Isotopic dependence of predissociation linewidths in the Schumann-Runge bands of oxygen

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Recent work on predissociation in the Schumann-Runge ($B\ ^3\Sigma_u^- - X\ ^3\Sigma_g^-$) system of oxygen[1–4] encourages a study of the isotopic dependence of the linewidths. It has been shown that the predissociation is dominated by the spin-orbit coupling to the repulsive $^5\Pi_u$ state with smaller contributions from couplings to the $^3\Pi_u$, $^1\Pi_u$ and $^3\Sigma_u^+$ states.[5] Additionally the orbit-rotation interaction between the $B\ ^3\Sigma_u^-$ and $^3\Pi_u$ states has been demonstrated to be important for lines emanating from high rotational states.[6] In this Note we describe the isotopic dependence of the linewidths.

In Mulliken's classification of predissociation[7] the crossing of the potential energy curves of the $^3\Pi_u$ and $B\ ^3\Sigma_u^-$ states is of type C^- (where the predissociating state crosses the repulsive branch of the bound state) and crossings of the $^5\Pi_u$, $^1\Pi_u$ and $^3\Sigma_u^+$ states are of type C^+ (where the predissociating states cross the attractive branch). Within the semiclassical approximation the width Γ_v, caused by one of the predissociating states, of the vibrational level with quantum number v and energy E_v (measured from the bottom of the well) satisfies[8–10]

$$\frac{\Gamma_v}{\hbar\omega_v} = \pi C(\tfrac{3}{2}\ \phi_v)^{1/3}(E_v - E_x)^{-1/2}\mathrm{Ai}^2[-(\tfrac{3}{2}\ \phi_v)^{2/3}], \tag{1}$$

where $\hbar\omega_v$ ($=dE_v/dv$) and C are given in Ref. 9, ϕ_v is a phase (given in Ref. 9 for a C^+ crossing and in Ref. 10 for a C^- crossing), E_x is the energy at the crossing point and Ai denotes the Airy function. We have previously described the calculation of the predissociation linewidths for oxygen based on a least squares fit to experimental data.[4] Letting μ denote the *ratio* of the reduced mass for any one of the three oxygen isotopomers $^{16}O_2$, $^{16}O^{18}O$ and $^{18}O_2$ to that of the reference isotopomer $^{16}O_2$ we see, from Ref. 9, that the phases ϕ_v and the quantity C scale as $\mu^{1/2}$. Hence the characteristic energy E^* in the (linear) approximation,[9]

$$\left(\frac{3}{2}\ \phi_v\right)^{2/3} \approx \frac{(E_v - E_x)}{E^*}, \tag{2}$$

scales as $\mu^{-1/3}$. Within this approximation we can determine the dependence of the width on the energy E_v. This dependence may be expressed as

$$\frac{\Gamma_v}{\hbar\omega_v} = K\mu^{2/3}\mathrm{Ai}^2\left[-\mu^{1/3}\frac{(E_v - E_x)}{\tilde{E}^*}\right], \tag{3}$$

where K is mass independent and $\tilde{E}^*$ is the value of E^* for the reference isotopomer $^{16}O_2$. Expression (1) goes over smoothly to sinusoidal variation for $E \gg E_x$.[9] Within approximation (2) this becomes

$$\frac{\Gamma_v}{\hbar\omega_v} = \frac{K(\tilde{E}^*)^{1/2}}{\pi}\mu^{1/2}(E_v - E_x)^{-1/2}\sin^2\left(\mu^{1/2}\tilde{\phi}_v + \frac{\pi}{4}\right), \tag{4}$$

where $\tilde{\phi}_v$ is the phase for the reference isotopomer $^{16}O_2$.

Equations (3) and (4) show that the widths depend in a complicated manner on the reduced mass and do not have the simple dependence we implied.[4] Figure 2 of Ref. 4 could be viewed to imply that the widths can be fitted to a universal curve for all isotopomers. However this conclusion is ambiguous because the points are fortuitously distributed so as to obscure the mass dependence. Furthermore Fig. 2 of Ref. 4 shows the *total* widths; more precise information is obtained from the individual linewidths. We illustrate this briefly with a sample of calculated widths for two predissociating states: the $^3\Pi_u$ of type C^- where the Airy fit (3) is appropriate and the $^5\Pi_u$ where the sine fit (4) is appropriate.

Figure 1 shows the $^3\Pi_u$ widths for each isotopomer. It is clear that all three sets of widths do not fit a universal curve. This was obscured in the plot in Ref. 4 of the *total* widths. The energies were calculated from equation (5) below.

Our $^5\Pi_u$ widths are in the sinusoidal region where equation (4) is appropriate. Because of the disposition of points it is not so obvious from a direct fit that each isotopomer fits a unique curve. However equation (4) clearly shows a mass dependence. To show that the linear semiclassical approximation, which leads to equation (4), is appropriate for our $^5\Pi_u$ calculations we have calculated the corresponding phases and we show, in Fig. 2, that the quantity $(\tfrac{3}{2}\ \phi_v)^{2/3}\mu^{-1/3}$ plotted as a function of energy E_v, has the linear form of equation (2). The energies were calculated from Ref. 11

$$E_v = (v+\tfrac{1}{2})\omega_e\mu^{-1/2} - (v+\tfrac{1}{2})^2\omega_e x_e\mu^{-1} + (v+\tfrac{1}{2})^3\omega_e y_e\mu^{-3/2}, \tag{5}$$

where ω_e was taken as 709.31 cm^{-1}, $\omega_e x_e$ as 10.65 cm^{-1} and $\omega_e y_e$ as -0.139 cm^{-1}. The dependence of E_v on μ, being dominated by the first term on the right hand side of equation (5), can be largely removed by simple scaling. Equation (5) also provides $\hbar\omega_v = dE_v/dv$. The values of

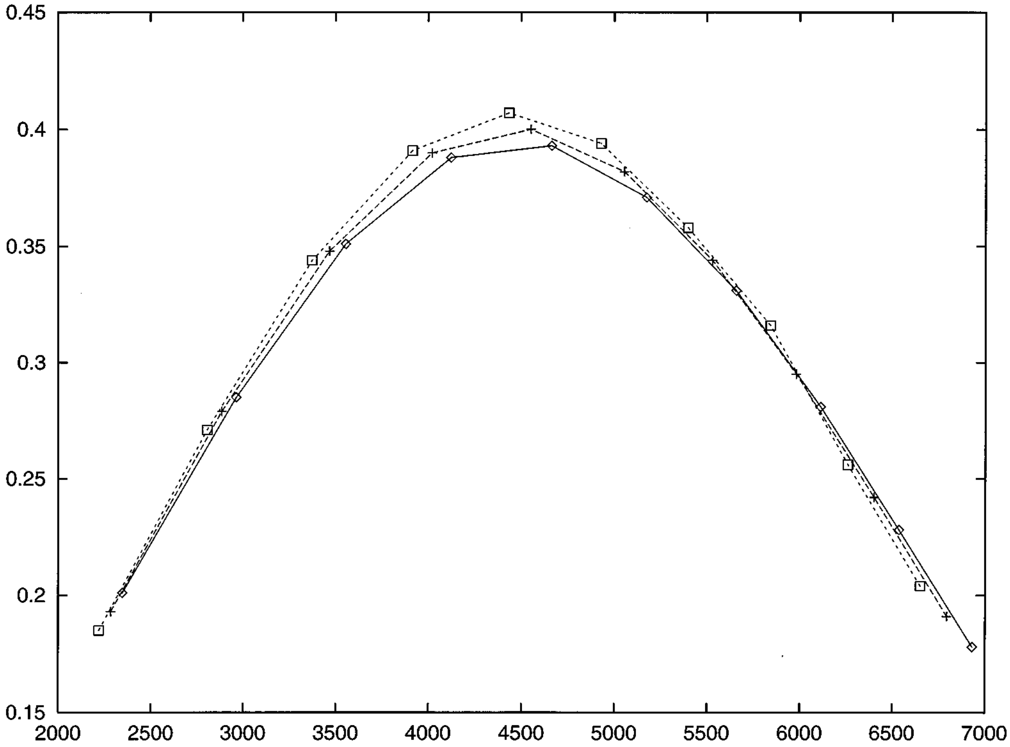

FIG. 1. Width (cm^{-1}) vs energy (cm^{-1}) in predissociation by the $^3\Pi_u$ state.

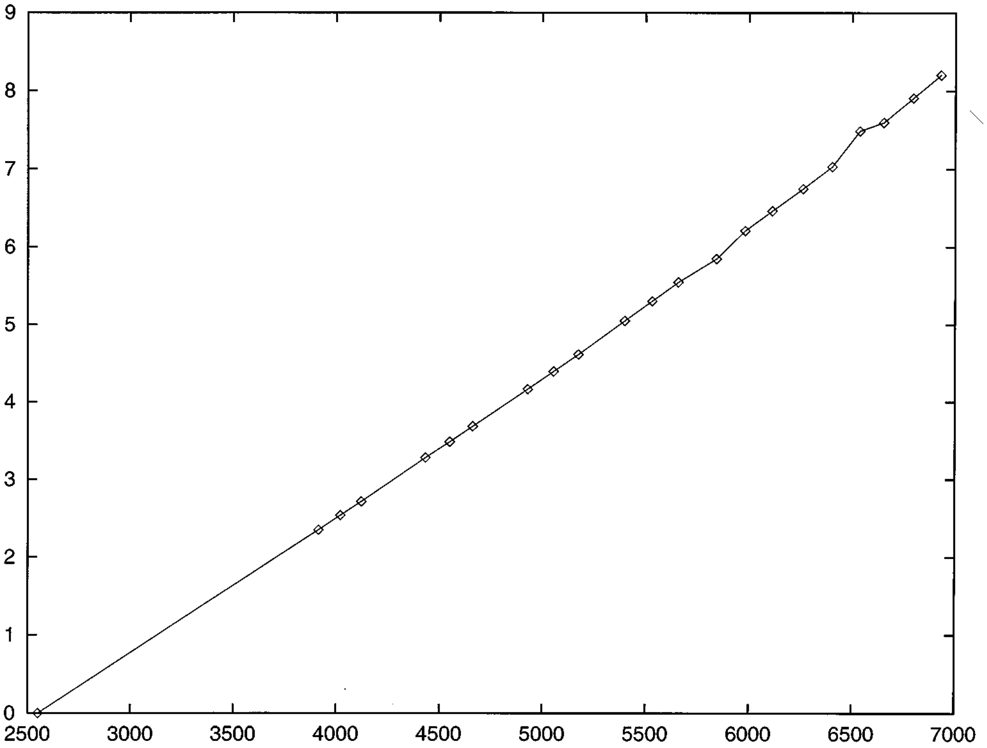

FIG. 2. $(\frac{3}{2}\,\phi_v)^{2/3}\mu^{-1/3}$ vs energy (cm^{-1}) in predissociation by the $^5\Pi_u$ state.

$\tilde{E}^* = 573\ \mathrm{cm}^{-1}$, $E_x = 2550\ \mathrm{cm}^{-1}$ and $K = 4.481$ were calculated by the method described in Ref. 10. The graph of the dimensionless quantities $(\frac{3}{2}\phi_v)^{2/3}\mu^{-1/3}$ and the intercept $(E_x,0)$ vs energies is expected to be a line with slope $1/\tilde{E}^*$. Figure 2 demonstrates this; from the slope we find the value of $\tilde{E}^*$ to be 557 cm^{-1} compared with 573 cm^{-1} given above. The $v=11$ vibrational level of $^{16}O_2$ and the $v=10$ vibrational level of $^{18}O_2$ are off line. This occurs because in each of these cases the phase is $\pi/4$ greater than a multiple of π causing its evaluation from the width to be ill conditioned.

In conclusion we see that in semiclassical theory, in contrast to the vibrational energy levels, the dependence of the predissociation widths on the reduced mass cannot be removed by a simple scaling procedure. We have shown this for previously calculated widths and we have resolved an ambiguity.

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