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<th><strong>Title</strong></th>
<th>Berry phase and its induced charge and spin currents in a ring of a double-exchange system</th>
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Doped manganite perovskite, such as La$_{1-x}$Ca$_x$MnO$_3$, was the first metallic ferromagnetic oxide extensively studied in the 1950s and has renewed recent research interests since the discovery of colossal magneto-resistance (CMR) effect. In the system, usually called double-exchange system, conduction electrons move in the background of the spin configuration of localized electrons. Strong Hunds rule coupling between the conduction and localized electrons in the same Mn ion forces the electrons to form high spin states, and makes the conduction electrons more mobile between the pairs of sites where the two localized spins align to parallel. The hopping term $t_{ij}$ between the two sites is determined by the two-spin configuration:

$$t_{ij} = -t \left( \cos \frac{\theta_i}{2} \cos \frac{\theta_j}{2} + \sin \frac{\theta_i}{2} \sin \frac{\theta_j}{2} e^{-i(\phi_i - \phi_j)} \right),$$

where the localized spins are parametrized by polar angles $\theta_i$ and $\phi_i$. It is noted that the Berry phase is acquired when $\phi_i \neq \phi_j$. A lot of theoretical efforts are made on the mechanism of ferromagnetism and anomalous transport properties of the systems. However, a profound understanding of this phase is highly desirable. Since the Berry phase comes from the strong interplay of electrons, it is anticipated that it will play an important role in the double-exchange system, especially when the route of the electron motion is closed.

Accumulation of the quantum phase in multiconnected geometry produces a quantum interference effect via the Aharonov-Bohm (AB) and/or Aharonov-Casher (AC) effect. Persistent currents in connection with AB, AC, and Berry phases in one-dimensional rings have been studied extensively. In particular, the AC effect, which is induced by the conventional spin orbit (SO) in the presence of disorder or external electric field, on persistent currents in mesoscopic rings were elucidated. Besides, it was found that Zeeman interaction between the electron spin and the texture couples spin and orbital motion in textured mesoscopic rings, and results in a Berry phase; as a consequence, the system supports persistent charge and spin currents. To explore the physical consequences of the Berry phase in Eq. (1), we here consider a ring of double-exchange system. Starting from a Kondo-like Hamiltonian, we derive an effective Hamiltonian by utilizing the projection technique. We investigate the AB and AC effects and find that persistent spin and charge currents can be induced spontaneously in the spin spiral state, in which the nontrivial geometric phase is accumulated. We expect that the contemporary nanotechnology makes it possible to observe this quantum phase effect experimentally.

Consider a clean ring of double-exchange system consisting of $N$ sites and $N_e$ electrons in the absence of external electromagnetic field. The momentum of an electron in the ring is

$$P + eA/c + \mu \hat{\sigma} \times E/c,$$

where $A$ is the vector potential induced by an electric current and $E$ is the electric field induced by a spin current. We point out that although the above term essentially represents spin-orbit coupling which had been addressed before, being significantly different from Refs. 11–13, the spin-orbit interaction considered here is related to the electric field induced by the spin current, which is supported by the Berry phase resulted from the Hunds coupling between the conduction and localized electrons in the double-exchange system. Hence the Hamiltonian to describe the ring is written as

$$H = -t \sum_{n=1}^{N} (e^{i(2\pi/N)(f_{AB}+f_{AC})} c_{n,\sigma} c_{n+1,\sigma} + h.c.)$$

$$- \frac{J_H}{2} \sum_{n,\sigma,\sigma'} S_n \cdot \sigma_{\sigma,\sigma'} c_{n,\sigma}^\dagger c_{n+1,\sigma'} + f_{AB} \sum_{n} S_n \cdot S_{n+1}.$$

$c_{n,\sigma}^\dagger$ and $c_{n,\sigma}$ are the creation and annihilation operators of conduction electron at site $n$ with spin $\sigma(=\pm 1)$, respectively. $S_n$ is the localized spin operator at site $n$. The conduction and localized electrons are coupled strongly by $J_H$, which is positive according to the Hunds rule. $f_{AB} = \Phi_{AB}/\Phi_0$ and $f_{AC} = \Phi_{AC}/\Phi_0 = \sigma f_{AC}$ where $\Phi_{AB} = 4\pi \Phi_{AB}$ is the AB magnetic flux and $\Phi_{AC} = (\mu/e) \oint (E \times dI) \cdot z$ is the AC flux with $\Phi_0 = h/e$. The last term in Eq. (2) is a tiny antiferromagnetic interaction between localized electrons. Note that the energy eigenvalue is a periodic function of $f_{AB}$ or $f_{AC}$ with a period of 1. It is thus sufficient to consider only the range of 1 for $f_{AB}$ or $f_{AC}$.

In the double-exchange system, the exchange integral between conduction electrons and localized electrons is so strong that the spins of conduction electron and localized
electrons at the same site are parallel and form a state with spin-(S + 1/2). Mathematically a relevant effective Hamiltonian can be derived approximately from Eq. (2) by introducing the projection operator
\[ P_n^+ = \sum_{\sigma, \sigma'} (P_n^+)^{\sigma \sigma'} \bar{c}_{n, \sigma} c_{n, \sigma'}, \]
where
\[ (P_n^+)^{\sigma \sigma'} = \frac{S_n^{\sigma \sigma'} + (S + 1) \delta_{\sigma \sigma'}}{2S + 1} \]
with \( c_{n, \sigma} = (1 - \bar{c}_{n, -\sigma}) \bar{c}_{n, \sigma} \). To simplify our problem further, we parametrize \( S_n \) by polar angles \( \theta_n \) and \( \phi_n \), and take \( S/(2S + 1) \approx (S + 1)/(2S + 1) = 1/2 \), which becomes exact in the large \( S \) limit. In this approximation, we obtain an effective Hamiltonian,
\[ H_{\text{eff}} = -\frac{N}{\sum_{n=1}} (t_n \alpha_n^{\dagger} \alpha_n + \text{H.c.}) + J_{\text{AF}} \sum_n (\sin \theta_n \sin \theta_{n+1} \cos (\phi_n - \phi_{n+1}) + \cos \theta_n \cos \theta_{n+1}), \]
of the ring with the periodic condition, where \( J_{\text{AF}} = J_{\text{AF}} S^2 \),
\[ \alpha_n^{\dagger} = \cos \frac{\theta_n}{2} \bar{c}_{n, \uparrow} + \sin \frac{\theta_n}{2} e^{i \phi_n} \bar{c}_{n, \downarrow} \]
and
\[ t_n = \left( \cos \frac{\theta_n}{2} \cos \frac{\theta_{n+1}}{2} e^{i (2\pi N) (f_{\text{AB}} + f_{\text{AC}})} + \sin \frac{\theta_n}{2} \sin \frac{\theta_{n+1}}{2} e^{i (2\pi N) (f_{\text{AB}} - f_{\text{AC}}) + \Delta \phi_n} \right). \]
Here \( \Delta \phi_n = \phi_n - \phi_{n+1} \). If we neglect the AB and AC phases in Eq. (4), the hopping terms go back to Eq. (1), \( \alpha^{\dagger} \) and \( \alpha \) operators satisfy the anticommutation relation and are of spinless fermion. Physically, the spin of conduction electrons is frozen to align to the localized spin, and \( \alpha \) describes the charge degree of freedom of electrons. The cost of transformation from \( \bar{c}_{n, \sigma} \) to \( \alpha_n^{\dagger} \) is that the renormalized hopping matrix acquires a quantum phase \( \Delta \phi \), which plays an important role in the present problem.

For a homogeneous system we take \( \theta_i = \theta \) and \( \Delta \phi_i = \phi \). The eigenvalues of Eq. (4) are obtained as
\[ E_l = -2t \left[ \cos^2 \frac{\theta}{2} \cos \left( \frac{2\pi}{N} (l + F_\pm + \lambda) \right) \right] + \sin^2 \frac{\theta}{2} \cos \left( \frac{2\pi}{N} (l + F_\pm + \phi) \right) + J_{\text{AF}} (\sin^2 \theta \cos \phi + \cos^2 \theta) \],
where \( F_\pm = f_{\text{AB}} \pm f_{\text{AC}} \) and \( l = 0, \pm 1, \pm 2, \ldots \).

The ground energy of the effective Hamiltonian with \( N_e \) electrons [Eq. (4)] is obtained,
evolves to an antiferromagnetic state. It is worth noting that there is no phase transition between these two states. Interestingly, $\phi<0$ for $N_e=71$ (odd number), while $\phi>0$ for $N_e=70$ (even number). Actually, for odd $N_e$ and $F_-=0$ the states with $\phi<0$ and $>0$ are degenerate for the energy in Eq. (7). For even $N_e$ or $F_\perp=\mp 0$ the states with different signs of $\phi$ are no longer degenerate. The sign of $\phi$ depends on the AB or AC flux $F_\perp$ and $\lambda$. Consequently, $\phi>0$ is expected for even $N_e$. Physically, the double-exchange ferromagnetism is predominant for a small antiferromagnetic coupling $J_{AF}$. With the increase of $J_{AF}$, the competition between the ferromagnetic double-exchange and the antiferromagnetic coupling drives electrons forming a spin spiral state, and an antiferromagnetic state eventually.

As is well known, the nonzero AB and/or AC fluxes induce charge and/or spin currents in a ring, which in turn stabilize the fluxes. The induced currents are given by

$$I_c = -\frac{e}{2\pi\hbar} \frac{\partial E_{\perp}}{\partial J_{AB}}$$

$$= -\frac{I_0^f}{\sin(\pi/N)} \left[ \cos^2 \frac{\theta}{2} \sin \frac{2\pi}{N} (F_\perp - \lambda) \right.$$

$$+ \sin^2 \frac{\theta}{2} \sin \frac{2\pi}{N} (F_\perp + \phi - \lambda) \biggr]\right],$$

$$I_s = \frac{I_0^f}{4\pi} \frac{\partial E_{\perp}}{\partial J_{AC}}$$

$$= \frac{I_0^f}{2\sin(\pi/N)} \left[ \cos^2 \frac{\theta}{2} \sin \frac{2\pi}{N} (F_\perp + \phi - \lambda) \right.$$

$$- \sin^2 \frac{\theta}{2} \sin \frac{2\pi}{N} (F_\perp + \phi - \lambda) \biggr]\right],$$

respectively, where $I_0^f = [2\pi t \sin(N_e \pi/N)]/N$ and $I_0^s = [2\pi / \sin(N_e \pi/N)]/N$. If only the AB effect is considered ($f_{AC}=0$) and $\phi=0$, the charge current is independent of $\theta$ and reduces to the result of tight-binding model. If only the AC effect is considered ($f_{AB}=0$) and $\phi=0$, the spin current is also independent of $\theta$. Actually, both $\theta$ and $\phi$ depend on the AB and AC fluxes and are determined by minimizing the total energy.

In reality, the ring is quasi-one-dimensional and the electromagnetic energies induced by the AB and AC fluxes should be taken into account. These electromagnetic energies can be written as $E_{AB} = (1/2B) f_{AB}^2$ and $E_{AC} = (1/2C) f_{AC}^2$, where $B = 2e^2/\hbar \Phi_0$ and $C$ is the inductance of the ring and $C = \mu^2 L/\pi^2 a^2 b^2 c^2$. Thus the total energy of the ring is

$$E_T = E_{gl} + E_{AB} + E_{AC}.$$  

(10)

By minimizing the total energy $E_T$, we obtain a set of equations to determine $\theta_0$, $\phi_0$ and the current-induced $f_{AB}^{(c)}$ and $f_{AC}^{(c)}$:

$$\frac{1}{2} \sin \theta_0 \cos F_1 - \cos F_2 + J_{AF} N_e \sin (2\theta_0)(\cos \phi_0 - 1) = 0,$$

$$2\pi N e \sin^2 \frac{\theta_0}{2} \sin F_2 - J_{AF} N_e \sin \theta_0 \sin \phi_0 = 0,$$

$$2\pi N e \left( \cos^2 \frac{\theta_0}{2} \sin F_1 + \sin^2 \frac{\theta_0}{2} \sin F_2 \right)$$

$$+ \frac{f_{AB}^{(c)}}{B} = 0,$$

$$2\pi N e \left( \cos^2 \frac{\theta_0}{2} \sin F_1 - \sin^2 \frac{\theta_0}{2} \sin F_2 \right)$$

$$+ \frac{f_{AC}^{(c)}}{C} = 0,$$

where $F_1 = (2\pi N)(f_{AB}^{(c)} + f_{AC}^{(c)} - \lambda)$ and $F_2 = (2\pi N)(f_{AB}^{(c)} + f_{AC}^{(c)} - \lambda)$. The AB and AC fluxes can be expressed approximately as

$$f_{AB}^{(c)} \approx \frac{(2\pi N)^2 \epsilon_0 (\lambda - \phi_0/2) \sin^2 \frac{\theta_0}{2} + (1/C)[\lambda - \phi_0 \sin^2 (\theta_0/2)]}{(2\pi N)^2 \epsilon_0 \sin^2 \frac{\theta_0}{2} + (1/B + 1/C) + (1/B C \epsilon_0)(N/2\pi)^2},$$

$$f_{AC}^{(c)} \approx \frac{(2\pi N)^2 \epsilon_0 (\phi_0/2) \sin^2 \frac{\theta_0}{2} + (1/B)[\lambda \cos (\theta_0/2) + (\phi_0 - \lambda) \sin^2 (\theta_0/2)]}{(2\pi N)^2 \epsilon_0 \sin^2 \frac{\theta_0}{2} + (1/B + 1/C) + (1/B C \epsilon_0)(N/2\pi)^2},$$

which indicates that the quantum phase $\phi$ can sustain both AB and AC effects. By ignoring the electromagnetic energies generated by the persistent current, i.e., $B = C \to \infty$, it is easy to obtain $f_{AB}^{(c)} \sim \lambda - \phi_0/2$ and $f_{AC}^{(c)} \sim \phi_0/2$. For general cases, we solve the set of Eq. (14) numerically.

Figure 2(a) presents both $\theta_0$ and $\phi_0$ dependence of $J_{AF}$ in different cases. Their behaviors are almost independent of $B$ and $C$ in the interested range. With the increase of $J_{AF}$, the ground state evolves from the ferromagnetic phase to the spiral phase, and to the antiferromagnetic phase eventually. The current-induced AB and AC fluxes as a function of $J_{AF}$ are plotted in Fig. 2(b), from which it is seen that there is a competition between AB and AC effects ($f_{AB}^{(c)}>0$ and $f_{AC}^{(c)}<0$). When $J_{AF}$ increases, $f_{AB}^{(c)}$ and $f_{AC}^{(c)}$ tend to saturation. The saturated values of $f_{AB}^{(c)}$ and $f_{AC}^{(c)}$ depend on different $B$ and $C$. In a typical mesoscopic system, $B \ll C$, and therefore $f_{AC}^{(c)} \ll f_{AB}^{(c)}$. The charge and spin persistent currents versus $J_{AF}$ are plotted in Figs. 2(c) and (d). Both currents vanish for $J_{AF} < 0.01$, but jump to finite values for $J_{AF} > 0.01$. In the cases of $1/B = 1/C = 0$, $I_c$ decreases with increasing of $J_{AF}$ due to the competition between $f_{AB}^{(c)}$, $f_{AC}^{(c)}$, and $\phi_0$. $I_s$ also shows a
similar behavior in the cases of $1/B = 1/C = 0$. When the electromagnetic energy is taken into account ($1/B = 1/C = 0.2$ and $1/B = 0.2$ with $1/C = 0.4$), $I_c$ and $I_s$ increase quickly at $J_{AF} \sim 0.01$ and tend to saturation whose value depends on $1/B$ and $1/C$, respectively.

It is noted that, in the case of even $N_e$, the phase shifts a half flux quantum ($\lambda = 1/2$). As a result, the topological effect may change as shown in Figs. 3(a)–(d). Since the sign of $\phi$ is opposite to that for $N_e = 71$ (odd number), the current-induced AB and AC effects are affected, even though the ground state properties do not change. It can be seen from Fig. 3(b) that $f_{AB}^c$ and $f_{AC}^c$ change in opposite directions to those for $N_e = 71$. $f_{AB}^c$ and $f_{AC}^c$ may still be nonzero in the ferromagnetic state ($J_{AF} < 0.01$), which sustains the charge and spin currents as shown in Figs. 3(c) and (d).

In summary, we have addressed an effective Hamiltonian for a ring of double-exchange system from an electronic model by considering both the Aharonov-Bohm and Aharonov-Casher effects. The motion of conduction electrons in the system acquires a Berry phase via the Hund’s coupling between these electrons and the localized electrons. This produces an observable quantum effect in a closed route of electron’s motion. In the classical double-exchange model, the phase is neglected at all.$^1$ Apart from the double-exchange ferromagnetism in the system, the superexchange antiferromagnetic coupling from the localized spins also plays an important role in stabilizing the magnetic structure. A spin spiral state can be stabilized for a moderate antiferromagnetic coupling and both charge and spin currents can be induced simultaneously via the geometric Berry phase in this state, which is a piece of physics emerging from the present analysis. This current-induced topological effect is expected to be observable in mesoscopic systems, which might provide a new way to investigate the magnetic structure in such double-exchange materials.

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