Multiplierless Perfect Reconstruction Modulated Filter Banks with Sum-of-Powers-of-Two Coefficients

S. C. Chan, W. Liu, and K. L. Ho

***Abstract*—This paper proposes an efficient class of perfect reconstruction (PR) modulated filter banks (MFB) using sum-of-powers-of-two (SOPOT) coefficients. This is based on a modified factorization of the DCT-IV matrix and the lossless lattice structure of the prototype filter, which allows the coefficients to be represented in SOPOT form without affecting the PR condition. A genetic algorithm (GA) is then used to search for these SOPOT coefficients. Design examples show that SOPOT MFB with a good frequency characteristic can be designed with very low implementation complexity. The usefulness of the approach is demonstrated with a 16-channel design example.**

***Index Terms*—Fast implementation, modulated filter bank, multiplierless, perfect reconstruction, sum-of-powers-of-two.**

I. Introduction

RECENTLY, there has been an increasing interest in designing filter banks with low implementation complexity. Such filter banks are useful in discrete multitone transmission (DMT) systems and many other applications. Approaches based on the sum-of-power-of-two (SOPOT) coefficients [2], [3] are particularly attractive because coefficient multiplications can be implemented with simple shifts and additions only. In this paper, a new family of modulated filter banks (MFBs) with SOPOT coefficients, called SOPOT MFB, is developed. The modulation matrix and prototype filter are derived from a fast DCT-IV algorithm of Wang [5] and the lattice structure in [1], respectively. The SOPOT coefficients are obtained using a genetic algorithm (GA). Design examples show that SOPOT MFB with good frequency characteristic can be designed with very low implementation complexity. An example of 16-channel SOPOT MFB with 33.60 dB stopband attenuation is given, which requires only 328 additions and 222 shifts for the analysis side. This paper is organized as follows. The theory of the proposed modulated filter banks is given in Section II. The construction of the SOPOT modulation matrix and the design of the SOPOT MFB are described in Sections III and IV, respectively, followed by a design example in Section V.

II. Theory of MFB

In cosine modulated filter banks (CMFB), the analysis filters $f_k(n)$ and synthesis filters $g_k(n)$ are derived from a prototype filter $h(n)$ by cosine modulation $c_{k,n}$ ($\overline{c}_{k,n}$)

$$f_k(n) = h(n)c_{k,n}, g_k(n) = h(n)\overline{c}_{k,n} \qquad k = 0, 1, \cdots M-1; \quad n = 0, 1, \cdots, N-1 \quad (1)$$

where M is the number of channels, and N is the length of the filter. Two different modulations (the CMFB in [1] and the extended lapped transform [ELT] in [6]) can be used. Here, we will consider the following modulation proposed in [1]:

$$c_{k,n} = 2\cos\left((2k+1)\frac{\pi}{2M}\left(n - \frac{N-1}{2}\right) + (-1)^k\frac{\pi}{4}\right)$$
$$\overline{c}_{k,n} = 2\cos\left((2k+1)\frac{\pi}{2M}\left(n - \frac{N-1}{2}\right) - (-1)^k\frac{\pi}{4}\right). \quad (2)$$

Let $H(z) = \sum_{q=0}^{2M-1} z^{-q}H_q(z^{2M})$ and $F_k(z)$ be the type-I polyphase decomposition of $h(n)$ and the z-transform of $f_k(n)$. It can be shown that the analysis filters can be expressed in matrix form as follows:

$$\begin{aligned}\boldsymbol{f}(z) &= [F_0(z), F_1(z), \cdots, F_{M-1}(z)]^T \\ &= \hat{\boldsymbol{C}}_{\boldsymbol{A}}\begin{bmatrix}\boldsymbol{h}_0(z^{2M}) \\ z^{-M}\boldsymbol{h}_1(z^{2M})\end{bmatrix}\boldsymbol{e}_M(z) = \boldsymbol{E}(z^M)\boldsymbol{e}_M(z) \qquad (3)\end{aligned}$$

where

$$\hat{\boldsymbol{C}}_{\boldsymbol{A}} = (-1)^q\sqrt{M}\boldsymbol{C}_M^{IV}[(\boldsymbol{I}_M + (-1)^{m-1}\boldsymbol{J}_M) \quad ((-1)^{m-1}\boldsymbol{I}_M - \boldsymbol{J}_M)]$$

($q = m/2$) for m even and $(m-1)/2$ for m odd [1])

$$\begin{aligned}\boldsymbol{e}_M^T(z) &= [1 \quad z^{-1} \quad \cdots \quad z^{-(M-1)}] \\ \boldsymbol{h}_0(z) &= \text{diag}\,[H_0(-z), H_1(-z), \cdots, H_{M-1}(-z)] \\ \boldsymbol{h}_1(z) &= \text{diag}\,[H_M(-z), H_{M+1}(-z), \cdots, H_{2M-1}(-z)].\end{aligned}$$

$\boldsymbol{I}_M$ and $\boldsymbol{J}_M$ are the identity and anti-identity matrices, respectively. $\boldsymbol{E}(z)$ is the polyphase matrix, and

$$[\boldsymbol{C}_M^{IV}]_{(k,n)} = \sqrt{2/M}\cos((k+1/2)(n+1/2)\pi/2M)$$

is the type-IV discrete cosine transform (DCT). Similarly, the polyphase matrix in the synthesis side can be written as

$$\boldsymbol{R}(z) = \boldsymbol{C}_S\begin{bmatrix}\boldsymbol{g}_0(z^2) \\ z^{-1}\boldsymbol{g}_1(z^2)\end{bmatrix}_{2M\times M} \quad \text{with}$$
$$\boldsymbol{C}_S = (-1)^{m+1}\boldsymbol{C}_A\boldsymbol{J}_{2M}$$

The system is a perfect reconstruction (PR) if $\boldsymbol{R}(z)\boldsymbol{E}(z) = z^{-d}\boldsymbol{I}_M$ for some positive integer d, which is equivalent to the following constraint on the prototype filter:

$$H_k(z)H_{2M-k-1}(z) + H_{M+k}(z)H_{M-k-1}(z) = cz^{-s_k} \quad (4)$$

Manuscript received July 28, 2000. This work was supported by RGC, Hong Kong SAR. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. R. Shenoy.

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Publisher Item Identifier S 1070-9908(01)03287-4.

where c is a nonzero constant and s_k are positive integers. It is interesting to note that the PR condition (4) remains unchanged if we replace $\hat{\boldsymbol{C}}_{\boldsymbol{A}}$ and $\hat{\boldsymbol{C}}_S$ by $\boldsymbol{M_A}$ and $\boldsymbol{M}_s$, respectively, as follows:

$$\begin{aligned}\boldsymbol{M_A} &= \sqrt{M}\boldsymbol{U}_M[\boldsymbol{I}_M \pm \boldsymbol{J}_M, \pm(\boldsymbol{I}_M \mp \boldsymbol{J}_M)],\\ \boldsymbol{M}_S &= \sqrt{M}(\boldsymbol{U}_M^{-1})^T[\boldsymbol{I}_m \pm \boldsymbol{J}_M, \pm(\boldsymbol{I}_M \mp \boldsymbol{J}_M)]\boldsymbol{J}_{2M}.\end{aligned} \tag{5}$$

$\boldsymbol{U}_M$ and $(\boldsymbol{U}_M^{-1})^T$ (nonsingular) are the prototype or modulation matrices of the analysis and synthesis banks, respectively. Without loss of generality, we will consider the case

$$\begin{aligned}\boldsymbol{M_A} &= \sqrt{M}\boldsymbol{U}_M[\boldsymbol{I}_M + \boldsymbol{J}_M, \boldsymbol{I}_M - \boldsymbol{J}_M],\\ \boldsymbol{M}_S &= \sqrt{M}(\boldsymbol{U}_M^{-1})^T[\boldsymbol{I}_M + \boldsymbol{J}_M, \boldsymbol{I}_M - \boldsymbol{J}_M]\boldsymbol{J}_{2M}.\end{aligned}$$

Using (5), it is possible to replace the type-IV DCT matrix with its SOPOT approximation $\boldsymbol{U}_M$, as we shall see in the next section.

III. SOPOT Modulation

The simplest way to derive the SOPOT modulation is to quantize directly the coefficients of the type-IV DCT in (3). Its inverse, however, cannot in general be expressed in SOPOT represenation. To overcome this problem, we first decompose the DCT-IV matrix using a fast algorithm of Wang [5] into a set of rotation-like matrices $\boldsymbol{R}_\theta$ as shown in the following:

$$\begin{aligned}\boldsymbol{R}_\theta = \begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix} = \boldsymbol{R}_\theta^{-1} = \begin{bmatrix}1 & -\tan(\theta/2)\\ 0 & 1\end{bmatrix}\\ \cdot\begin{bmatrix}1 & 0\\ \sin\theta & 1\end{bmatrix}\cdot\begin{bmatrix}1 & \tan(\theta/2)\\ 0 & -1\end{bmatrix}.\end{aligned} \tag{6}$$

$\boldsymbol{R}_\theta$ is related to the conventional rotation matrix by multiplying the second column by -1. Using the fact a complex rotation can be implemented as three shears in three multiplications and three additions, a similar factorization of $\boldsymbol{R}_\theta$, as shown in (6), can also be obtained. This factorization can also be used in constructing other SOPOT sinusoidal transforms [7]. It can be seen that the proposed factorizations for $\boldsymbol{R}_\theta$ and $\boldsymbol{R}_\theta^{-1}$ involve the same set of coefficients, i.e., $\tan(\theta/2)$ and $\sin\theta$. They can therefore be quantized directly into SOPOT representation without affecting their inverse relationship. As mentioned earlier, our SOPOT modulation is based on the fast algorithm of [5]. It factorizes a DCT-IV matrix of size $(2^\gamma \times 2^\gamma)$ with $M = 2^\gamma$ into a product of $2\gamma + 1$ sparse matrices, as shown in the following:

$$\begin{aligned}\boldsymbol{C}_M^{IV} &= \boldsymbol{Q}_M\boldsymbol{V}_M(\gamma)\left[\prod_{i=1}^{\gamma-1}\boldsymbol{K}_M(\gamma-i)\boldsymbol{V}_M(\gamma-i)\right]\boldsymbol{H}_M,\\ &\gamma = \log_2 M\end{aligned} \tag{7}$$

where

$$\begin{aligned}\boldsymbol{H}_M =& \boldsymbol{P}_M(\boldsymbol{P}_{M/2} \oplus \overline{\boldsymbol{P}}_{M/2})\\ &\cdot \text{diag}(\boldsymbol{P}_{M/4}, \overline{\boldsymbol{P}}_{M/4}, \boldsymbol{P}_{M/4}, \overline{\boldsymbol{P}}_{M/4})\\ &\cdots \text{diag}(\boldsymbol{P}_4, \overline{\boldsymbol{P}}_4, \cdots, \boldsymbol{P}_4, \overline{\boldsymbol{P}}_4), \qquad N \geq 4\end{aligned}$$

and

$$\boldsymbol{P}_M = \begin{bmatrix}1 & 0 & . & . & . & . & . & .\\ 0 & . & . & . & . & . & . & 1\\ 0 & 1 & 0 & . & . & . & . & .\\ 0 & . & . & . & . & . & 1 & 0\\ . & . & . & . & . & . & . & .\\ 0 & . & . & . & 1 & 0 & . & .\\ 0 & . & 1 & 0 & . & . & . & .\\ 0 & . & . & 1 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\boldsymbol{Q}_M = \begin{bmatrix}1 & 0 & . & . & . & . & . & 0\\ 0 & . & . & . & . & . & 0 & 1\\ 0 & 0 & 1 & 0 & . & . & . & 0\\ . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & .\\ 0 & 0 & 0 & 1 & 0 & . & . & 0\\ 0 & 0 & . & . & . & . & 1 & 0\\ 0 & 1 & 0 & 0 & . & . & . & 0\end{bmatrix}.$$

$\overline{\boldsymbol{P}}_M$ is obtained by reversing both the rows and columns of $\boldsymbol{P}_M$. $\boldsymbol{Q}_M$ is a permutation matrix that changes the odd-numbered components of the vector into a reversed order. $\boldsymbol{K}_M(i)$, $i = 1, 2, \cdots, \gamma - 1$ are block diagonal matrices given by

$$\boldsymbol{K}_M(i) = (1/\sqrt{2}) \cdot \text{diag}\,(\boldsymbol{B}(i), \boldsymbol{B}(i), \cdots, \boldsymbol{B}(i), \boldsymbol{B}(i))$$

where

$$\boldsymbol{B}(i) = \begin{bmatrix}\boldsymbol{I}_{2^i} & \boldsymbol{I}_{2^i}\\ \boldsymbol{I}_{2^i} & -\boldsymbol{I}_{2^i}\end{bmatrix}.$$

$\boldsymbol{V}_M(\gamma)$ is given by

$$\boldsymbol{V}_M(\gamma) = \text{diag}\,(\boldsymbol{T}_{1/4M}, \boldsymbol{T}_{5/4M}, \cdots, \boldsymbol{T}_{(2M-3)/4M})$$

where

$$\boldsymbol{T}_r = \begin{bmatrix}\cos r\pi & \sin r\pi\\ \sin r\pi & -\cos r\pi\end{bmatrix}.$$

The other matrices $\boldsymbol{V}_M(i)$, $i = 1, 2, \cdots, \gamma - 1$, are obtained by alternating the submatrices $\boldsymbol{I}_{2^i}$ and $\boldsymbol{T}(i)$ in the main diagonal as follows:

$$\boldsymbol{V}_M(i) = \text{diag}\,(\boldsymbol{I}_{2^i}, \boldsymbol{T}(i), \boldsymbol{I}_{2^i}, \cdots, \boldsymbol{T}(i))$$

where

$$\boldsymbol{T}(i) = \text{diag}\,(\boldsymbol{T}_{1/2^{i+1}}, \boldsymbol{T}_{5/2^{i+1}}, \cdots, \boldsymbol{T}_{(2^{i+1}-3)/2^{i+1}}).$$

In order to obtain $\boldsymbol{U}_M$, the SOPOT approximation of the DCT-IV matrix, we can replace $\boldsymbol{T}_r$ in the matrix $\boldsymbol{V}_M$s with

$$\boldsymbol{S}_r = \begin{bmatrix}1 & -\beta_r\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \alpha_r & 1\end{bmatrix}\begin{bmatrix}1 & \beta_r\\ 0 & -1\end{bmatrix}$$

as in (6), with α_r and β_r in SOPOT form. Let $\boldsymbol{R}_M(i)$ be the matrix obtained by replacing $\boldsymbol{T}_r$ in $\boldsymbol{V}_M(i)$ with $\boldsymbol{S}_r$. As $\boldsymbol{S}_r^{-1} = \boldsymbol{S}_r$ and $\boldsymbol{R}_M^{-1}(i) = \boldsymbol{R}_M(i)$, the SOPOT prototype matrix $\boldsymbol{U}_M$ and its inverse can be derived from (7) and are given as follows:

$$\begin{aligned}\boldsymbol{U}_M &= \boldsymbol{Q}_M\boldsymbol{R}_M(\gamma)\left[\prod_{i=1}^{\gamma-1}\boldsymbol{K}_M(\gamma-i)\boldsymbol{R}_M(\gamma-i)\right]\boldsymbol{H}_M,\\ \boldsymbol{U}_M^{-1} &= \boldsymbol{H}_M^T\left[\prod_{i=1}^{\gamma-1}\boldsymbol{R}_M(i)\boldsymbol{K}_M(i)\right]\boldsymbol{R}_M(\gamma)\boldsymbol{Q}_M.\end{aligned} \tag{8}$$

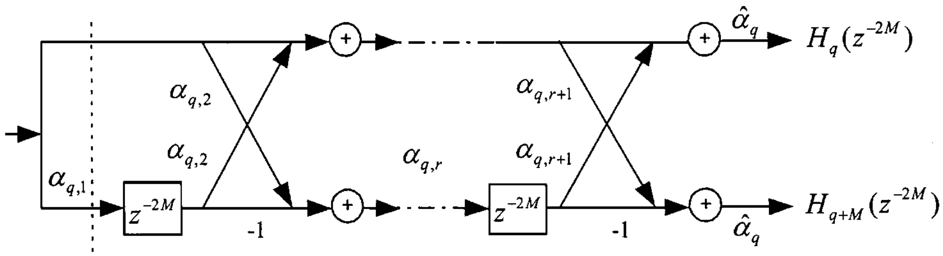

Fig. 1. Lossless lattice structure of the prototype filter. Only the qth lattice is shown ($\hat{\alpha}_q = \Pi_{i=1}^{r+1} (1+\alpha_{q,i}^2)^{-0.5}$).

IV. Design of the MFB

To design the prototype filter with SOPOT coefficients, we use the lossless lattice structure proposed in [1] (Fig. 1). N is therefore equal to $2mM$. The lattice coefficients are represented in SOPOT form as follows:

$$\alpha_{q,n} = \sum_{k=1}^{p_n} a_k 2^{b_k}, \qquad a_k \in \{-1, 1\}, \qquad b_k \in \{l, \cdots, 1, 0, -1, \cdots, -l\}. \tag{9}$$

l is a positive integer, which determines the range of the coefficient, and p_n is the number of terms used in each coefficient. The following minimax objective function is used in the design of the prototype filter:

$$D_h = \max \| H(e^{j\omega})| - |H_d(e^{j\omega})\| \tag{10}$$

where $H(z)$ and $H_d(z)$ are, respectively, the actual and ideal frequency responses of the prototype filter. The modulation matrix is designed using a similar minimax objective function

$$D_M = \max_{0 \le k \le M-1} \{E_k = \max \| F_k(e^{j\omega})| - |\hat{F}_k(e^{j\omega})\|\} \tag{11}$$

where $F_k(e^{j\omega})$ and $\hat{F}_k(e^{j\omega})$ are, respectively, the actual and ideal frequency responses of the kth analysis filter. The SOPOT coefficients of the prototype filter are first obtained by minimizing D_h using the GA. The SOPOT coefficients for the modulation is then obtained by minimizing D_M using again the GA. In our GA, the bit-string representation is used because it is easier to represent the SOPOT coefficients. Also, for simplicity, single-point crossover with a randomly generated crossover point is employed. The mutation rate is set to 0.5 (a fairly high value to introduce more randomness into the GA) and the gene chosen for mutation is determined randomly.

V. Design Examples

Table I shows the coefficients of the prototype filter for an 16-channel SOPOT MFB. The cutoff frequency of the prototype filter is $\omega_s = 0.075\pi$. Because the prototype filter is linear-phase, there are altogether eight different lattices. Polyphase components $H_q(z)$ and $H_{q+16}(z)$ are derived from the qth lattice, and all of them have two lattice coefficients $\alpha_{q,1}$ and $\alpha_{q,2}$, i.e., $(m = 2)$. The prototype matrix $\boldsymbol{U}_{16}$ is given by

$$\boldsymbol{U}_{16} = \boldsymbol{Q}_{16}\boldsymbol{R}_{16}(4)\boldsymbol{K}_{16}(3)\boldsymbol{R}_{16}(3)\boldsymbol{K}_{16}(2)\boldsymbol{R}_{16}(2) \cdot \boldsymbol{K}_{16}(1)\boldsymbol{R}_{16}(1)\boldsymbol{H}_{16} \tag{12}$$

TABLE I
SOPOT Coefficients for the Prototype Filter of the 16-Channel MFB

q	$\alpha_{q,1}$,	$\alpha_{q,2}$
0	$2^2 - 2^{-1}$	$-2^2 + 2^{-4}$
1	$2^1 + 2^{-1} + 2^{-5}$	$-2^2 - 2^0 - 2^{-2}$
2	$2^1 + 2^{-1}$	$-2^2 - 2^0 - 2^{-1} - 2^{-3}$
3	2^1	$-2^3 - 2^0$
4	$2^0 + 2^{-1} + 2^{-4}$	$-2^4 - 2^2$
5	$2^0 + 2^{-1} - 2^{-6}$	$-2^5 + 2^1$
6	$2^0 + 2^{-2}$	$-2^4 - 2^1$
7	$2^0 + 2^{-4}$	$-2^5 - 2^4$

TABLE II
SOPOT Coefficients for the Prototype Matrix of the 16-Channel MFB

$r(\mathbf{S}_r)$	α_r	β_r
1/64	2^{-4}	2^{-5}
5/64	$2^{-2} - 2^{-6} - 2^{-7}$	$2^{-3} - 2^{-7}$
9/64	$2^{-1} - 2^{-4}$	$2^{-2} - 2^{-6} - 2^{-7}$
13/64	$2^{-1} + 2^{-4} + 2^{-6} + 2^{-7}$	$2^{-2} + 2^{-4} + 2^{-6}$
17/64	$2^{-1} + 2^{-2} + 2^{-6} + 2^{-7}$	$2^{-1} - 2^{-4} + 2^{-7}$
21/64	$2^0 - 2^{-3} - 2^{-7}$	$2^{-1} + 2^{-4} - 2^{-7}$
25/64	$2^0 - 2^{-4}$	$2^{-1} + 2^{-2} - 2^{-5}$
29/64	$2^0 - 2^{-7}$	$2^0 - 2^{-3} - 2^{-6}$
1/16	$2^{-3} + 2^{-4} + 2^{-6} + 2^{-7}$	$2^{-3} - 2^{-6}$
5/16	$2^{-1} + 2^{-2} + 2^{-4}$	$2^{-1} + 2^{-5}$
1/8	$2^{-2} + 2^{-3}$	$2^{-3} + 2^{-4} + 2^{-7} + 2^{-8}$
1/4	$2^{-1} + 2^{-2} - 2^{-4} + 2^{-6}$	$2^{-1} - 2^{-4} - 2^{-6} - 2^{-7}$

where

$$\boldsymbol{R}_{16}(4) = \text{diag}\,\{\boldsymbol{S}_{1/64}, \boldsymbol{S}_{5/64}, \boldsymbol{S}_{9/64}, \boldsymbol{S}_{13/64}, \cdot \boldsymbol{S}_{17/64}, \boldsymbol{S}_{21/64}, \boldsymbol{S}_{25/64}, \boldsymbol{S}_{29/64}\},$$
$$\boldsymbol{R}_{16}(3) = \text{diag}\,\{\mathbf{I}_8, \boldsymbol{S}_{1/16}, \boldsymbol{S}_{5/16}, \boldsymbol{S}_{9/16}, \boldsymbol{S}_{13/16}\},$$
$$\boldsymbol{R}_{16}(2) = \text{diag}\,\{\mathbf{I}_4, \boldsymbol{S}_{1/8}, \boldsymbol{S}_{5/8}, \mathbf{I}_4, \boldsymbol{S}_{1/8}, \boldsymbol{S}_{5/8}\}, \quad \text{and}$$
$$\boldsymbol{R}_{16}(1) = \text{diag}\,\{\mathbf{I}_2, \boldsymbol{S}_{1/4}, \mathbf{I}_2, \boldsymbol{S}_{1/4}, \mathbf{I}_2, \boldsymbol{S}_{1/4}, \mathbf{I}_2, \boldsymbol{S}_{1/4}\}.$$

As mentioned earlier, each $\boldsymbol{S}_r$ is characterized by two parameters α_r and β_r. Noting that

$$\boldsymbol{T}_{x+1/2} = \boldsymbol{T}_x \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{S}_{x+1/2} = \boldsymbol{S}_x \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

x = 1/16, 5/16, 1/8, $\boldsymbol{S}_{9/16}$, $\boldsymbol{S}_{13/16}$, and $\boldsymbol{S}_{5/16}$ can be derived from $\boldsymbol{S}_{1/16}$, $\boldsymbol{S}_{5/16}$, and $\boldsymbol{S}_{1/8}$, respectively. Table II shows the result for the SOPOT prototype matrix, and Fig. 2 shows the frequency responses of the analysis filters. The stopband attenuation is 33.60 dB. The arithmetic complexity of the analysis side is 328 additions and 222 shifts. On the other hand, the

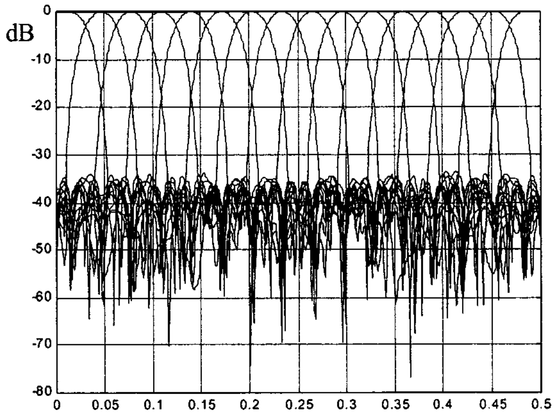

Fig. 2. Frequency responses of the 16-channel SOPOT MFB: analysis filters, stopband attenuation $A_s = 33.60$ dB. Prototype filter length is 64, $\omega_s = 0.075\pi$, and stopband attenuation $A_s = 37$ dB).

corresponding orthogonal CMFB requires 168 additions and 80 real multiplications. Its stopband attenuation, designed by constrained optimization using mathetmatics and staistics libraries (IMSLs), is 34.39 dB. Other examples (not shown here for page limitation) also suggest that the low-complexity SOPOT MFB can achieve similar stopband attenuation as their CMFB counterparts.

VI. Conclusion

An efficient class of multiplierless PR modulated filter banks using SOPOT coefficients is presented. It is based on a modified factorization of the DCT-IV matrix and the lossless lattice structure of the prototype filter, which allows the coefficients to be represented as SOPOT form without affecting the PR condition. A GA is used to search for these SOPOT coefficients. Design examples show that SOPOT MFB with good frequency characteristic can be obtained. The usefulness of the approach is demonstrated with a 16-channel example.

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