}^z is exactly equal to zero according to the symmetry. It shows that both electric and spin Hall conductances can survive in weak disorder, but be suppressed in strong disorder. We also did calculation for several other cases, and obtained similar results. The case of pure Dresselhaus coupling is in agreement with the work of Sheng *et al.* for pure Rashba coupling.[23]

Physically the spin Hall conductance can be well understood from the spin transverse force caused by the spin-orbit coupling in Eq. (2). The electric field drives electrons moving along the field such that the electrons with spin-up or spin-down experience opposite the transverse force and thus a nonzero spin current is induced perpendicular to the field. The relative ratio of the two coupling strength determines the direction of the spin Hall current as the spinomotive force changes its sign around $\lambda = \beta$ and vanishes at the point. All calculated results are consistent with this. However, the spinomotive force is not a direct origin of the nonzero G_{cH}, since G_{cH} arises only when two couplings are present simultaneously. From the symmetry properties of the system we notice that the Hamiltonian with pure Rashba coupling is invariant under the transformation: $k_x \rightarrow k_x$, $k_y \rightarrow -k_y$, and $\sigma^x \rightarrow -\sigma^x$, $\sigma^y \rightarrow \sigma^y$, $\sigma^z \rightarrow -\sigma^z$. Similarly the Hamiltonian with pure Dresselhaus coupling is invariant under the transformation: $k_x \rightarrow k_x$, $k_y \rightarrow -k_y$, and $\sigma^x \rightarrow \sigma^x$, $\sigma^y \rightarrow -\sigma^y$, $\sigma^z \rightarrow -\sigma^z$. This is why the electric Hall current vanishes in these two cases, while the spin Hall current circulates because there is no symmetry constraint on it as both k_y and σ^z change their signs under such transformation. On the other hand, the Hamiltonian with both Rashba and Dresselhaus couplings does not possess the reflection symmetry of $k_x \rightarrow k_x$, $k_y \rightarrow -k_y$. Therefore the coexistence of both couplings breaks the reflection symmetry of the system, which makes the electric current not parallel to the electric field such that it gives rise to a nonvanishing Hall conductance G_{cH}. This unconven-

tional Hall conductance may be related to some discussions in terms of the anomalous Hall effect due to parity anomaly and additional band crossing.[5] Moreover, since the diagonal spin conductance is nonzero in this case,[26] the diagonal spin current along leads 1 and 2 might generate a charge Hall current via the reciprocal spin Hall effect.[32]

In conclusion, we studied the electric Hall conductance as well as the spin Hall conductance for a finite-size system with four leads. Both electric and spin Hall conductances are nonzero when both Rashba and Dresselhaus couplings are present, thus the current is actually spin polarized. Unlike the anomalous Hall effect, the present electric Hall current is driven by the spin-orbit coupling, not by the exchange coupling with the magnetic impurities.[33] This effect also differs from the one resulted from a spin polarized current via the Rashba coupling.[34] Though the incident current is not spin polarized, the Hall current is polarized in our case.

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