Mixed H_2/H_∞ Filtering for Uncertain Systems with Regional Pole Assignment

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The mixed H_2/H_∞ filtering problem for uncertain linear continuous-time systems with regional pole assignment is considered. The purpose of the problem is to design an uncertainty-independent filter such that, for all admissible parameter uncertainties, the following filtering requirements are simultaneously satisfied: 1) the filtering process is asymptotically stable; 2) the poles of the filtering matrix are located inside a prescribed region that compasses the vertical strips, horizontal strips, disks, or conic sectors; 3) both the H_2 norm and the H_∞ norm on the respective transfer functions are not more than the specified upper bound constraints. We establish a general framework to solve the addressed multiobjective filtering problem completely. In particular, we derive necessary and sufficient conditions for the solvability of the problem in terms of a set of feasible linear matrix inequalities (LMIs). An illustrative example is given to illustrate the design procedures and performances of the proposed method.

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I. INTRODUCTION

State estimation of dynamic systems in the presence of both process and measurement noises is one of the important problems in control engineering [1]. Among various state estimation methods, the celebrated Kalman filtering (also known as H_2 filtering) approach minimizes the H_2 norm of the estimation error, under the assumptions that the system parameters are well posed and the noise processes have exactly known power spectral densities. The application areas of Kalman filtering range from control engineering to signal processing, such as depth estimation in machine vision development (see e.g. [15]).

It is now well known that (see e.g. [1]) the traditional Kalman filtering approach may have poor performance against modeling error and noises with uncertain spectral densities. This situation has resulted in the rapid developments of H_∞ filtering and cost-guaranteed robust filtering. For instance, the estimation problem was reformulated in [22] in terms of H_∞ norm constraints. An example was proposed by de Souza et al. [10] to demonstrate that the H_∞ filtering is more robust against plant uncertainties than the H_2 filtering. So far, there have been many approaches to dealing with the H_∞ filtering problem, such as the game-theoretic method [23], the linear matrix inequality (LMI) approach [8, 18], the model matching approach [14], to name just a few.

On the other hand, the H_∞ filtering typically leads to a large intolerable estimation error variance when the system is driven by white noise signals. Therefore, the mixed H_2/H_∞ filtering problem, which simultaneously considers the presence of two sets of exogenous signal inputs (i.e., the deterministic input with bounded energy and the stochastic input with known statistics), was first introduced in [3] as an attempt to capture the benefits of both the pure H_2 and H_∞ filters. The mixed H_2/H_∞ filtering theory allows us to trade off between the best performances of the H_2 and H_∞ filters. As is well known, the individual H_2 or H_∞ filtering problem has readily computable solutions. Unfortunately, this is no longer such a case for the mixed H_2/H_∞ problem as there is no known "nice" solution.

So far, there have been several approaches to tackling the mixed H_2/H_∞ filtering problem. For example, Bernstein and Haddad [3] transformed the mixed H_2/H_∞ filtering problem into an auxiliary minimization problem. Then, by using the Lagrange multiplier technique, they gave the solutions in terms of an upper bound on the H_2 filtering error. In [5] and [23], a time domain game approach was proposed to solve the mixed H_2/H_∞ filtering problem through a set of coupled Riccati equations. Khargonekar et al. [17] and Rotstein et al. [20] exploited the convex optimization method to obtain the solutions involving affine symmetric matrix inequalities. Furthermore, when there exist parameter uncertainties, the robust H_2 and/or H_∞ filtering problem has recently received much research attention, see e.g. [9, 12, 17, 24–26, 28] and references therein.

On the other hand, the standard mixed H_2/H_∞ filter design primarily concerns with the stability and frequency-domain performance specifications of the filter, and considers very little about the transient property of the estimation dynamics. As is well known, the dynamics of a linear system is closely related to the location of its poles. By constraining the filter's poles to lie inside a prescribed region in the open left-half plane, the filter designed would have the expected transient performance. Besides, regional pole assignment can also provide indirect tolerance against plant uncertainties. It is worth emphasizing that, in the past few years, the controller design problem with regional poles placement has been extensively studied. In particular, Chilali and Gahinet [6] studied in detail the design of state- or output-feedback H_∞ controllers that satisfy additional constraints on the regional pole location, and the results were further extended in [7] and [21] to the uncertain system that was described by a polytopic state-space model. In [2], the mixed H_2/H_∞ control problem with regional pole assignment was considered for deterministic continuous-time systems. Recently, in [13], the H_2/H_∞ robust filtering problem was investigated for convex bounded uncertain systems by using an LMI approach. It should be pointed out that, compared with the control design case, the corresponding filtering design problem with pole assignment in a desired region has gained much less attention, not to mention the case when mixed H_2/H_∞ performance is also the required filtering objectives. This situation motivates our present investigation.

We deal here with the multiobjective H_2/H_∞ filtering problem with regional pole assignment. The approach developed is different from that proposed in [3], where the Lagrange multiplier technique was used for solving a set of highly coupled Riccati and Lyapunov equations. Instead, the LMI approach is employed, which is based on the change-of-variable technique introduced in [21] and used in [13]. Since LMIs intrinsically reflect constraints rather than optimality, they tend to offer more flexibility for combining several constraints. LMIs can now be solved efficiently via interior-point optimization algorithms, such as those described in [4], [11], and [16]. Moreover, software like MATLAB LMI Toolbox is now available to solve such LMIs efficiently. Specifically, we transform all the performance specifications (that is, the constraints on H_2 norm bound, H_∞ norm bound, and pole clustering as well) into unified LMI formulations. Therefore, the overall problem remains convex, and the desired filter parameters can be obtained by solving the LMIs.

The rest of this work is organized as follows. The mixed H_2/H_∞ filtering problem with regional pole assignment for uncertain continuous-time system is formulated in Section II. In Section III, necessary and sufficient conditions are developed for solving the mixed H_2/H_∞ filtering problem with regional pole assignment. An illustrative example is given in Section IV. Section V contains some concluding remarks.

The notation used here is fairly standard. $\otimes$ denotes the Kronecker product. $\|\bullet\|_p$ stands for H_p-norm in Hardy space. $\mathrm{Tr}(M)$ represents the trace of matrix M. In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. $\bar{\lambda}$ means the conjugate of λ. The block (m,n) of a block matrix is the submatrix with respect to the mth row and nth column. The shorthand $\mathrm{diag}\{M_1,M_2,\ldots,M_N\}$ is denoted by

$$\mathrm{diag}\{M_1,M_2,\ldots,M_N\} = \begin{bmatrix} M_1 & 0 & \cdots & 0 \\ 0 & M_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & M_N \end{bmatrix}.$$

Sometimes, the arguments of an identity matrix are omitted in the analysis when no confusion can arise.

II. PROBLEM FORMULATION

Consider a linear uncertain continuous-time system described by

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + B_1 w(t) + B_2 v(t) \\ y(t) &= (C + \Delta C)x(t) + D_1 w(t) + D_2 v(t) \\ z_\infty(t) &= L_\infty x(t) \\ z_2(t) &= L_2 x(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $y(t) \in R^p$ is the measured output, $z_\infty(t) \in R^{\mathrm{m1}}$ is a combination of the states to be estimated (with respect to H_∞-norm constraints), and $z_2(t) \in R^{\mathrm{m2}}$ is another combination of the states to be estimated (with respect to H_2-norm constraints). $w(t) \in R^{\mathrm{p1}}$ is a disturbance input with bounded energy and stationary power, which is assumed to belong to $L_2[0,\infty]$, and $v(t) \in R^{\mathrm{p2}}$ is a zero-mean Gaussian white noise process with unit covariance. A, C, B_1, B_2, D_1, D_2, L_∞, and L_2 are known real matrices with appropriate dimensions, whereas ΔA and ΔC are perturbation matrices representing parameter uncertainties, and are assumed to be time-invariant matrices of the form

$$\begin{bmatrix} \Delta A \\ \Delta C \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \Gamma E \tag{2}$$

where H_1, H_2, and E are known constant matrices of appropriate dimensions, and $\Gamma \in R^{i\times j}$ is an uncertain matrix that satisfies

$$\Gamma^{\mathrm{T}}\Gamma \leq I. \tag{3}$$

The uncertainties ΔA and ΔC are said to be admissible if they meet conditions (2) and (3). It is also assumed that the initial state $x(0)$ is known. Without loss of generality, we take $x(0) = 0$.

REMARK 1 The parameter uncertainty structure as in (2)–(3) has been widely used in the problems of robust control and robust filtering of uncertain systems (see, e.g., [24–28] and the references therein). Many practical systems possess parameter uncertainties that can be either exactly modeled or overbounded by (3). Note that when $z_\infty(t)$ and $z_2(t)$ are identical, the system (1) without parameter uncertainties will be reduced to those described in [20, 23].

We make the following assumption throughout this work.

Assumption 1 The system matrix A is stable, that is, all eigenvalues are located in the left-half complex plane.

REMARK 2 Assumption 1 is necessary for the robust filtering problem to be meaningful.

Now consider the following filter for the system (1):

$$\begin{aligned} \dot{\hat{x}}(t) &= F\hat{x}(t) + Gy(t) \\ \hat{z}_\infty(t) &= \hat{L}_\infty \hat{x}(t) \\ \hat{z}_2(t) &= \hat{L}_2\hat{x}(t) \end{aligned} \tag{4}$$

where $\hat{x}(t) \in R^n$ is the estimated state, $\hat{z}_\infty(t) \in R^{m1}$ is an estimate for $z_\infty(t)$, $\hat{z}_2(t) \in R^{m2}$ is an estimate for $z_2(t)$, and F, G, $\hat{L}_\infty$, and $\hat{L}_2$ are filter parameters to be determined. Notice that the filter structure (4) is not dependent upon the parameter uncertainties.

Define

$$x_e(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \tag{5}$$

the augmented system formed from the system (1) and the filter (4) can now be expressed as

$$\begin{aligned} \dot{x}_e(t) &= (A_e + \Delta A_e)x_e(t) + B_{e1}w(t) + B_{e2}v(t) \\ e_\infty(t) &= z_\infty(t) - \hat{z}_\infty(t) = C_\infty x_e(t) \\ e_2(t) &= z_2(t) - \hat{z}_2(t) = C_2 x_e(t) \end{aligned} \tag{6}$$

where

$$A_e = \begin{bmatrix} A & 0 \\ GC & F \end{bmatrix}, \qquad \Delta A_e = \begin{bmatrix} H_1 \\ GH_2 \end{bmatrix} \Gamma [E \quad 0] =: H_e \Gamma E_e \tag{7a}$$

$$B_{e1} = \begin{bmatrix} B_1 \\ GD_1 \end{bmatrix}, \qquad B_{e2} = \begin{bmatrix} B_2 \\ GD_2 \end{bmatrix}$$
$$C_\infty = [L_\infty \quad -\hat{L}_\infty], \quad C_2 = [L_2 \quad -\hat{L}_2]. \tag{7b}$$

Let

$$T_\infty(s) = C_\infty(sI - A_e - \Delta A_e)^{-1}B_{e1}$$
$$T_2(s) = C_2(sI - A_e - \Delta A_e)^{-1}B_{e2}$$

be, respectively, the transfer function from $w(t)$ to the error state $e_\infty(t)$ (corresponding to the H_∞-norm consideration), and the transfer function from $v(t)$ to the error state $e_2(t)$ (corresponding to the H_2-norm consideration).

For the purpose of regional pole assignment, we now recall the concept of LMI region proposed in [7]. An LMI region is any subset D in the open left-half complex plane that can be described as follows:

$$D = \{\lambda \in C : f_D(\lambda) = L + \lambda M + \bar{\lambda} M^{\mathrm{T}} < 0\} \tag{8}$$

where L and M are real matrices such that $L^{\mathrm{T}} = L$. The matrix-valued function $f_D(\lambda)$ is called the characteristic function of D. As explained in [7], with different choices of the matrices L and M, the LMI region D defined in (8) can be used to represent many kinds of popular pole regions, such as left half-plane, disk, vertical strips, horizontal strips, conic sector, etc.

Now, we are in the position to introduce the notions of "quadratic stability" and "quadratic D-stability" for the uncertain system (6).

DEFINITION 1 The uncertain augmented system (6) is said to be quadratically stable if there exists a symmetric positive definite matrix X such that for all admissible perturbations ΔA and ΔC, the following inequality

$$(A_e + \Delta A_e)^{\mathrm{T}} X + X(A_e + \Delta A_e) < 0 \tag{9}$$

holds.

DEFINITION 2 [6, 7] The uncertain augmented system (6) is said to be quadratically D-stable if there exists a symmetric positive definite matrix X such that for all admissible perturbations ΔA and ΔC, the following matrix inequality

$$L \otimes X + M \otimes (X(A_e + \Delta A_e)) + M^{\mathrm{T}} \otimes ((A_e + \Delta A_e)^{\mathrm{T}} X) < 0 \tag{10}$$

is true where the LMI region D is defined in (8).

REMARK 3 It has been revealed in [6] and [7] that, if (10) is satisfied, then all poles of the uncertain time-invariant matrix $A_e + \Delta A_e$ are constrained to lie within the specified LMI region D.

The following well-known lemmas, which give the bounded realness results for the addressed H_2 and H_∞-norm constraints, are needed in the derivation of our main results.

LEMMA 1 [11, 18] *Let the constant $\gamma > 0$ be given. The uncertain augmented system* (6) *is quadratically stable and $\|T_\infty(s)\|_\infty < \gamma$, if and only if there exists a symmetric positive definite matrix X such that for all* *admissible perturbations ΔA and ΔC, the following inequality*

$$\begin{bmatrix} (A_e+\Delta A_e)^{\mathrm{T}}X + X(A_e+\Delta A_e) & XB_{e1} & C_\infty^{\mathrm{T}} \\ B_{e1}^{\mathrm{T}}X & -\gamma I & 0 \\ C_\infty & 0 & -\gamma I \end{bmatrix} < 0 \tag{11}$$

is true.

LEMMA 2 [19, 21] *Let the constant $\beta > 0$ be given. The uncertain augmented system* (6) *is quadratically stable and $\|T_2(s)\|_2 < \beta$, if and only if there exist symmetric positive definite matrices X and Q such that for all admissible perturbations ΔA and ΔC, the following three inequalities*

$$\begin{bmatrix} (A_e+\Delta A_e)^{\mathrm{T}}X + X(A_e+\Delta A_e) & XB_{e2} \\ B_{e2}^{\mathrm{T}}X & -I \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} X & C_2^{\mathrm{T}} \\ C_2 & Q \end{bmatrix} > 0 \tag{13}$$

$$\mathrm{Tr}(Q) < \beta^2 \tag{14}$$

hold.

Now, we are ready to state the multiobjective filtering problem investigated in the work presented here. For the linear time-invariant uncertain systems (1) with given LMI pole region D, H_2-norm upper bound β and H_∞-norm upper bound γ, we are interested in seeking the filter parameters F, G, $\hat{L}_\infty$ and $\hat{L}_2$, such that for all admissible parameter uncertainties ΔA and ΔC, the augmented system (6) satisfies the following four robust performance requirements simultaneously.

1) All poles of the augmented system (6) are constrained to lie inside a prescribed LMI region D, that is, (6) is quadratically D-stable.

2) The transfer function from the deterministic disturbance input $w(t)$ to the error state $e_\infty(t)$ meets the H_∞-norm upper bound constraint

$$\|T_\infty(s)\|_\infty = \|C_\infty(sI - A_e - \Delta A_e)^{-1}B_{e1}\|_\infty < \gamma.$$

3) The transfer function from the white noise input $v(t)$ to the error state $e_2(t)$ meets the H_2-norm upper bound constraint

$$\|T_2(s)\|_2 = \|C_2(sI - A_e - \Delta A_e)^{-1}B_{e2}\|_2 < \beta.$$

4) The upper bound of the H_2 guaranteed cost is minimized, that is, $\mathrm{Tr}(Q)$ is minimized.

In the above, we aim to minimize the upper bound of the H_2 performance that is of direct physical significance. On the other hand, if the solution exists, then there should be a solution set for achieving the H_∞-norm upper bound constraint 2 above, and the H_2-norm upper bound constraint 3 above. In this case, we have a design freedom to minimize either the upper bound of H_2 performance, or the upper bound of H_∞ performance, or play the tradeoff between them.

In the light of Definition 1, Definition 2, Lemma 1 and Lemma 2, the multiobjective filtering problem addressed above can be recast into the following optimization problem:

$$\min_{X>0,Q>0,F,G,\hat{L}_2,\hat{L}_\infty} \mathrm{Tr}(Q) \qquad \text{subject to} \quad (10)\text{–}(13). \tag{15}$$

The problem (15) is referred to as the mixed H_2/H_∞ filtering problem with regional pole assignment. Note that at this stage, such a problem is not a convex one yet, since the parameter uncertainties ΔA and ΔC are involved, which makes the problem more complicated. Our goal in the next section is to derive necessary and sufficient conditions, in the form of LMIs, for the solutions of the aforementioned filter design problem. Therefore, the powerful LMI Toolbox could be utilized to solve the overall convex optimization problem efficiently.

We point out that the filtering problems as well as the system descriptions presented here are quite different from those presented in [13]. Specifically, in this paper, 1) we consider an additional performance requirement, the regional pole assignment, which is used to guarantee the transient performance of the filtering process, 2) we consider two different kinds of exogenous signal inputs (i.e., the deterministic input with bounded energy and the stochastic input with known statistics), and 3) the norm-bounded uncertainties are considered here whereas the convex-bounded uncertainties are treated in [13].

III. SOLUTION TO MULTIOJECTIVE H_2/H_∞ FILTERING PROBLEM

In this section, we give the solution to the mixed H_2/H_∞ filtering problem with regional pole assignment based on an LMI approach. Before giving our main results, the following lemmas are needed.

LEMMA 3 (Schur Complement) *Given constant matrices Ω_1, Ω_2, and Ω_3 where $\Omega_1 = \Omega_1^{\mathrm{T}}$ and $0 < \Omega_2 = \Omega_2^{\mathrm{T}}$. Then $\Omega_1 + \Omega_3^{\mathrm{T}}\Omega_2^{-1}\Omega_3 < 0$ if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^{\mathrm{T}} \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0$$

or equivalently

$$\begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^{\mathrm{T}} & \Omega_1 \end{bmatrix} < 0.$$

LEMMA 4 *Let M, H, and E be real matrices of appropriate dimensions, with Γ satisfying* (3)*, then*

$$M + H\Gamma E + E^{\mathrm{T}}\Gamma^{\mathrm{T}}H^{\mathrm{T}} < 0 \tag{16a}$$

if and only if there exists a positive scalar $\varepsilon > 0$ such that

$$M + \varepsilon E^{\mathrm{T}} E + \frac{1}{\varepsilon} H H^{\mathrm{T}} < 0 \tag{16b}$$

or equivalently

$$\begin{bmatrix} M & H & \varepsilon E^{\mathrm{T}} \\ H^{\mathrm{T}} & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{17}$$

PROOF The first conclusion is Lemma 2.4 of [27]. The equivalence between (16b) and (17) follows immediately from the Schur complement lemma (Lemma 3).

REMARK 4 Lemma 4 is also known as the S-procedure technique, which is often utilized to convert the inequality involving norm-bounded uncertainty like (16a) into an equivalent LMI with an extra scalar parameter ε.

It is shown in the following theorem that the addressed multiobjective filtering problem can be solved if and only if the solutions to certain LMIs are known to exist.

THEOREM 1 *Let D be an arbitrary LMI region contained in the open left-half plane and let* (8) *be its characteristic function. The problem* (15) *is solvable, if and only if there exist symmetric positive definite matrices R, S, Q, matrices Q_1, Q_2, Q_3, Q_4 and positive scalars ε_1, ε_2, and ε_3 such that the following LMIs*

$$\begin{bmatrix} \Theta & M_1^{\mathrm{T}} \otimes \begin{bmatrix} SH_1 \\ RH_1 + Q_2 H_2 \end{bmatrix} & M_2^{\mathrm{T}} \otimes \begin{bmatrix} \varepsilon_1 E^{\mathrm{T}} \\ \varepsilon_1 E^{\mathrm{T}} \end{bmatrix} \\ M_1 \otimes [H_1^{\mathrm{T}} S \quad (RH_1 + Q_2 H_2)^{\mathrm{T}}] & -\varepsilon_1 I & 0 \\ M_2 \otimes [\varepsilon_1 E \quad \varepsilon_1 E] & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} A^{\mathrm{T}} S + SA & * & * & * & * & * \\ RA + Q_2 C + Q_1 + A^{\mathrm{T}} S & RA + Q_2 C + A^{\mathrm{T}} R + C^{\mathrm{T}} Q_2^{\mathrm{T}} & * & * & * & * \\ B_1^{\mathrm{T}} S & (RB_1 + Q_2 D_1)^{\mathrm{T}} & -\gamma I & * & * & * \\ L_\infty - Q_3 & L_\infty & 0 & -\gamma I & * & * \\ H_1^{\mathrm{T}} S & (RH_1 + Q_2 H_2)^{\mathrm{T}} & 0 & 0 & -\varepsilon_2 I & * \\ \varepsilon_2 E & \varepsilon_2 E & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} A^{\mathrm{T}} S + SA & * & * & * & * \\ RA + Q_2 C + Q_1 + A^{\mathrm{T}} S & RA + Q_2 C + A^{\mathrm{T}} R + C^{\mathrm{T}} Q_2^{\mathrm{T}} & * & * & * \\ B_2^{\mathrm{T}} S & (RB_2 + Q_2 D_2)^{\mathrm{T}} & -I & * & * \\ H_1^{\mathrm{T}} S & (RH_1 + Q_2 H_2)^{\mathrm{T}} & 0 & -\varepsilon_3 I & * \\ \varepsilon_3 E & \varepsilon_3 E & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix} -S & -S & L_2^{\mathrm{T}} - Q_4^{\mathrm{T}} \\ -S & -R & L_2^{\mathrm{T}} \\ L_2 - Q_4 & L_2 & -Q \end{bmatrix} < 0 \tag{21}$$

where

$$\Theta := L \otimes \begin{bmatrix} S & S \\ S & R \end{bmatrix} + M \otimes \begin{bmatrix} SA & SA \\ RA + Q_2 C + Q_1 & RA + Q_2 C \end{bmatrix} + M^{\mathrm{T}} \otimes \begin{bmatrix} SA & SA \\ RA + Q_2 C + Q_1 & RA + Q_2 C \end{bmatrix}^{\mathrm{T}}$$

are feasible. Here, the constant matrices M_1, M_2 are obtained from the factorization of $M = M_1^{\mathrm{T}} M_2$ where M_1 and M_2 have full column rank. Moreover, if the LMIs (18)–(21) *are solvable, the desired filter parameters can be determined by*

$$F = X_{12}^{-1} Q_1 (S - R)^{-1} X_{12} \tag{22}$$

$$G = X_{12}^{-1} Q_2 \tag{23}$$

$$\hat{L}_\infty = Q_3 (S - R)^{-1} X_{12} \tag{24}$$

$$\hat{L}_2 = Q_4 (S - R)^{-1} X_{12} \tag{25}$$

where the matrix X_{12} comes from the following factorization

$$I - RS^{-1} = X_{12} Y_{12}^{\mathrm{T}} \tag{26}$$

with both X_{12} and Y_{12} being nonsingular square matrices.

PROOF Factorize the matrix M as $M = M_1^{\mathrm{T}} M_2$, where M_1, M_2 have full column rank. Such a factorization can be obtained easily through the singular value decomposition (SVD) technique. The purpose of this factorization is to guarantee that the block (1,2) and block (1,3) have the same row by Schur Complement Lemma (Lemma 3).

Applying Lemma 4 to (10)–(13), respectively, we obtain the following LMIs on the positive definite matrix $X > 0$ and the positive scalar parameters ε_1, ε_2, and ε_3:

$$\begin{bmatrix} L\otimes X + M\otimes (XA_e) + M^{\mathrm{T}}\otimes (A_eX)^{\mathrm{T}} & M_1^{\mathrm{T}}\otimes (XH_e) & \varepsilon_1 M_2^{\mathrm{T}}\otimes E_e^{\mathrm{T}} \\ M_1\otimes (H_e^{\mathrm{T}}X) & -\varepsilon_1 I & 0 \\ \varepsilon_1 M_2\otimes E_e & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \tag{27}$$

$$\begin{bmatrix} A_e^{\mathrm{T}}X + XA_e & XB_{e1} & C_\infty^{\mathrm{T}} & XH_e & \varepsilon_2 E_e^{\mathrm{T}} \\ B_{e1}^{\mathrm{T}}X & -\gamma I & 0 & 0 & 0 \\ C_\infty & 0 & -\gamma I & 0 & 0 \\ H_e^{\mathrm{T}}X & 0 & 0 & -\varepsilon_2 I & 0 \\ \varepsilon_2 E_e & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{28}$$

$$\begin{bmatrix} A_e^{\mathrm{T}}X + XA_e & XB_{e2} & XH_e & \varepsilon_3 E_e^{\mathrm{T}} \\ B_{e2}^{\mathrm{T}}X & -I & 0 & 0 \\ H_e^{\mathrm{T}}X & 0 & -\varepsilon_3 I & 0 \\ \varepsilon_3 E_e & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} X & C_2^{\mathrm{T}} \\ C_2 & Q \end{bmatrix} > 0. \tag{30}$$

Recall that our goal is to derive the expressions of the filter parameters from (27)–(30). To do this, we partition X and X^{-1} as

$$X = \begin{bmatrix} R & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}, \qquad X^{-1} = \begin{bmatrix} S^{-1} & Y_{12} \\ Y_{12}^{\mathrm{T}} & Y_{22} \end{bmatrix} \tag{31}$$

where the partitioning of X and X^{-1} is compatible with that of A_e defined in (7a), i.e., $R \in R^{n\times n}$, $X_{12} \in R^{n\times n}$, $X_{22} \in R^{n\times n}$, $S \in R^{n\times n}$, $Y_{12} \in R^{n\times n}$, $Y_{22} \in R^{n\times n}$.

We define

$$T_1 := \begin{bmatrix} S^{-1} & I \\ Y_{12}^{\mathrm{T}} & 0 \end{bmatrix}, \qquad T_2 := \begin{bmatrix} I & R \\ 0 & X_{12}^{\mathrm{T}} \end{bmatrix}. \tag{32}$$

It follows directly from $XX^{-1} = I$ that

$$XT_1 = T_2 \tag{33}$$

$$T_1^{\mathrm{T}} X T_1 = T_1^{\mathrm{T}} T_2 = \begin{bmatrix} S^{-1} & I \\ I & R \end{bmatrix} > 0 \tag{34}$$

and

$$I - S^{-1}R = Y_{12}X_{12}^{\mathrm{T}}. \tag{35}$$

From (34), we have $R - S > 0$, which implies that $I - S^{-1}R$ is nonsingular. Hence, X_{12} and Y_{12} are also nonsingular, i.e., invertible.

Furthermore, let us define the changes of the filter parameters as follows:

$$\begin{aligned} Q_1 &:= X_{12}FY_{12}^{\mathrm{T}}S, \\ Q_2 &:= X_{12}G \\ Q_3 &:= \hat{L}_\infty Y_{12}^{\mathrm{T}}S, \\ Q_4 &:= \hat{L}_2 Y_{12}^{\mathrm{T}}S. \end{aligned} \tag{36}$$

Applying the congruence transformations $\mathrm{diag}\{I\otimes T_1, I, I\}$ to (27), $\mathrm{diag}\{T_1, I, I, I, I\}$ to (28), $\mathrm{diag}\{T_1, I, I, I\}$ to (29), $\mathrm{diag}\{T_1, I\}$ to (30), respectively, we obtain

$$\begin{bmatrix} \Theta & M_1^{\mathrm{T}}\otimes \begin{bmatrix} H_1 \\ RH_1 + Q_2H_2 \end{bmatrix} & M_2^{\mathrm{T}}\otimes \begin{bmatrix} \varepsilon_1 S^{-1}E^{\mathrm{T}} \\ \varepsilon_1 E^{\mathrm{T}} \end{bmatrix} \\ M_1\otimes [H_1^{\mathrm{T}} \quad (RH_1 + Q_2H_2)^{\mathrm{T}}] & -\varepsilon_1 I & 0 \\ M_2\otimes [\varepsilon_1 ES^{-1} \quad \varepsilon_1 E] & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \tag{37}$$

$$\begin{bmatrix} AS^{-1} + S^{-1}A^{\mathrm{T}} & * & * & * & * & * \\ RAS^{-1} + Q_2CS^{-1} + Q_1S^{-1} + A^{\mathrm{T}} & RA + Q_2C + A^{\mathrm{T}}R + C^{\mathrm{T}}Q_2^{\mathrm{T}} & * & * & * & * \\ B_1^{\mathrm{T}} & (RB_1 + Q_2D_1)^{\mathrm{T}} & -\gamma I & * & * & * \\ L_\infty S^{-1} - Q_3S^{-1} & L_\infty & 0 & -\gamma I & * & * \\ H_1^{\mathrm{T}} & (RH_1 + Q_2H_2)^{\mathrm{T}} & 0 & 0 & -\varepsilon_2 I & * \\ \varepsilon_2 ES^{-1} & \varepsilon_2 E & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{38}$$

$$
\begin{bmatrix}
AS^{-1}+S^{-1}A^{\mathrm{T}} & * & * & * & * \\
RAS^{-1}+Q_2CS^{-1}+Q_1S^{-1}+A^{\mathrm{T}} & RA+Q_2C+A^{\mathrm{T}}R+C^{\mathrm{T}}Q_2^{\mathrm{T}} & * & * & * \\
B_2^{\mathrm{T}} & (RB_2+Q_2D_2)^{\mathrm{T}} & -I & * & * \\
H_1^{\mathrm{T}} & (RH_1+Q_2H_2)^{\mathrm{T}} & 0 & -\varepsilon_3 I & * \\
\varepsilon_3 ES^{-1} & \varepsilon_3 E & 0 & 0 & -\varepsilon_3 I
\end{bmatrix} < 0 \tag{39}
$$

$$
\begin{bmatrix}
-S^{-1} & -I & S^{-1}L_2^{\mathrm{T}}-S^{-1}Q_4^{\mathrm{T}} \\
-I & -R & L_2^{\mathrm{T}} \\
L_2S^{-1}-Q_4S^{-1} & L_2 & -Q
\end{bmatrix} < 0 \tag{40}
$$

where

$$
\begin{aligned}
\Theta := {} & L\otimes\begin{bmatrix} S^{-1} & I \\ I & R\end{bmatrix} \\
& + M\otimes\begin{bmatrix} AS^{-1} & A \\ RAS^{-1}+Q_2CS^{-1}+Q_1S^{-1} & RA+Q_2C\end{bmatrix} \\
& + M^{\mathrm{T}}\otimes\begin{bmatrix} AS^{-1} & A \\ RAS^{-1}+Q_2CS^{-1}+Q_1S^{-1} & RA+Q_2C\end{bmatrix}^{\mathrm{T}}.
\end{aligned}
$$

Again, applying the congruence transformations $\mathrm{diag}\{I\otimes\mathrm{diag}\{S,I\},I,I\}$ to (37), $\mathrm{diag}\{S,I,I,I,I,I\}$ to (38), $\mathrm{diag}\{S,I,I,I,I\}$ to (39), $\mathrm{diag}\{S,I,I\}$ to (40), respectively, leads to the inequalities (18)–(21). This concludes the proof.

It follows from Theorem 1 that the problem (15) is now successfully recast as the following convex optimization problem:

$$
\min_{R>0,S>0,Q>0,Q_1,Q_2,Q_3,Q_4,\varepsilon_1,\varepsilon_2,\varepsilon_3} \mathrm{Tr}(Q) \qquad \text{subject to} \quad (18)\text{–}(21). \tag{41}
$$

On the other hand, in view of (22)–(25), we make the linear transformation on the state estimate

$$
\vec{x}(t) = X_{12}\hat{x}(t)
$$

and then obtain a new representation form of the filter as follows:

$$
\begin{aligned}
\dot{\vec{x}}(t) &= Q_1(S-R)^{-1}\vec{x}(t) + Q_2 y(t) \\
\hat{z}_\infty(t) &= Q_3(S-R)^{-1}\vec{x}(t) \\
\hat{z}_2(t) &= Q_4(S-R)^{-1}\vec{x}(t).
\end{aligned} \tag{42}
$$

We can now see from (42) that, the filter parameters can be obtained directly by solving the problem (41) without performing QR factorization for the identity (26).

REMARK 5 The problem (41) is a standard LMI problem. Note that in recent years LMIs have gained much attention for their computational tractability and usefulness in control engineering as the so-called interior point method has been proved to be numerically very efficient for solving the LMIs [4, 11, 16]. Theorem 1 shows that the addressed multiobjective problem also lies in the LMI framework. The change-of-variable technique utilized in the proof of Theorem 1 was introduced in [21], and similar to that used in [13].

REMARK 6 LMI regions are often specified as the intersection of elementary regions, such as vertical strips, horizontal strips, disks, or conic sectors. Given LMI regions $D_1, D_2, \ldots, D_N$, the intersection

$$
D = D_1 \cap D_2 \cap \cdots \cap D_N
$$

has characteristic function

$$
f_D(\lambda) = \mathrm{diag}\{f_{D_1}(\lambda), f_{D_2}(\lambda), \ldots, f_{D_N}(\lambda)\}
$$

and is still an LMI region [6, 7]. Therefore, the LMIs of $f_{D_1}(\lambda), f_{D_2}(\lambda), \ldots, f_{D_N}(\lambda)$, which can be derived, respectively, from (18), must be feasible so that the corresponding LMI for the intersection of the regions $D_1, D_2, \ldots, D_N$ is solvable. This point is illustrated by an example in the next section.

REMARK 7 Note that LMIs (18)–(21) are affine in the scalar positive parameters ε_1, ε_2, and ε_3. Hence, unlike the results in [24–26], [28], these parameters do not need to be tuned when solving the LMIs. Furthermore, these three parameters can be viewed as additional LMI variables, which could be exploited to reduce the possible conservatism when using Lemma 4. The optimal solutions can be obtained conveniently by solving LMIs (18)–(21) using Matlab LMI Toolbox.

IV. ILLUSTRATIVE EXAMPLE

Consider linear continuous-time system described by (1) with

$$
A = \begin{bmatrix} -3 & 2 & -2 \\ 3 & -5 & 1 \\ 4 & 1 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}
$$

$$B_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \qquad C = [-1 \quad 1 \quad 2]$$

$$D_1 = 1, \qquad D_2 = 2$$

$$L_\infty = [0.5 \quad 1 \quad 2], \qquad L_2 = [1 \quad 2 \quad 3].$$

The parameter uncertainties are given by

$$\Delta A = H_1 \Gamma N = \begin{bmatrix} 0.5 \\ 0.6 \\ 0.8 \end{bmatrix} \Gamma [1 \quad 0.5 \quad 0.5]$$

$$\Delta C = H_2 \Gamma N = 0.5 \Gamma [1 \quad 0.5 \quad 0.5]$$

where Γ is a perturbation matrix satisfying $\Gamma^{\mathrm{T}}\Gamma \le I$. We wish to design a filter such that the upper bound $\mathrm{Tr}(Q)$ of $\|T_2(s)\|_2^2$ is minimized subject to $\|T_\infty(s)\|_\infty < \gamma = 8$, and pole assignment to the shadow part in Fig. 1, where $\alpha = 0.2$, $r = 8$, $\alpha_1 = 0.5$, $\alpha_2 = 7$.

We first consider the disk centered at $(-\alpha, 0)$ with radius r, that is, in terms of the LMI region,

$$L = \begin{bmatrix} -r & \alpha \\ \alpha & -r \end{bmatrix}, \qquad M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M_1 = [1 \quad 0], \qquad M_2 = [0 \quad 1].$$

In this case, (18) can be rewritten as follows:

$$\begin{bmatrix} -rS & -rS & S(A+\alpha I) & S(A+\alpha I) & SH_1 & 0 \\ * & -rR & RA + Q_2 C + Q_1 + \alpha S & R(A+\alpha I) + Q_2 C & RH_1 + Q_2 H_2 & 0 \\ * & * & -rS & -rS & 0 & \varepsilon_1 E^{\mathrm{T}} \\ * & * & * & -rR & 0 & \varepsilon_1 E^{\mathrm{T}} \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0. \tag{43}$$

In the case of a vertical strip $\mathrm{Re}(\lambda) < -\alpha_1$, i.e., $L = 2\alpha_1$, $M = 1$, $M_1 = 1$, and $M_2 = 1$, the corresponding inequality (18) can be rewritten as follows:

$$\begin{bmatrix} (A+\alpha_1 I)^{\mathrm{T}} S + S(A+\alpha_1 I) & S(A+\alpha_1 I) + A^{\mathrm{T}} R + C^{\mathrm{T}} Q_2^{\mathrm{T}} + Q_1^{\mathrm{T}} + \alpha_1 S & SH_1 & \varepsilon_4 E^{\mathrm{T}} \\ * & (A+\alpha_1 I)^{\mathrm{T}} R + C^{\mathrm{T}} Q_2^{\mathrm{T}} + R(A+\alpha_1 I) + Q_2 C & RH_1 + Q_2 H_2 & \varepsilon_4 E^{\mathrm{T}} \\ * & * & -\varepsilon_4 I & 0 \\ * & * & * & -\varepsilon_4 I \end{bmatrix} < 0. \tag{44}$$

Furthermore, for the vertical strip $\mathrm{Re}(\lambda) > -\alpha_2$, i.e., $L = -2\alpha_2$, $M = -1$, $M_1 = -1$, and $M_2 = 1$, (18) can be rewritten as follows:

$$\begin{bmatrix} -(A+\alpha_2 I)^{\mathrm{T}} S - S(A+\alpha_2 I) & -S(A+\alpha_2 I) - A^{\mathrm{T}} R - C^{\mathrm{T}} Q_2^{\mathrm{T}} - Q_1^{\mathrm{T}} - \alpha_2 S & -SH_1 & \varepsilon_5 E^{\mathrm{T}} \\ * & -(A+\alpha_2 I)^{\mathrm{T}} R - C^{\mathrm{T}} Q_2^{\mathrm{T}} - R(A+\alpha_2 I) - Q_2 C & -RH_1 - Q_2 H_2 & \varepsilon_5 E^{\mathrm{T}} \\ * & * & -\varepsilon_5 I & 0 \\ * & * & * & -\varepsilon_5 I \end{bmatrix} < 0. \tag{45}$$

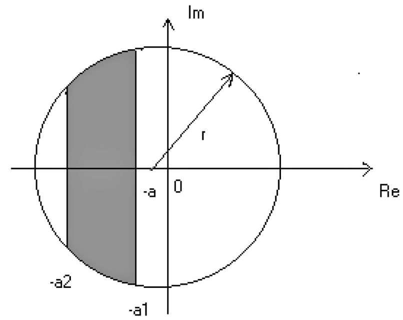

Fig. 1. Intersection of vertical strips and circle.

According to (41), the desired filter design problem can be transformed into the following convex problem:

$$\min_{R>0, S>0, Q>0, Q_1, Q_2, Q_3, Q_4, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5} \mathrm{Tr}(Q)$$

$$\text{subject to} \quad (19)\text{–}(21) \text{ and } (43)\text{–}(45). \tag{46}$$

By using the Matlab LMI Toolbox, the optimal solution to the convex problem (46) is given by $\mathrm{Tr}(Q) = 2.5324$ with the filter parameters

$$F = \begin{bmatrix} -3.8342 & 2.5490 & -3.3582 \\ 1.7257 & -5.2803 & -0.8384 \\ 2.1521 & -0.1680 & -0.3074 \end{bmatrix}$$

$$G = \begin{bmatrix} 1.4759 \\ -0.4552 \\ -2.5692 \end{bmatrix}$$

$$\hat{L}_\infty = [-0.0965 \quad -0.3192 \quad -0.3888]$$

$$\hat{L}_2 = [0.0248 \quad -0.4532 \quad -0.5504]$$

and the scalar parameters $\varepsilon_1 = 9.7576$, $\varepsilon_2 = 8.2645$, $\varepsilon_3 = 3.1529$, $\varepsilon_4 = 5.5567$, $\varepsilon_5 = 1.7829$.

If $\gamma = 4.8$, the optimal solution is given by $\mathrm{Tr}(Q) = 3.5319$ with the filter parameters

$$F = \begin{bmatrix} -4.1712 & 2.5874 & -3.7071 \\ 1.8171 & -5.1073 & -0.1456 \\ 2.2749 & -0.0785 & -0.1776 \end{bmatrix}$$

$$G = \begin{bmatrix} 1.2962 \\ -0.3377 \\ -2.1584 \end{bmatrix}$$

$$\hat{L}_\infty = [-0.2540 \quad -0.4357 \quad -0.5625]$$

$$\hat{L}_2 = [-0.0615 \quad -0.5444 \quad -0.7938]$$

and the scalar parameters $\varepsilon_1 = 6.7113$, $\varepsilon_2 = 5.3312$, $\varepsilon_3 = 2.2130$, $\varepsilon_4 = 3.5058$, $\varepsilon_5 = 1.3566$.

It is evident from this example that, by using the proposed LMI algorithm, there exists much flexibility in compromising both the H_2 performance and H_∞ performance.

V. CONCLUSION

In this paper, we have considered the mixed H_2/H_∞ filtering problem with regional pole assignment for uncertain continuous-time systems. Necessary and sufficient conditions for the solvability of the problem have been given in terms of LMIs that can be solved efficiently and reliably. Finally, in our opinion, the idea introduced here can also be applied to design robust filters for more complex systems such as sampled-data systems and stochastic parameter systems.

REFERENCES

[1] Anderson, B. D. O., and Moore, J. B.
Optimal Filtering.
Englewood Cliffs, NJ: Prentice-Hall, 1979.

[2] Bambang, R., Shimemura, E., and Uchida, K.
Mixed H_2/H_∞ control with pole placement in a class of regions.
Optimal Control Applications & Methods, **15**, 3 (1994), 151–173.

[3] Bernstein, D. S., and Haddad, W. M.
Steady-state Kalman filtering with an H_∞ error bounded.
Systems & Control Letters, **12**, 1 (1989), 9–16.

[4] Boyd, S., Ghaoui, L. E., Feron, E., and Balakrishnan, V.
Linear Matrix Inequalities in System and Control Theory.
SIAM Studies in Applied Mathematics, Philadelphia, PA, 1994.

[5] Chen, X., and Zhou, K.
Multiobjective filtering design.
In *Proceedings of the IEEE Canadian Conference on Electrical and Computer Engineering*, Alberta, Canada, 1999, 708–713.

[6] Chilali, M., and Gahinet, P.
H_∞ design with pole placement constraints: An LMI approach.
IEEE Transactions on Automatic Control, **41** (Mar. 1996), 358–367.

[7] Chilali, M., Gahinet, P., and Apkarian, P.
Robust pole placement in LMI regions.
IEEE Transactions on Automatic Control, **44** (Dec. 1999), 2257–2270.

[8] de Oliveira, M. C., Geromel, J. C., and Bernussou, J.
An LMI optimization approach to multiobjective controller design for discrete-time systems.
In *Proceedings of IEEE Conference on Decision Control*, Phoenix, AZ, 1999, 3611–3616.

[9] de Souza, C. E., and Shaked, U.
Robust H_2 filtering for uncertain systems with measurable inputs.
IEEE Transactions on Signal Processing, **47**, 8 (Aug. 1999), 2286–2292.

[10] de Souza, C. E., Shaked, U., and Fu, M.
Robust H_∞ filtering for continuous time varying uncertain systems with deterministic input signals.
IEEE Transactions on Signal Processing, **43** (Mar. 1995), 709–719.

[11] Gahinet, P., and Apkarian, P.
A linear matrix inequality approach to H_∞ control.
International Journal of Robust Nonlinear Control, **4**, 4 (1994), 421–448.

[12] Geromel, J. C.
Optimal linear filtering under parameter uncertainty.
IEEE Transactions on Signal Processing, **47** (Jan. 1999), 168–175.

[13] Geromel, J. C., and De Oliveira, M. C.
H_2/H_∞ robust filtering for convex bounded uncertain systems.
IEEE Transactions on Automatic Control, **46** (Jan. 2001), 100–107.

[14] Hung, Y. S.
A model matching approach to H_∞ filtering.
IEE Proceedings, Pt. D, *Control Theory and Applications*, **140**, 2 (1993), 133–139.

[15] Hung, Y. S., and Ho, H. T.
A Kalman filter approach to direct depth estimation incorporating surface structure.
IEEE Transactions on Pattern Analysis and Machine Intelligence, **21** (June 1999), 570–575.

[16] Iwasaki, T., and Skelton, R. E.
All controllers for the general H_∞ control problem: LMI existence conditions and state space formulas.
Automatica, **30**, 8 (1994), 1307–1317.

[17] Khargonekar, P. P., Rotea, M. A., and Baeyens, E.
Mixed H_2/H_∞ filtering.
International Journal of Robust Nonlinear Control, **6**, 4 (1996), 313–330.

[18] Li, H., and Fu, M.
A linear matrix inequality approach to robust H_∞-filtering.
IEEE Transactions on Signal Processing, **45** (Sept. 1997), 2338–2350.

[19] Masubuchi, I., Ohara, A., and Suda, N.
LMI-based controller synthesis: A unified formulation and solution.
International Journal of Robust Nonlinear Control, **8**, 8 (1998), 669–686.

[20] Rotstein, H., Sznaier, M., and Idan, M.
Mixed H_2/H_∞ filtering theory and an aerospace application.
International Journal of Robust Nonlinear Control, **6**, 4 (1996), 347–366.

[21] Scherer, C., Gahient, P., and Chilali, M.
Multiobjective output-feedback control via LMI optimization.
IEEE Transactions on Automatic Control, **42** (July 1997), 896–911.

[22] Shaked, U., and Theodor, Y.
H_∞-optimal estimation: A tutorial.
In *Proceedings of the IEEE Conference on Decision Control*, Tucson, AZ, 1992, 2278–2286.

[23] Theodor, Y., and Shaked, U.
A dynamic game approach to mixed H_2/H_∞ estimation.
International Journal of Robust Nonlinear Control, **6**, 4 (1996), 331–345.

[24] Wang, Z., and Huang, B.
Robust H_2/H_∞ filtering for linear systems with error variance constraints.
IEEE Transactions on Signal Processing, **48** (Aug. 2000), 2463–2467.

[25] Wang, Z., and Unbehauen, H.
Robust H_2/H_∞ state estimation for systems with error variance constraints: The continuous-time case.
IEEE Transactions on Automatic Control, **44** (May 1999), 1061–1065.

[26] Wang, Z., and Qiao, H.
Robust filtering for bilinear uncertain stochastic discrete-time systems.
IEEE Transactions on Signal Processing, **50** (Mar. 2002), 560–567.

[27] Xie, L.
Output feedback H_∞ control of systems with parameter uncertainty.
International Journal of Control, **63** (1996), 741–750.

[28] Yang, F., Wang, Z., and Hung, Y. S.
Robust Kalman filtering for discrete time-varying uncertain systems with multiplicative noises.
IEEE Transactions on Automatic Control, **47** (July 2002), 1179–1183.

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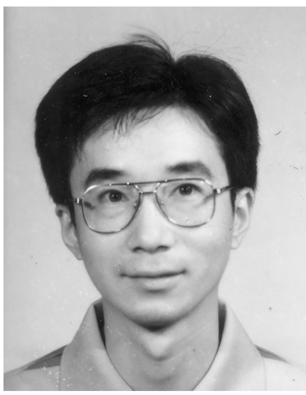

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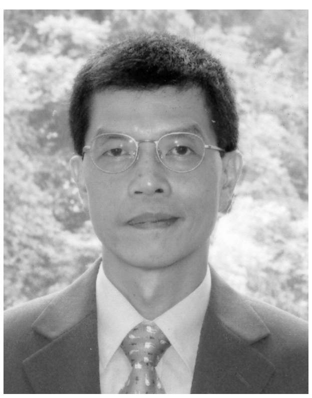

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