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Spin current carried by magnons

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A spin current is usually carried by electrons and generated due to the imbalance of up-spin and down-spin. Here we investigate another type of spin current, which is carried by magnons. Using nonequilibrium Green's-function technique, we have derived a Landauer-Büttiker-type formula for spin current transport. The spin current satisfies conservation condition and can be expressed in terms of the magnon Green's functions of the mesoscopic ferromagnetic isolating system. As an application of this theory, we study the magnon transport properties of a two-level magnon quantum dot in the presence of the magnon-magnon scattering. By solving the self-consistent equations, we obtain the nonlinear spin current as a function of the magnetochemical potential. The spin current generated by using a parametric quantum pumping mechanism is also discussed.

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Mesoscopic electronic transport has been widely studied[1] for the last two decades. Many novel features including the quantized conductance through a ballistic point contact discovered by van Wees *et al.*[2] and Wharam *et al.*[3] are due to the following two reasons. First, the length of the mesoscopic system is smaller than that of electronic phase breaking so that electrons can keep their phase memory and electron wave coherence can play an important role. Second, electrons are confined in certain dimensions, which gives the quantized electronic energy level. It is natural to look for the bosonic analogies of various mesoscopic electronic phenomena. Indeed, many efforts have been predicted in this regard, for example, Rego and Kirczenow[4] theoretically predicted quantized thermal conductance of dielectric quantum wires. Schwab *et al.*[5] found that the thermal conductance of the nanobridge had an upper limit, set by the laws of quantum mechanics. Sun *et al.*[6] studied the four-terminal thermal conductance, which exhibited a set of Onsager relations. All these results demonstrate that the behavior of the lattice degree of freedom in mesoscopic structures is nonclassical. In this paper, we investigate another type of wave, the spin wave, which can propagate coherently through a nanostructure. The relevant problem is the magnetization transport in all kinds of magnetic systems,[7–10] which attract wide interests both theoretically and experimentally. In particular, Meier and Loss[11] investigated the magnon transport in both ferromagnetic and antiferromagnetic materials. They found that the spin conductance was quantized in the units of order $(g\mu_B)^2/h$ in the antiferromagnetic isotropic spin-1/2 chains. Schütz *et al.*[12] studied the persistent spin current in mesoscopic Heisenberg rings. Although the spin current is widely studied by many authors,[13] it is mostly limited to the mesoscopic electronic systems by making use of the imbalance of spin-up and spin-down electrons. Here we will focus on the magnon transport under the nonequilibrium condition. The flow of magnons gives rise to the spin current. Suppose a mesoscopic ferromagnetic system connected to two ferromagnetic leads, which can be described by a spin ferromagnetic Heisenberg model. The difference in magnetic field between two ferromagnetic leads plays the role of the electric chemical potential difference in the electric charge transport and generates the spin current carried by the magnon, while the electronic degree of freedom is totally frozen. We have derived a Landauer-Büttiker-type formula for spin current driven by the magnetochemical potential. The formula goes beyond the linear-response limit and is also valid in the presence of the interaction between magnons. We have also considered the spin current generated by quantum pumping mechanism. Numerical calculations show that the pumped spin current increases with increasing of the temperature.

Let us begin with the following model Hamiltonian:

$$H=\sum_{k\alpha=L,R}[\epsilon_k^0+g\mu_B B_\alpha]a_{k\alpha}^\dagger a_{k\alpha}+H_{cen}\{d_n,d_n^\dagger\}$$
$$+\sum_{kLn}[T_{k\alpha}a_{k\alpha}^\dagger d_n+\text{c.c.}], \tag{1}$$

where $a_{k\alpha}^\dagger$ and $d_n^\dagger$ are the creation operators for the spin-wave excitation magnon in the lead α and the central quantum scattering region, respectively. They are bosonic operators and obey the commutation relations. The first term is the Hamiltonian for lead α, in which $\epsilon_k^0=|J|Sk^2$ is the magnon excitation spectrum,[14] g is the gyromagnetic ratio, and μ_B is the Bohr magneton. B_α is the constant magnetic field in the lead α and can be turned conveniently from the experimental point of view. Note that $g\mu_B B_\alpha$ plays the role of the electrochemical potential in the charge transport, and naturally we call it magnetochemical potential. Similar to Ref. 11, we use the notation $B_\alpha=B_0+\Delta B_\alpha$ with $B_0\gg\Delta B_\alpha$. We do not write down the explicit Hamiltonian form H_{cen} for the central scattering region because our derivation in the following can include the magnon-magnon interaction and has nothing to do with the detailed interaction form. The third term stands for the magnon tunneling part between the lead α and the central region with the matrix element $T_{k\alpha}$. By using Holstein-Primakoff transformation,

$$S_{iz}=S-a_i^\dagger a_i, \tag{2}$$

the spin current can be calculated from the left lead $\hbar=1$,[15]

$$I_{sL}=\left\langle \frac{dS_{zL}}{dt}\right\rangle =-\left\langle \frac{d\hat{N}_L}{dt}\right\rangle, \tag{3}$$

where $\hat{N}_L=\Sigma_k a^\dagger_{kL}a_{kL}$. From Heisenberg equation of motion, the spin current can be rewritten as

$$I_{sL}(t)=i\sum_{kn}\left[T_{kLn}\langle a^\dagger_{kL}(t)d_n(t)\rangle-\text{c.c.}\right]. \tag{4}$$

Define the following lesser Green's function:

$$G^<_{nkL}(t,t')=-i\langle a^\dagger_{kL}(t')d_n(t)\rangle,$$

we can express the spin current as

$$I_{sL}(t)=-\sum_{kn}\left[T_{kLn}G^<_{nkL}(t,t)+\text{c.c.}\right].$$

Just like the derivation in Ref. 16, we can obtain

$$\begin{aligned}I_{sL}=i\int_{B_m}^\infty \frac{dE}{2\pi}&\text{Tr}\{\Gamma_L(E-g\mu_B\Delta B_L)\\ &\times[(G^R(E)-G^A(E))f_L(E)-G^<(E)]\},\end{aligned} \tag{5}$$

where $B_m=\max\{g\mu_B(B_0+\Delta B_L),g\mu_B(B_0+\Delta B_R)\}$ and $\Gamma_\alpha(E)$ is the linewidth function, which is defined as $[\Gamma_\alpha(E)]_{mn}\equiv 2\pi\Sigma_k T^*_{k\alpha m}T_{k\alpha n}\delta(E-\epsilon_{k\alpha})$. $G^{R,A,<}(E)$ are the Fourier transformations of the retarded, advanced, and lesser Green's functions for the quantum scattering regime, respectively:

$$G^{R,A}_{mn}(t,t')\equiv\mp i\theta(\pm t\mp t')\langle[d_m(t),d^\dagger_n(t')]\rangle,$$

$$G^<_{mn}(t,t')\equiv -i\langle d^\dagger_n(t')d_m(t)\rangle,$$

$f_\alpha(E)=1/(\exp[(E-g\mu_B\Delta B_\alpha)/k_BT]-1)$ is the Bose-Einstein distribution function of the lead α. Equation (4) is one of the main results in this paper. First, this formulation goes beyond the linear-response limit[11] and expresses the spin current in terms of the local magnon Green's functions of quantum scattering region. Second, this formulation is also valid in the presence of the magnon-magnon interaction. Although this formula looks similar to that of the charge transport, we must note that the distribution function obeys Bose-Einstein statistics. In addition, since the number of magnons is not conserved, there is a lower limit in the energy integral. In the mean-field approximation, we can further write the spin current as follows by using the Keldysh equation $G^<=G^R\Sigma^<G^A$. The self-energy $\Sigma^{r,a,<}$ are related to the free-magnon Green's functions $g^{r,a,<}_{k\alpha}$ in lead α by the following relation:

$$[\Sigma^{r,a,<}(E)]_{mn}=\sum_{k\alpha}T^*_{k\alpha m}T_{k\alpha n}g^{r,a,<}_{k\alpha}(E)$$

in which $g^{r,a,<}_{k\alpha}(E)$ are the Fourier transformation of $g^{r,a,<}_{k\alpha}(t)$,

$$\begin{aligned}g^{r,a}_{k\alpha}(t)&=\mp i\theta(\pm t)\langle[a_{k\alpha}(t),a^\dagger_{k\alpha}(0)]\rangle\\ &=\mp i\theta(\pm t)\exp(-i\epsilon_{k\alpha}t),\end{aligned}$$

$$g^<_{k\alpha}(t)=-i\langle a^\dagger_{k\alpha}(0)a_{k\alpha}(t)\rangle=-if_\alpha\exp(-i\epsilon_{k\alpha}t).$$

A straight calculation gives the lesser self-energy $\Sigma^<(E)=-i\Sigma_\alpha\Gamma_\alpha(E-g\mu_B\Delta B_\alpha)\ f_\alpha(E)$. We must point out that the lesser self-energy sign for magnons is very different from that of electrons. This reflects the fact that magnon is boson, while electron is fermion. They satisfy the different commutation relations. With this lesser self-energy, we have

$$\begin{aligned}I_{sL}&=\int_{B_m}^\infty\frac{dE}{2\pi}\text{Tr}\{\Gamma_L(E-g\mu_B\Delta B_L)G^R(E)\\ &\quad\times\Gamma_R(E-g\mu_B\Delta B_R)G^A(E)\}[f_L(E)-f_R(E)]\\ &\equiv\int_{B_m}^\infty\frac{dE}{2\pi}T(E)[f_L(E)-f_R(E)],\end{aligned} \tag{6}$$

with

$$\begin{aligned}T(E)\equiv\text{Tr}\{&\Gamma_L(E-g\mu_B\Delta B_L)G^R(E)\\ &\times\Gamma_R(E-g\mu_B\Delta B_R)G^A(E)\}\end{aligned}$$

being the transmission coefficient of the magnon. Obviously, the spin current satisfies the conservation condition: $\Sigma_{\alpha=L,R}I_{s\alpha}=0$, this is because we do not consider the spin relaxation in our model calculation.

Magnetospin current. As the application of this theory, we first consider the linear-response limit, which means that the magnetochemical difference $B_L-B_R\equiv\Delta B\to 0$. In this case, Eq. (6) may be rewritten as

$$I_{sL}=-g\mu_B\Delta B\int_{B_m}^\infty\frac{dE}{2\pi}T(E)\frac{\partial f(E)}{\partial E}, \tag{7}$$

which demonstrates that the magnetospin current is proportional to the linear term of the external magnetochemical bias ΔB. Especially, when the system is in ballistic regime, the transmission coefficient satisfies $T(E)\equiv 1$, and the spin current reduces to a very simple form

$$I_{sL}=g\mu_B f(B_0)\Delta B/2\pi. \tag{8}$$

Hence the spin conductance $G=g\mu_B f(B_0)/2\pi$, which is exactly same as a result in Ref. 11.[15] Next, we consider a single-magnon quantum dot[17] with two energy levels and calculate the spin current using Eq. (6). Suppose there is the magnon-magnon interaction due to the nonlinear effect, we have the following Hamiltonian:[18]

$$H_{cen}=\epsilon_1 d^\dagger_1 d_1+\epsilon_2 d^\dagger_2 d_2+\gamma(d^\dagger_1 d^\dagger_2 d^2_2+d^\dagger_2 d^\dagger_1 d^2_1+\text{c.c.}),$$

where $\epsilon_{1,2}$ are the magnon energy levels, which can be turned by an external magnetic field. The last term stands for the magnon-magnon scattering with strength γ. To calculate the retarded Green's function for the quantum scattering region, we treat the magnon-magnon interaction using the mean-field procedure, i.e., $\gamma(d^\dagger_1 d^\dagger_2 d^2_2+d^\dagger_2 d^\dagger_1 d^2_1+\text{c.c.})\approx 2\gamma(\langle n_1\rangle+\langle n_2\rangle)(d^\dagger_1 d_2+\text{c.c.})$, where $\langle n_{1,2}\rangle$ are the mean occupation numbers in the two discrete levels, which can be calculated self-consistently. By introducing the 2×2 matrix retarded Green's function

$$G^r(t)=-i\theta(t)\langle[\Psi(t),\Psi^\dagger(0)]\rangle,$$

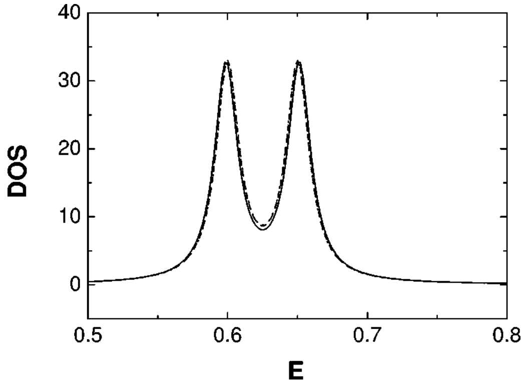

FIG. 1. The DOS dependence of the energy for the different temperatures: $\beta=1$ (solid line), $\beta=2$ (dashed line), and 4 (dotted line).

with $\Psi=\binom{d_1}{d_2}$, $\Psi^\dagger=(d_1^\dagger,d_2^\dagger)$, one can easily obtain the retarded Green's function in Fourier space with the wideband limit:[16]

$$G^r(E)=\frac{1}{\begin{pmatrix} E-\epsilon_1+i\Gamma/2 & -2\gamma(\langle n_1\rangle+\langle n_2\rangle) \\ -2\gamma(\langle n_1\rangle+\langle n_2\rangle) & E-\epsilon_2+i\Gamma/2 \end{pmatrix}}.$$

As for $\langle n_i\rangle$, with $i=1,2$, we have the following self-consistent equation:

$$\langle n_i\rangle=i\int\frac{dE}{2\pi}G_{ii}^<(E).$$

The above two equations and the expression of the spin current form a closed solution to the magnon transport problem. To get more physical insight, we now perform some numerical calculations. We set two discrete energy levels $\epsilon_1=0.6$, $\epsilon_2=0.65$, the linewidth functions are $\Gamma_L=\Gamma_R=0.01$ and the scattering strength of the magnon-magnon is $\gamma=0.02$. In Fig. 1 we plot the density of state $(\mathrm{DOS})\equiv-(1/\pi)\mathrm{Tr}\{\mathrm{Im}\,G^r(E)\}$ versus the energy at different temperatures $\beta=1$ (solid line), 2 (dashed line), and 4 (dotted line). The parameters of the lower limit in the integral are set $B_0=0.2$, $\Delta B_L=0.02$, and $\Delta B_R=0$. We find two resonant peaks at $E=0.6$ and 0.65, which correspond to the original two bare levels ϵ_1 and ϵ_2, respectively. In addition, the width and height of the peaks are not sensitive to the temperature.[19] In the inset of Fig. 2 we depict the self-consistently calculated occupation numbers of the discrete energy levels as the function of the temperature β (bigger β corresponds to smaller temperature). When the temperature decreases, the occupation number decays rapidly. In Fig. 2, we plot the spin current as the function of the bias ΔB for the different temperatures $\beta=1$ (solid line), 2 (dashed line), and 4 (dotted line). Other parameters are chosen the same as those in Fig. 1. The spin current increases with the bias monotonously and exhibits the typical nonlinear feature. In addition, we can get the larger spin current for the higher temperature because the more magnons participate in the transport.

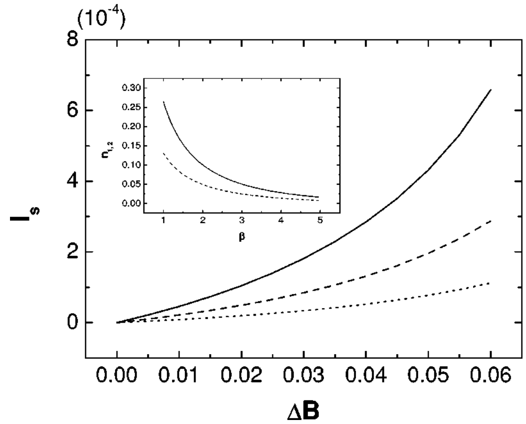

FIG. 2. The nonlinear spin current I_s vs the magnetochemical bias ΔB. Inset: the self-consistently calculated occupation numbers $\langle n_1\rangle$ (solid line) and $\langle n_2\rangle$ (dashed line) vs the temperature β.

Pumped spin current. Finally, we will discuss the quantum pumping mechanism[20] to generate the spin current by setting $B_L=B_R=B_0$. Similar to the charge quantum pumping,[21] although there are no magnetochemical potential and temperature differences between two leads, a dc spin current through a mesoscopic ferromagnetic scattering region can be pumped out by periodically varying two independent system parameters X_1 and X_2. For instance, we can use the magnetic field as parameters, which can be realized and readily controlled experimentally. In the slow harmonic variation of parametric magnetic fields $X_1(t)=g\mu_BB_{p1}\sim\cos\omega t$ and $X_2(t)=g\mu_BB_{p2}\cos(\omega t+\Phi)$, the adiabatic pumped spin current in a period τ can be written as[20]

$$I_\alpha=\frac{-i}{\tau}\int_0^\tau dt\int\frac{dE}{2\pi}\sum_\beta\,[\partial_tS_{\alpha\beta}(E,t)S_{\alpha\beta}^\dagger(E,t)]\partial_Ef(E), \tag{9}$$

where $S_{\alpha\beta}(E,t)$ is the instantaneous magnon scattering matrix element, $\tau=2\pi/\omega$, and $f(E)$ is the Bose-Einstein distribution. In particular, when the pumping amplitude is small, we have

$$I_{ps}=\omega g\mu_BB_{p1}B_{p2}\sin\Phi\int_{B_0}^\infty\frac{dE}{2\pi}[-\partial_Ef(E)] \times\mathrm{Im}\left\{\frac{\partial S_{11}^*(E)}{\partial X_1}\frac{\partial S_{11}(E)}{\partial X_2}+\frac{\partial S_{12}^*(E)}{\partial X_1}\frac{\partial S_{12}(E)}{\partial X_2}\right\}, \tag{10}$$

where $S_{\alpha\beta}$ are the magnon scattering matrix elements. In the following, we will consider a spin current parametric quantum pump which consists of a double barrier structure and the units are fixed by setting $g\mu_B=(|J|S)^{-1}=k_B=1$.[22] This double barrier structure is modeled by the potential $V(x)=V_1\delta(x+a)+V_2\delta(x-a)$ with $V_1=B_1+B_p\cos\omega t$, and $V_2=B_2+B_p\cos(\omega t+\Phi)$. Because the motion of spin wave obeys the Schrödinger equation, the advantage of this model is that the retarded Green's function $G^r(x,x')$ can be obtained exactly from the electron point of view,[23] and therefore the scattering matrix can be calculated from the Fisher-Lee relation.[24] $S_{\alpha\beta}=-\delta_{\alpha\beta}+ivG^r_{\alpha\beta}$ with $v=2k=2\sqrt{E}$ the magnon velocity:

$$S_{11}(E)=-1+\frac{(1-r_1)[1-r_2\exp(4ika)]}{1-r_1r_2\exp(4ika)},$$

$$S_{12}(E)=\frac{(1-r_1)(1-r_2)\exp(2ika)}{1-r_1r_2\exp(4ika)},$$

where $r_j=iB_j/(2k+iB_j)$ with $j=1,2$. Having these scattering matrix elements, we can get the pumped spin current for the symmetry barriers ($B_1=B_2=B$) after a little algebra:

$$I_{ps}=\omega B_p^2\sin\Phi\int_{B_0}^{\infty}\frac{dE}{2\pi}\left(-\frac{\partial f(E)}{\partial E}\right)\frac{|t|^2}{|1-r^2\exp(4ika)|^4}$$
$$\times\mathrm{Im}\{(1-r)^2\exp(4ika)[1-r^*\exp(-4ika)]^2\}, \tag{11}$$

where $r=iB/(2k+iB)$ and $t=2ik/(2k+iB)^2$. In Fig. 3, we plot the pumped spin current as a function of the temperature β at the different lower limit in the integral $B_0=0.1$ and $B_0=0.5$. Here we have set the strength of the double barrier $B=20B_0$, the width of the double barrier $2a=200$, and the amplitude of the pumping $B_p=0.1$. The pumping frequency is set $\omega=0.01$ and the difference of the phase is chosen to be $\pi/2$. We find that the spin current decreases very quickly when β is increased. Physically, this is because the excited magnon numbers decay exponentially as the temperature decreases. Especially, when the temperature tends to zero, there is no magnon excitation, and therefore the pumped spin current becomes zero. In addition, we find that only the lower-energy magnons contribute to the pumped current due to the presence of the factor $\partial_E f(E)$. Therefore, when we change the lower limit in the integral from $B_0=0.1$ to $B_0=0.5$, the pumped spin current becomes much smaller. Beyond the adiabatic approximation and for the finite parametric amplitude case, we can use the real-space Green's-function approach in Ref. 25. This is beyond the scope of this work.

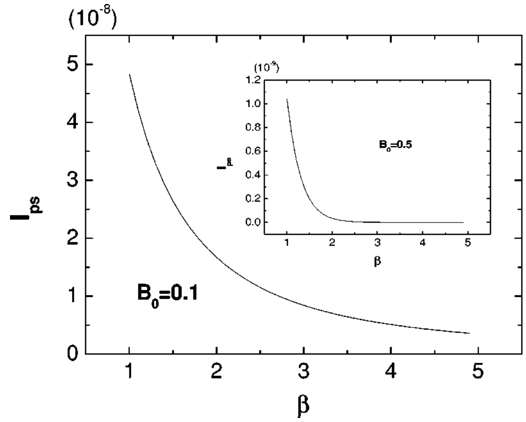

FIG. 3. The pumped spin current I_{ps} vs the temperature β for the different integral limits: $B_0=0.1$ and $B_0=0.5$.

In summary, we have given a quantum transport theory of spin current for the mesoscopic ferromagnetic system. The spin current is carried by the magnon and can be expressed by the local magnon Green's functions of the mesoscopic ferromagnetic system. The formula is also valid in the presence of the interaction between magnons in the quantum scattering region. The magnetochemical potential difference and a quantum pumping mechanism are proposed to generate the spin current. Because the spin current can be detected by its induced electric field[11,26] or by spin quantum Hall effect proposed by Hirsch,[27] we believe the study of the magnon transport will open a new subfield in mesoscopic systems.

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