Šuchov's Bent Networks: the impact of network curvature on Šuchov's gridshell designs

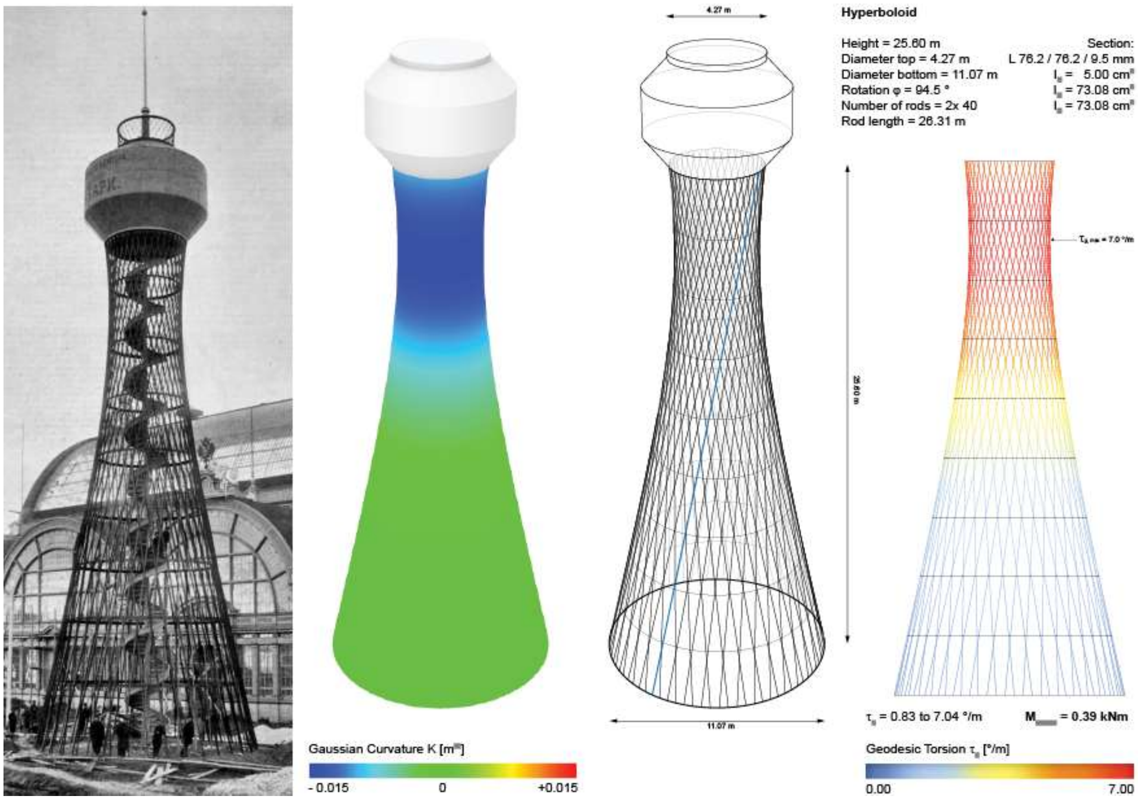
Eike Schling
Dr.-Ing. Univ., Architect, Assistant Professor HKU
corresponding author

Department of Architecture
Faculty of Architecture
The University of Hong Kong
Knowles Building 4/F 413
Pokfulam Road, Hong Kong
schling@hku.hk

Rainer Barthel
Prof. Dr.-Ing. Univ. TUM

Lehrstuhl für Tragwerksplanung
Fakultät für Architektur
Technische Universität München
Theresianum, Arcisstr. 21
80333 München, Germany
rainer.barthel@tum.de

Graphical Abstract



ABSTRACT

At the turn of the twentieth century, the Russian engineer Vladimir Šuchov developed an internationally acclaimed design language for steel construction. His filigree constructions – from lattice shell to the hanging roof – are timeless in their economy, lightness and, not least, simplicity. By combining a profound geometric and mechanical knowledge, Šuchov succeeded in deducing astonishingly pragmatic designs. A decisive strategy for simplification was the deliberate, elastic and plastic deformation of Z, L and flat steel profiles into continuously curved structural elements. As a result, Šuchov was able to assemble complex doubly curved grids using only repetitive components and simple riveted connections.

In this paper we conduct a purely geometrical analysis of these structures looking at the curvature behaviour of four exemplary structures with increasing complexity: the hyperbolic tower, the rotational hanging roof, the cylindrical lattice vault, and the parabolic, double-curved gridshell. By systematically deriving their geometry and examining the three axes of curvature of each rod within those grids, we discover previously undescribed deformations of the steel members and deduce their associated bending moments and stresses.

The results reveal Šuchov's systematic approach to utilizing elastic deformation to simplify construction, and offer conclusions about the prefabrication, the construction process, and the design approach he used for his lattice constructions. This has a direct impact not only on specific repair measures and structural analysis of his structures, but also on current research and the design and construction of similar gridshells today.

Keywords

steel; gridshell; curvature analysis; elastic deformation; repetitive design; construction process; Vladimir Šuchov

Highlights

- Using theories of differential geometry to analyse the curvature and bending of elastic gridshells
- Systematic analysis and interpretation of four structural typologies by Vladimir Šuchov
- Deduction of fundamental design and construction strategies based on prefabricated and installed states
- Impact of elastic steel construction on gridshell design.

1. Introduction

The constructions of the Russian engineer Vladimir Šuchov were ground-breaking, not least because of the great skill in realizing complex geometries in a simple and economical way (Graefe et al. 1990). They are of current importance, not only with regard to sustainability, but also to digital design, the desire for efficient forms and the associated building complexity (Edemskaya and Agkathidis 2016).

Most modern gridshells consist of discrete mesh constructions, i.e. they are built from straight steel elements which support flat panes of glass. In this case, geometric complexity arises in the joints, where complex spatial angles and offsets need to be resolved.

Šuchov pursued a fundamentally different strategy. His lattice elements are continuously curved and twisted to meet tangentially at the intersection points and thus allow the use of simple riveted connections. Furthermore, Šuchov's networks are based almost exclusively on regular, rotational surfaces, and thus allow the use of repetitive components.

When examining their geometry more closely, it becomes apparent that the individual structural elements follow neither a constant curvature nor twist. A straight steel profile, as used in the hyperbolic lattice towers (Fig. 1A), needs to be twisted at different intensities along its length in order to facilitate the tangential connections with the lattice structure. Even a simple arched member that is assembled diagonally into a lattice vault (Figure 1C), exhibits varying values of curvature about its three local section axes. These effects are even more complex in the case of the hanging roofs in Nizhny (Fig. 1B) and the double-curved gridshell in Vyksa (Fig 1D)

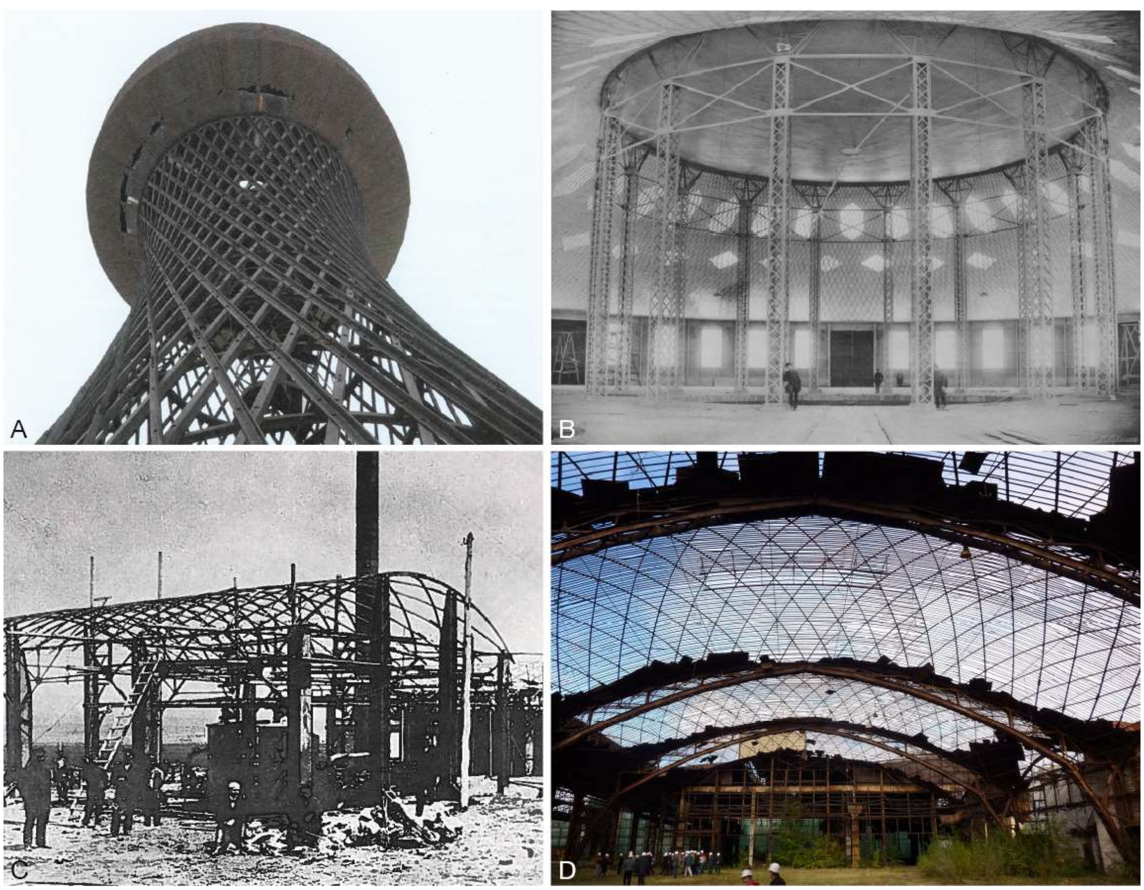


Figure 1: Lattice structures of Vladimir Šuchov: (A) Hyperbolic lattice structure of the water tower in Nizhny Novgorod, 1896. (B) Tensile steel roof of the Pavilion of Construction in Nizhny Novgorod, 1896. (C) The lattice barrel vault of the pump station in Grosny, 1890. (D) The doubly curved gridshell of the plate-rolling workshop in Vyksa, 1897

Šuchov used rolled L- or Z-profiles that were formed plastically in the factory with a constant curvature, and bent elastically into their final shape on site by applying appropriate force. The question arises to what extent Šuchov was aware of these additional deformations and if he anticipated them in his design and the construction process. While it is certain that, in the case of the hyperbolic lattice towers, the elastic deformation of the bars was part of the assembly strategy, his written archives show no comments on this process, and there is little or no written evidence mentioning this strategy in his other building typologies.

In the following article, we begin by explaining the fundamentals of curvatures, comparing a curve in space and on a surface. We then deduce the corresponding deformations of members in a dense grid structure. This theory is systematically applied to four building types used by Šuchov. We conduct a geometrical analysis of these geometric networks by tracking the curvature deviations between the prefabricated and installed state and derive the resulting bending moments. Hence we can draw conclusions about the construction process, the required fabrication, and thus the geometrical approach that Šuchov developed and used.

2. Geometrical and Mechanical Fundamentals

In order to understand the geometric properties of Šuchov's gridshells, we must first differentiate the curvatures of a curve in space, and on a surface and clarify their mathematical formulation.

The **curvature k of a curve in space** can be described at any point along this curve using the local circle of curvature. It touches the curve tangentially and lies in the osculating plane in which the curve shows the greatest deviation from the straight line. The curvature k is calculated using the inverse of the radius of this circle r_k ($k = 1/r_k$).

To determine the **curvature of a surface**, we look at the local intersection curves with the normal planes of the surface. There are two orthogonal cutting directions which create the highest and lowest normal curvature at this point (Fig. 2). The Gaussian curvature K is calculated from the product of these two principal curvatures k_1 and k_2 ($\mathrm{K} = \mathrm{k}_1 \times \mathrm{k}_2$). If the curvature circles of k_1 and k_2 lie on the same side of the surface, as in a dome, they have the same sign. Their product is then positive and the surface is described as positively curved, or synclastic, such as the doubly curved gridshells in Vyksa (Fig. 2A).

If the two curvatures have opposite orientation, they are measured with opposite signs and one speaks of a negatively curved of anticlastic surface. This is the case for the hanging roof and the hyperbolic towers in Nizhny (Fig 2 B, C). In the case of a cylinder, such as the barrel vault in Grozny (Fig 2D), the surface is curved in only one direction. The second principal curvature is zero, and so is the surface curvature K.

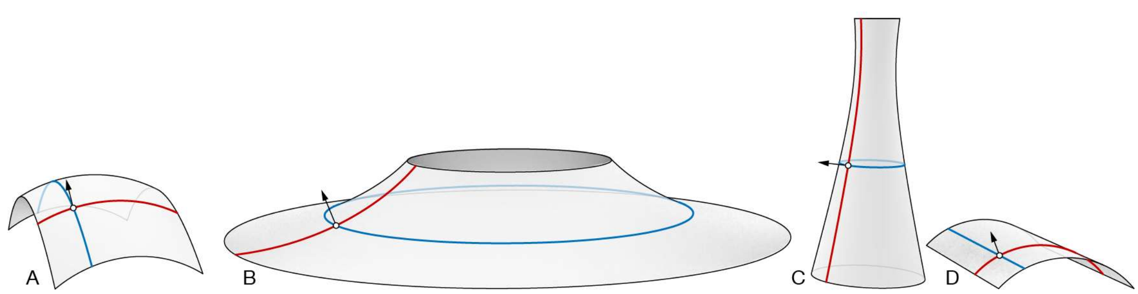


Figure 2: Illustration of surface curvatures using our four examples: The positively curved sweep surface of the gridshell in Vyksa (A), the negatively curved surfaces of revolution of the hanging roof (B), the hyperbolic lattice towers (C) in Nizhny, and the cylindrical surface of the arched lattice vault in Grozny (D).

Looking at a **curve on a surface**, we can determine a local coordinate system, the Darboux frame (Strubecker 1969), from the normal vector **n** of the surface, the tangent vector **t** of the curve and its cross-product **u** (Fig. 3, left). If we imagine moving along this path, the rotations of these three axes can be measured in radians/m. The rotation around **n** corresponds to a change in direction left or right and is called *geodesic curvature* $\mathrm{k_g}$. This curvature is independent of the surface geometry. The rotation around **u** describes the change in inclination, up or down. It is called the *normal curvature* $\mathrm{k_n}$. The rotation around **t** describes the twist along the path and is called *geodesic torsion* τ_g. Both $\mathrm{k_n}$ and τ_g are linked to the surface curvature, and are dependent on the local angle of deviation μ between the path's tangent and the principal curvature directions.

$$\mathrm{k_n}(\mu) = \mathrm{k_1}(\cos\mu)^2 + \mathrm{k_2}(\sin\mu)^2 \tag{1}$$

$$\tau_\mathrm{g}(\mu) = \frac{1}{2}\,(\mathrm{k_2} - \mathrm{k_1})\ \sin 2\mu \tag{2}$$

It is particularly important for our investigations that we distinguish between the spatial curvature k of a curve and its specific curvatures $\mathrm{k_g}$ and $\mathrm{k_n}$ on a surface. The spatial curvature k can be projected to the tangent plane **u,t** (for $\mathrm{k_g}$) or the normal plane **n,t** (for $\mathrm{k_n}$) of the Darboux Frame. The values are related through the Pythagorean theorem.

$$\mathrm{k} = \sqrt{\mathrm{k_n}^2 + \mathrm{k_g}^2} \tag{3}$$

Only when one of the two partial curvatures is zero does the other correspond to the spatial curvature.

This is the case for so-called "**geodesic curves**". They are the paths on a surface that do not exhibit any geodesic curvature. In other words, they never sway left or right (and are thus used to describe the shortest distance between two points on a curved surface). In this case $\mathrm{k_g} = 0$, and $\mathrm{k_n} = k$. In the following investigations, we use geodesics as reference curves to illustrate the geodesic curvature within Šuchov's networks.

Why are we so interested in these theoretical mathematical definitions?

Šuchov used curved steel rods in his construction. The alignment of these flat, Z- or L-profiles to the remaining grid is enforced through frequent tangential riveted connections. We can thus assume that the geometric axes x, y and z along the section resemble the Darboux Frame of a theoretical curve on a design surface. Because of this conformity, the local deformations of the steel (the 'lateral' curvature κ_z, the 'upright' curvature κ_y and the twist κ_x) are equivalent to the mathematical values $\mathrm{k_g}$, $\mathrm{k_n}$ and τ_g (Fig. 2) (Schling 2018).

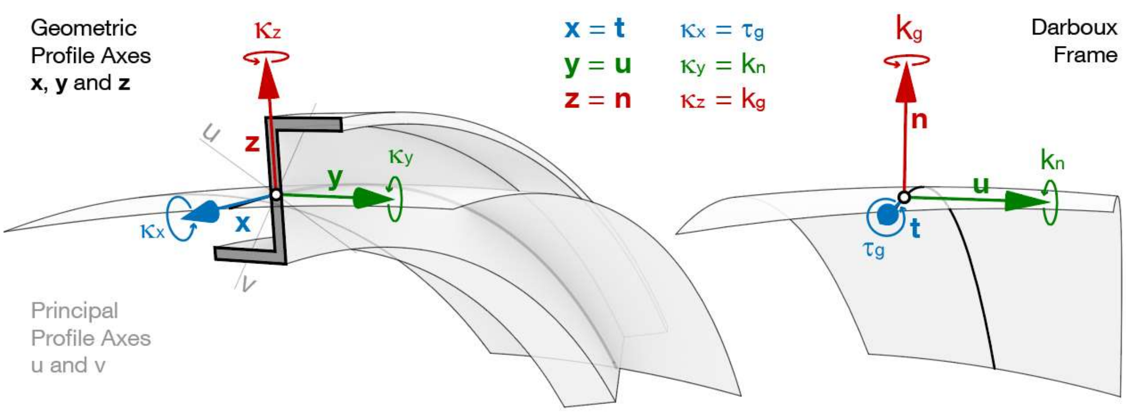


Figure 3: Geometric curvature and mechanical deformation: The geometric curvature of a surface curve can be assumed equivalent to the physical deformation of steel profiles.

If we include the mechanical properties of the steel profiles, we can use Euler's and Saint-Venant's theorem to derive the bending and torsional moments M_Z, M_Y, M_T that are necessary to deform a straight beam into the designated curvature. For our calculations, we use the material-specific properties' elasticity (E = 21 000 kN/cm²) and shear modulus (G = 8 100 kN/cm²), as well as the section-specific properties - moment of inertia I and section modulus W for each axis, which are given separately for each example. Thus we can calculate the resulting normal and shear stresses σ_z, σ_y and τ_x, which result from such a restraint.

$$k_g = \kappa_z = \frac{1}{r} = \frac{M_z}{E \cdot I_z} = \frac{\sigma_z \cdot W_z}{E \cdot I_z} \left[\frac{\text{rad}}{\text{m}}\right] \tag{4}$$

$$k_n = \kappa_y = \frac{1}{r} = \frac{M_y}{E \cdot I_y} = \frac{\sigma_y \cdot W_y}{E \cdot I_y} \left[\frac{\text{rad}}{\text{m}}\right] \tag{5}$$

$$\tau_g = \kappa_x = \frac{M_T}{G \cdot I_T} = \frac{\tau_x \cdot W_T}{G \cdot I_T} \left[\frac{\text{rad}}{\text{m}}\right] \tag{6}$$

For our calculations we differentiate between two deformed states: The *prefabricated state* in which a steel section is formed plastically to a given radius, and the *installed state*, in which the steel is bent elastically and twisted into its final position through riveted connections. Their comparison reveals unforeseen elastic behaviour, e.g. that some pre-bent profiles needed to be straightened slightly to fit the final structure.

Šuchov prefabricated the standard steel profiles either straight (in the case of the hyperbolic tower) or with a constant curvature around the y-axis, and had the rivet holes punched out in the factory before the components were assembled on site (The Engineer 1897). This procedure is essential because it implies that the final position of joints, and thus the whole geometry of the grid, was determined to a high level of precision during fabrication, and this resulted in some substantial enforced deformations and local stresses in the final structures.

We do not conduct an in-depth Finite Element-analysis, but simply use the curvature values to deduce their corresponding bending moments, in order to highlight fundamental effects and interpret Šuchov's construction strategies. We are aware of the simplifications this entails:

- The alignment of the curvature Darboux Frame, with the local profile axes of bending, is a theoretical assumption, which is true for an infinitely dense grid. In reality, we cannot eliminate the possibility of out-of plane deformations between the rivet connections. Here an evasive geometry to weaker bending axes is thinkable.
  This is especially relevant for L- and Z-profiles, where the principal axes **u** and **v** do not coincide with the geometric axis used in our calculations. The unsymmetrical bending stiffness can have an influence on the individual elastic behaviour of steel members in-between rivet connections.
  However, Šuchov's grid structures are dens and exhibit continuously smoothly curved structural members. Apart from the hyperbolic towers, the steel grids are covered with a roof cladding that further enforces the tangentially orientation. We therefore consider it safe to assume a general alignment with the design surface.
- The members in Šuchov's structures are assembled laterally on two levels, above and below the theoretical design surface, which results in a minor eccentricity of the centre of gravity to our centre line. We list the eccentricity for each example, but use the original network on the design surface for our calculation of curvature and bending moments.

3. The Hyperbolic Tower – Water tower in Nizhny Novgorod, 1896

The hyperbolic lattice towers are ideal for our first and simplest geometric analysis, as they are assembled from straight lengths of steel sections. Their spatial curvature k and their network curvatures k_n and k_g are zero in the prefabricated as well as the installed state. Our investigation focuses exclusively on their geodesic torsion and the corresponding mechanical twisting. Let us first look at a unitized rotational hyperboloid and the geometric curvature of its surface and rulings (Fig. 4):

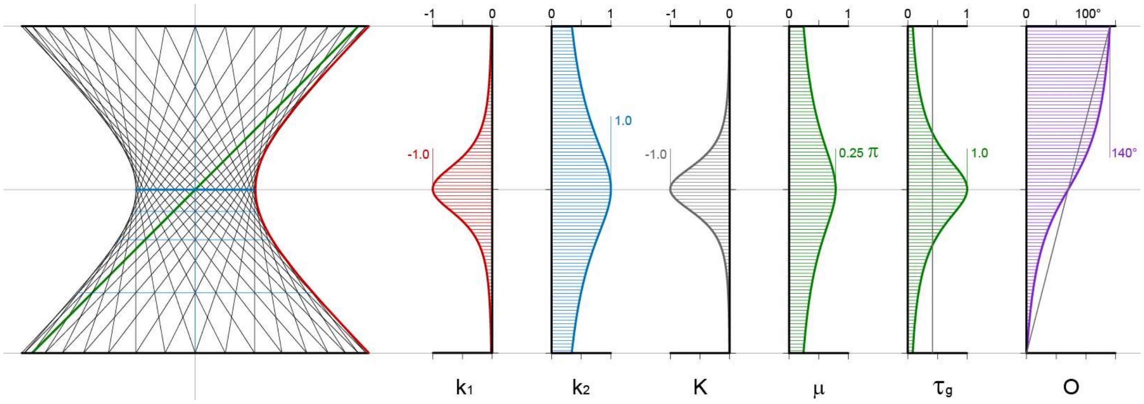


Figure 4: Hyperboloid: Curvatures of a unitized hyperboloid ($x^2 + y^2 - z^2 = 1$) and the resulting twist of its rulings.

A unitised hyperboloid is described by the equation $x^2 + y^2 - z^2 = 1$. The meridian hyperbola (k_1, red) has a maximum curvature of -1.0 at the waist and an opening angle that is approaching 45° at the extremities. The horizontal rings have a maximal curvature of 1.0 m^{-1} (k_2, blue), at the waist and increases towards the top and bottom.

This results in a maximum Gaussian curvature ($K = k_1 \times k_2$) of -1.0 along the waist line. The angle μ between the rulings and the principal curvature direction (in the meridian) is 45 ° at the waist and decreases towards the extremities. The geodesic torsion τ_g of the rulings can be calculated from k_1, k_2 and μ (see equation 2). Its integral, O, measures the accumulated twist of the steel section at a certain height. The torsion is far from linear and increases significantly at the waist. For τ_g and O, we indicated a theoretical constant torsion (grey line) for comparison. It illustrates the substantial deviation to the real twisting behaviour.

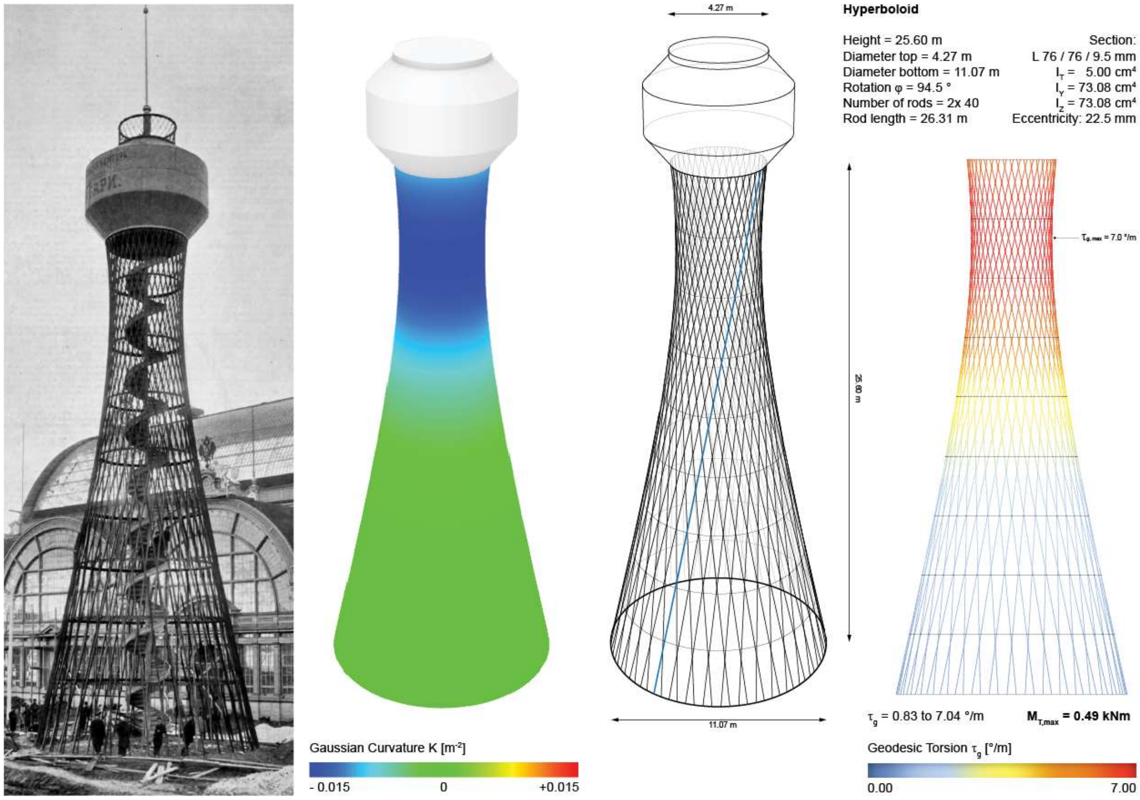


Figure 5: The Hyperbolic Tower in Nizhny (The Engineer 1897): The Gaussian curvature is a good indicator of the twisting of the steel sections, and both are a maximum at the waist.

We selected the first water tower in Nizhny Novgorod as an example to examine the effects of geodesic torsion. The geometry has been well documented (Graefe et al. 1990; Beckh and Hoheisel 2010). The torsion has even been analysed statically by Beckh: In his dissertation, he approximated a torsional moment of 0.24 kNm, based on a simplified constant elastic twist of 90° over 26.28 m length (3.4°/m) (Beckh 2012, p. 38).

Our analysis shows that the torsion is not constant, and exhibits a maximum twist at the waist, just as predicted by the unitized hyperboloid (Fig. 5). In water tower in Nizhny Novgorod, the waist is not at the mid-height of the tower, but is shifted upwards between the second and third horizontal rings. This is due to the fact that the top ring is smaller (d = 4.27 m) than the bottom ring (d = 11.07 m), and we are thus looking only at the lower portion of a symmetrical hyperboloid. The curvature analysis reveals that the steel profiles have a twist of up to 7.0°/m at the waist – more than double the amount previously suspected. Even though this twist is still in the elastic region for this particular L-section (76 / 76 / 9.5 mm), it does require a torsional moment of up to 0.49 kNm. The construction workers on site had to anticipate this work and be well equipped with the appropriate tools. Such strenuous work might even have affected the choice of geometry in Šuchov's later hyperbolic designs, with the goal to limit the elastic twisting required on site.

4. The Hanging Roof – Pavilion of Construction in Nizhny Novgorod, 1896

Let us now take a look at the rotational hanging roofs of Šuchov. Simple flat steel strips, 50.8 x 4.8 mm, were hung diagonally at regular intervals between an upper tension-ring and a lower compression-ring. We assume that the low stiffness about the y-axis had little effect on the hanging shape, and thus we use a catenary, i.e. the hyperbolic cosine (Fig. 5) in our 3D model. In order to guarantee drainage, the straps were installed in such a way that no slack formed below the lower ring, i.e. before the local minimum of the diagonal hyperbolic cosine.

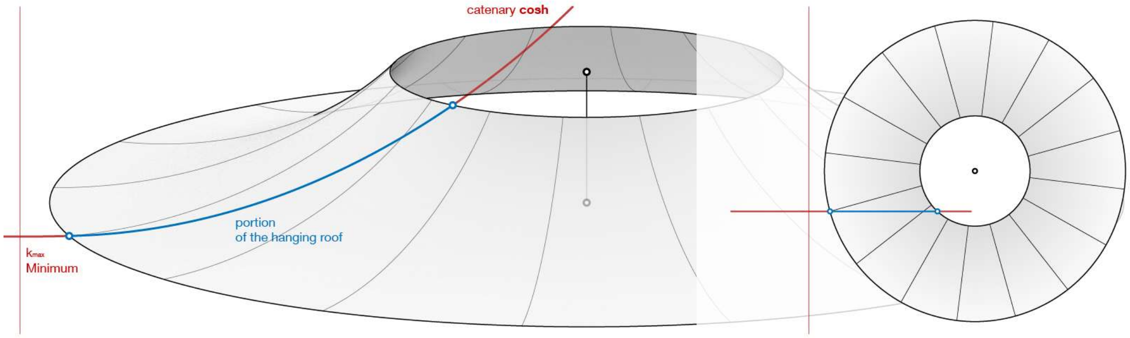


Figure 6: Geometry of the Pavilion of Construction in Nizhny Novgorod, 1896: The roof geometry of Šuchov's rotational hanging structures is derived from a diagonal catenary curve.

For the rotunda in Nizhny (Fig. 7), we use the height difference of the two rings, their radii, the angle of rotation given by Graefe (Graefe et al. 1990, p. 30) and estimate a realistic length of the steel strips. Two layers of strips were arranged in opposite directions and thus form the typical diagonal grid, which can be drawn with straight lines in plan (Fig. 8). The overlap points were calculated by Šuchov according to this geometry and the rivet holes were punched into the strips before installation (Nozhova 2016).

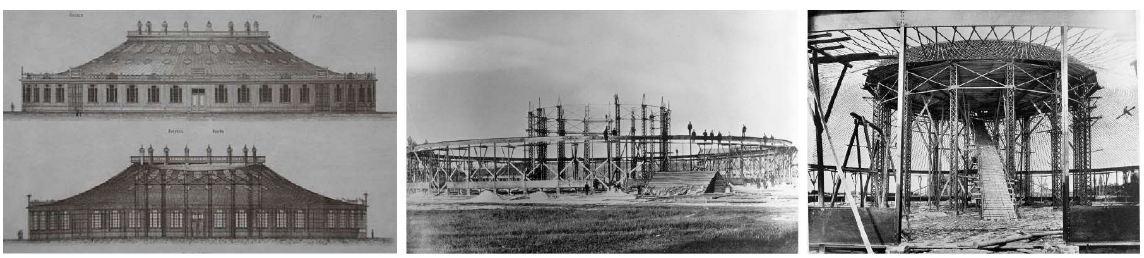


Figure 7: Pavilion of Construction in Nizhny Novgorod, 1896: Plan drawings and construction of the rotational hanging roof (Graefe et al. 1990).

When assembling this structure, the free-hanging steel strips were initially bent solely about their weak y-axis. However, once the strips were riveted together, a tangential alignment was enforced, which resulted in additional curvatures in all three axes (Fig. 9). First of all, the strips are twisted 0.4 °/m at the lower and more than 3.0 °/m at the upper edge. This torsion itself does not cause any major concerns. Based on our calculation, less than 0.01 kNm is necessary to twist the slender steel section.

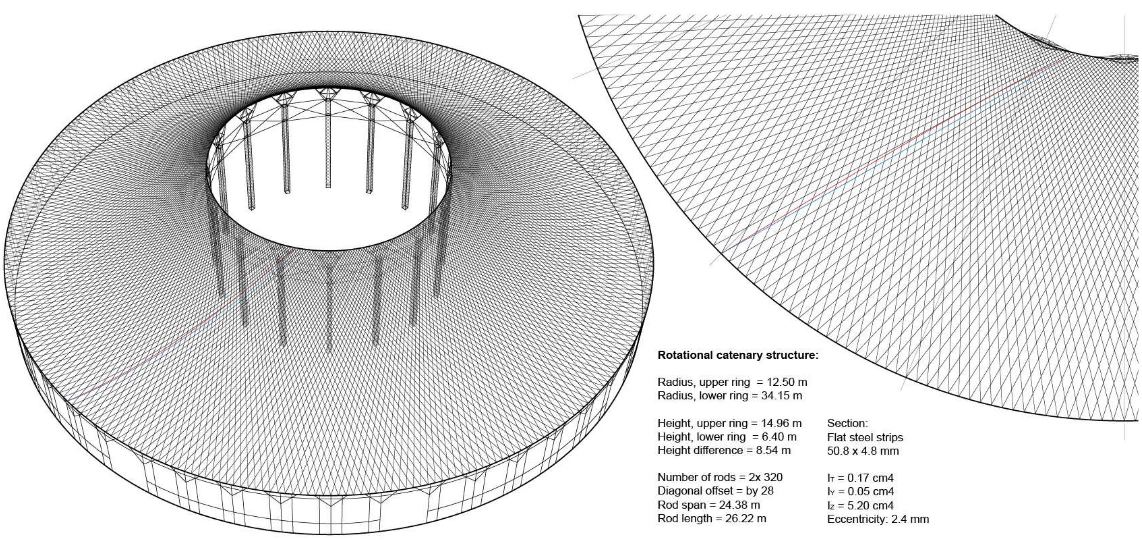


Figure 8: Geometry of the hanging roof in Nizhny: The geodesic curve (red) illustrates that the given geometry (blue) does not follow a geodesic path and will thus be subject to geodesic curvature.

The twist, however, creates further challenges, as it naturally changes the path of the steel strips. As a consequence, both normal and geodesic curvatures need to be adjusted so that the steel profiles stay on the designated path and align with the rivet holes. Specifically, the geodesic curvature (i.e. the bending around the strong z-axis) results in a bending moment of close to 0.1 kNm, a deformation which would certainly be noticeable during construction.

If we draw a geodesic line on the rotational surface connecting the start and end point of a strip (Fig. 8, right, in red), we see that it does not follow a catenary path (in blue). In the upper third, the geodesic path deviates to the degree that it touches the neighbouring gridline. It is this deviation that needs to be forced back on track by the riveted connections, leading to local deformation around the z-axis. Unfortunately, there are no published sources that comment on such a behaviour or suggest a different construction strategy, and none of the built examples have survived for us to confirm our findings.

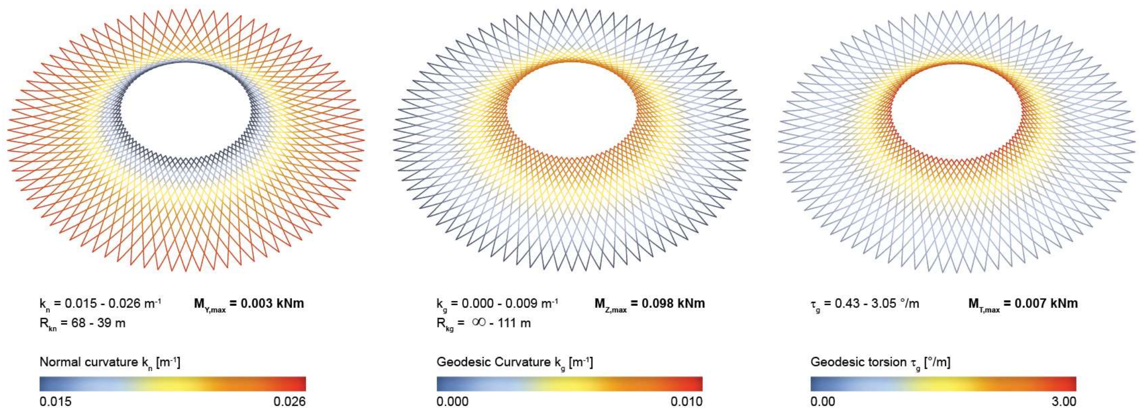


Figure 9: Curvature analysis of the hanging roof in Nizhny: The steel strips are bent and twisted around all axes to follow the catenary path. The illustration shows every fourth element.

5. The Lattice Barrel Vault - Pump Station in Grosny, 1890

Let us now look at Šuchov's arched lattice vaults. There are two plausible geometric approaches for this construction, which we will illustrate first using a semi-circular vault.

The typical plan of Šuchov's barrel vaults show a linear diamond grid, which suggests that the lattice was planned as planar arched members positioned diagonally (Fig. 10, left). However, what appears to be graphically simple reveals complexity in detail. A circular arch positioned at an angle in plan results in a slight elliptical camber in cross section. We can describe this network geometrically as a vertical projection onto an (elliptical) cylinder, which naturally results in a distortion at the steeper sides. The intersection points are therefore not at a constant distance from one another. If we unroll such a pattern, the flattened grid reveals substantial elongation and significant geodesic curvature.

A much simpler solution is the geodesic path of a helix (Fig. 10, right). Imagine a straight strip wrapping around a cylinder at an incline (visible on the cardboard centre of any toilet role). This strip naturally follows a regular spiral with constant slope and curvature. Since this curve is not planar, it is depicted as an S-curve in plan. The flattened grid, however, illustrates the regularity of the network with constant node spacing and no geodesic curvature.

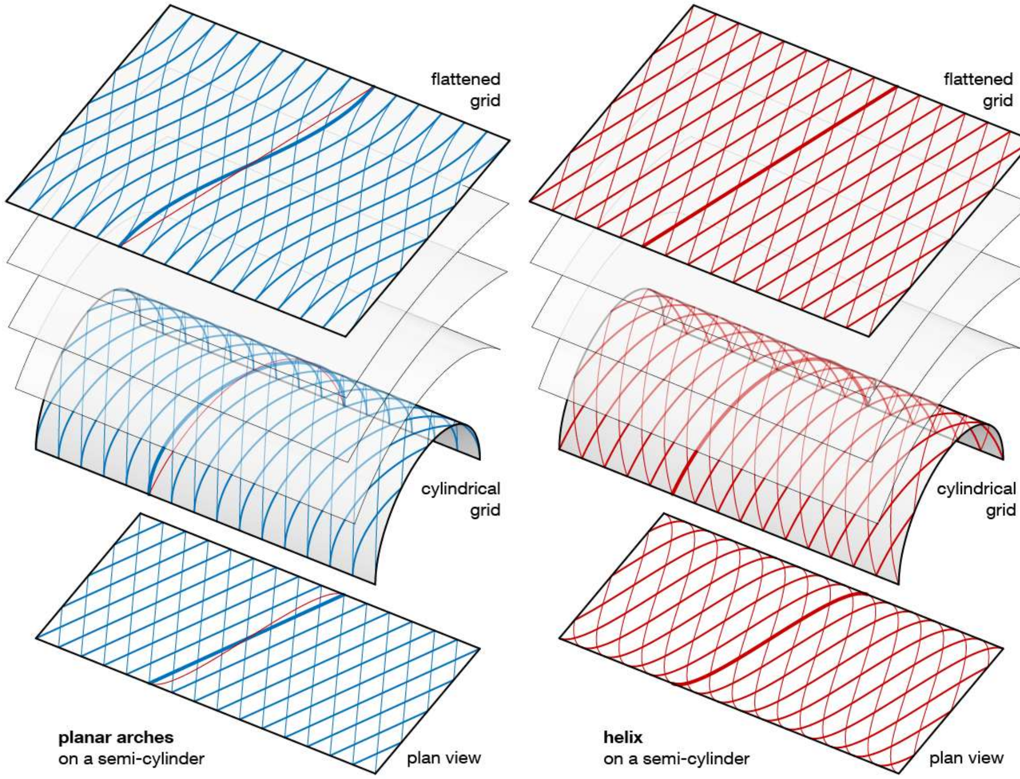


Figure 10: Comparison of a semi-cylinder with flat arches (blue, left) and a helix (red, right): The flat arches create a straight diamond grid, while the helix is seen as an s-curve in plan. The flattening of both grid reveals the geodesic behaviour with high curvature of the flat arches, and straight paths of the helix (illustration based on Tutsch 2020, p. 97).

The lattice vault of the pumping station in Grozny was built in 1890. The Z-profiles were prefabricated with constant curvature and installed in two diagonal layers spanning from edge beam to edge beam (Fig. 11).

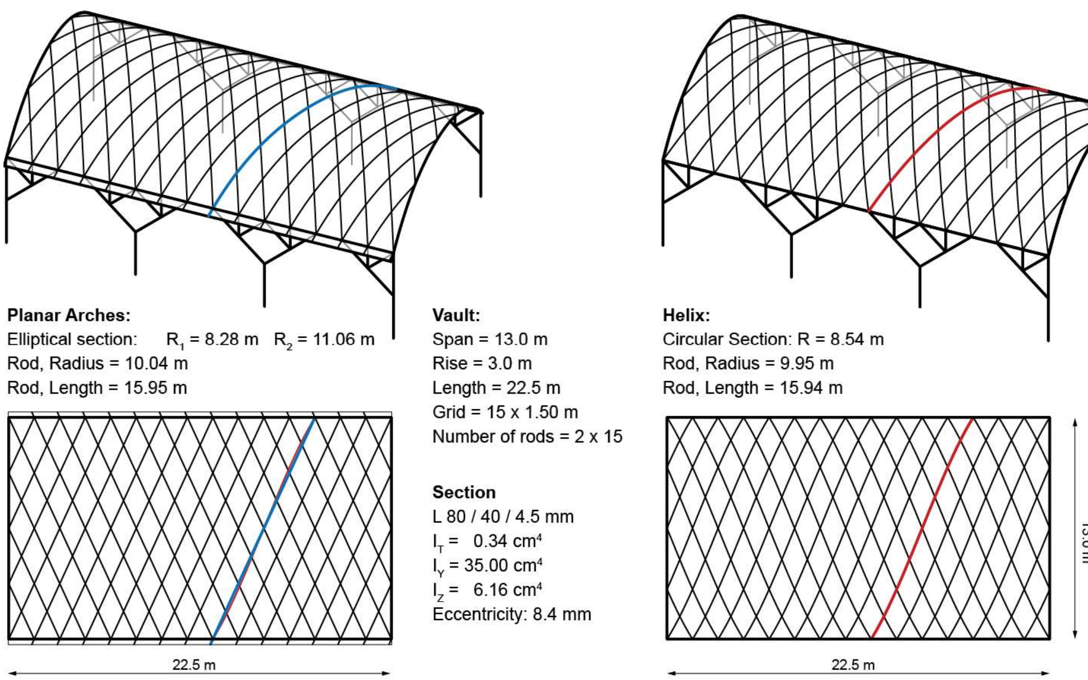


Figure 11: Two possible layouts of the lattice vault in Grozny: Straight arches (left), create a straight plan view but might complicate construction. A helix (right) will be drawn as s-curves in plan.

If Šuchov really did use planar arches as a basis for his construction planning, including the rivet holes, we can expect there would have been complex deformational challenges during construction (Fig. 12, left): In this case, the two diagonal steel members do not meet tangentially, i.e. the L-profiles exhibit an open angle between the neighbouring flanges at the intersection points. In order to press these two flanges together and close the rivets, an external torsion between 1.7 and 2.6°/m must be applied. For the L-profiles (80 / 40 / 4.5 mm) this is easily achieved with a torsional moment of just above 0.01 kNm. However, just like in the hanging roofs, the torsion leads to a change in direction, which is compensated by a geodesic curvature of up to 33 m radius ($k_g = 0.03\ m^{-1}$). The construction workers needed to apply a bending moment of up to 0.4 kNm to connect the steel sections to the eaves line. The third effect is most counterintuitive. Following the Pythagorean theorem (equation 3), the normal curvature behaves reciprocally to the geodesic curvature, thus decreasing towards the support, which necessitates a slight straightening of the circular segments and results in a negative bending moment of up to -0.36 kNm.

The helical layout on the other hand (Fig. 12, right), would have allowed for constant distance between rivet holes, as well as constant curvature around the y-axes, rolled during prefabrication. Construction workers would have simply had to twist each profile at a low, constant torsion of 2.3°/m on site.

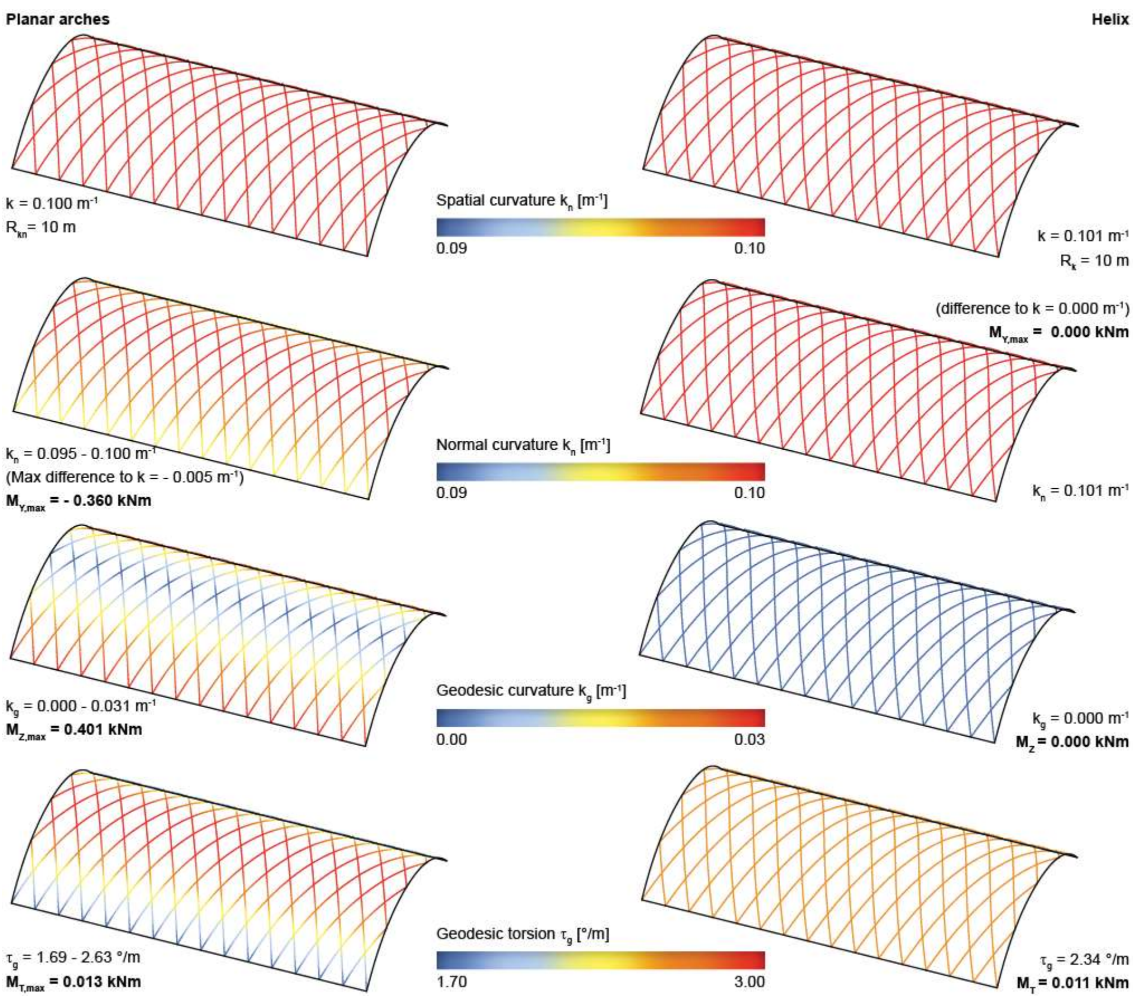


Figure 12: Curvature analysis of Šuchov's lattice vaults constructed with flat circular arches (left) or along a helix (right): The comparison shows that the construction with flat arches requires a high level of complexity and geometrical compatibility. The helix, on the other hand, enables constant normal curvature, torsion and rivet spacing, and avoids geodesic curvature altogether.

It is hard to imagine that Šuchov would not have chosen the helical layout. He was familiar with the geometry and has utilized its potential for simple prefabrication and assembly, e.g, in his helical staircases (Fig. 4, left). Several drawings by Šuchov (Graefe et al. 1990) also indicate this knowledge in respect to the lattice vaults (Fig. 13):

- A preliminary draft for patents shows two plan drawings that broach the issue of a cylindrical layout, illustrating the plan view of a helical network (top left) and another drawing that shows resemblance to the flattened grid of planar circular arch (top right).
- A sketch by Šuchov addresses the deformation of the barrel lattice (bottom, left). It shows two L-profiles in their riveted condition and marks their previous, un-deformed state. Corresponding changes in location are also marked in the plan, which might depict the geodesic curvature.
- Finally, a construction drawing of the All-Russian Factory Exhibition (bottom, right) clearly shows the expected S-curve in the plan. It is thus certain that, for this structure, the construction along the helix was chosen.

All other plan drawings show a straight diamond grid and thus contradict the helical layout. These drawings may have been deliberately simplified for a smaller scale. Sources discussing the original rivet spacing would provide a final proof of this matter.

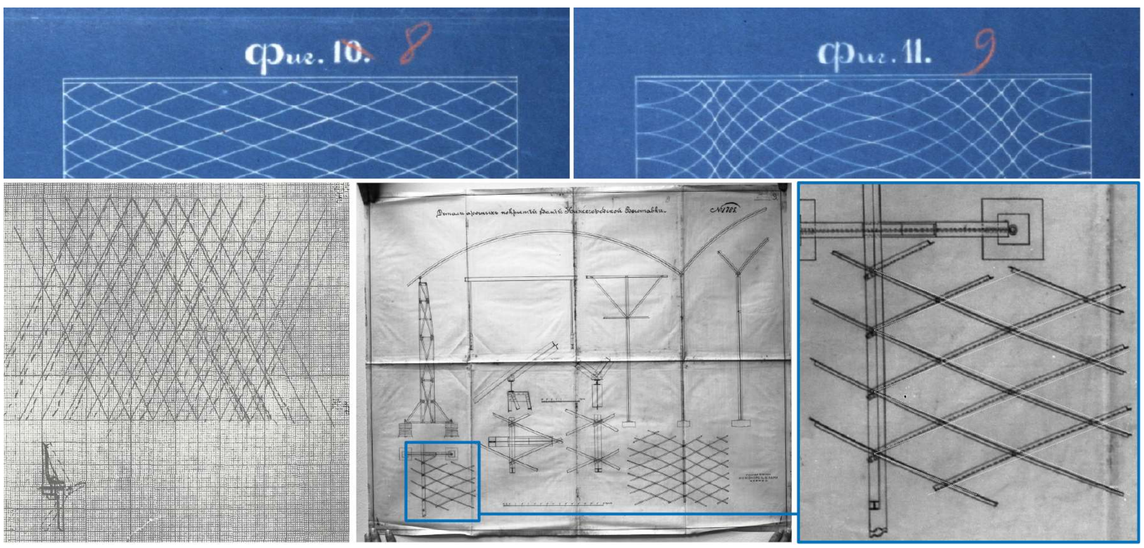


Figure 13: Drawings from Šuchov's archives addressing the helical layout and torsion in his lattice vaults (Graefe et al. 1990).

6. The Doubly Curved Gridshell – The Plate-Rolling Workshop in Vyksa, 1897

The doubly curved lattice shells in Vyksa are a further development of the barrel vaults discussed in the previous section. Instead of straight, horizontal supports, Šuchov designed a trussed frame in the shape of a parabolic arch (Barthel et al. 2009).

Matthias Beckh and Rainer Barthel analysed the geometry and construction method (Beckh and Barthel 2009) following a site visit and concluded that the most practical and therefore most likely approach, was an assembly from planar, circular segments with a constant radius and spacing along the two parallel, parabolic frames, thus creating a sweep surface. We adopt this approach for our curvature studies.

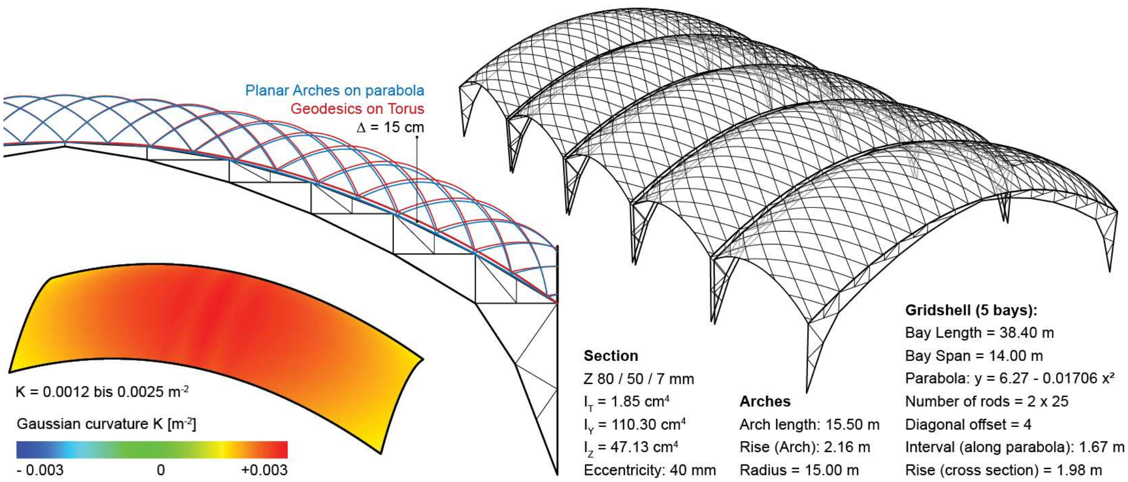
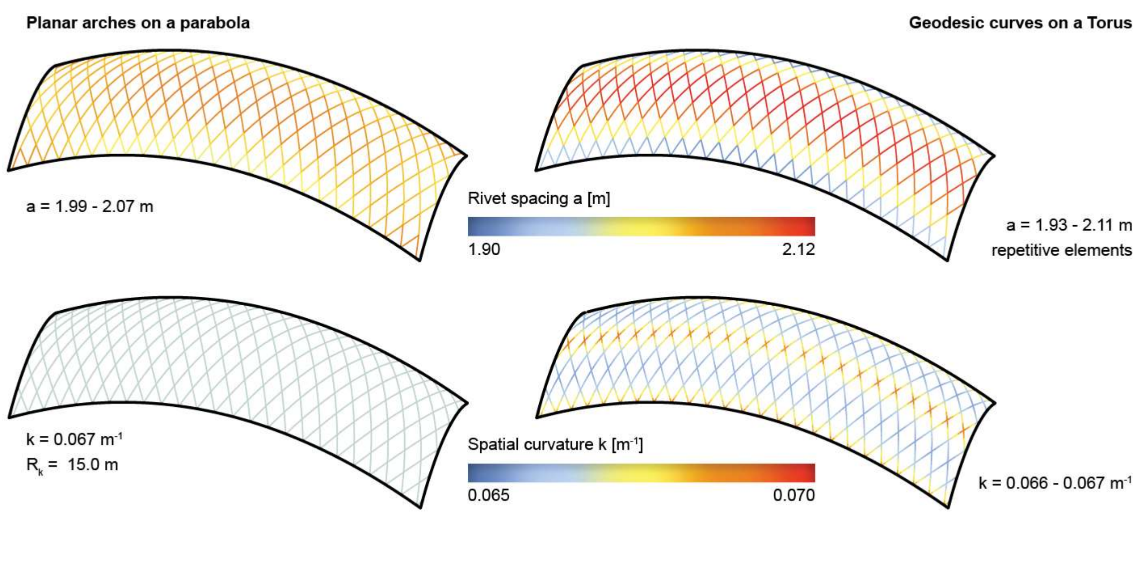


Figure 14: Gridshell in Vyksa: The planar arches are arranged at 1.67 m intervals and a diagonal layout skipping 4 bars along two parabolic curves. The elevation above shows a comparison of these arches on a parabola (blue) and geodesic curves on a torus surface (red) (see also Fig. 15).

The network shows strong similarities with the planar arch system described in the previous section. Again, there is a slight elliptical camber in cross section. The additional curvature of the parabolic base curve creates a maximum Gaussian curvature at the apex. On closer examination this network harbours many open questions.

If the riveted connections were designed and prepared in advance, then the Z-profiles had to be twisted and bent into a tangential alignment (Fig. 15, left). The maximum torsion of approx. 1.3°/m appears at the corners and can easily be introduced into the open Z-profiles. The normal curvature decreases slightly compared to the prefabricated state but gives rise to considerable negative bending moments of up to -0.27 kNm. The greatest stresses, however, are caused by the geodesic curvature. The Z-profiles have a relatively high moment of inertia about their z-axis, so that a torque of up to 1.22 kNm was required to adjust the direction of the steel at the supports – a great challenge and laborious task. Another complexity arises from the variable distance between connection points. The rivet spacing for this layout is variable not only within one arch (up to 20 cm), but also changes in adjacent arches (with 3.5 cm difference between the lowest and highest arch).



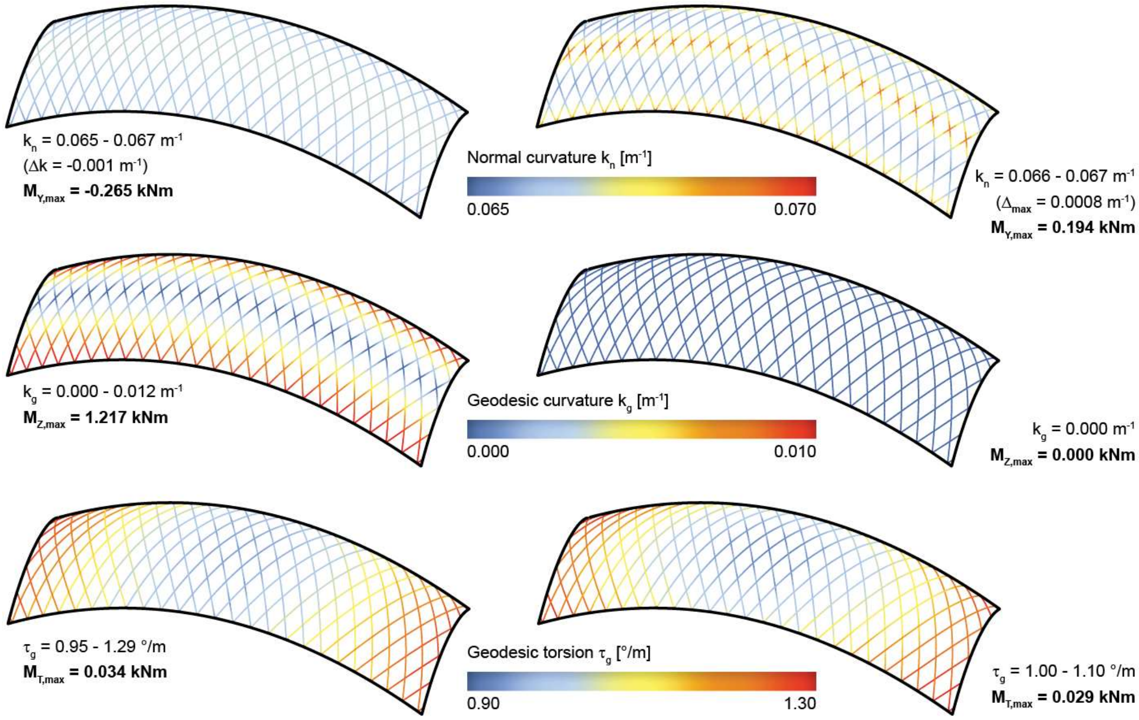


Figure 15: Curvature analysis of the Gridshell in Vyksa: Left: Planar arches on a parabola. Right: Geodesic curves on torus. The torus geometry enables repetitive prefabrication (same rivet spacing of subsequent rods), reduces torsion and normal curvature, and avoids geodesic curvature.

These observations lead to a number of questions:

- Was Šuchov aware of the complexity and individual fabrication caused by this layout?
- Were the connection points calculated beforehand or simply marked and punched on site?
- Did the network really follow the same parabola as described by the boundary frames, or was it subject to its own geometric rules?

If it was Šuchov's goal (as in all other examples) to use repetitive parts, i.e. using constant curvature elements and a common template for punching holes, and to reduce the elastic deformations that had to be applied on site, he would have considered a different geometric approach.

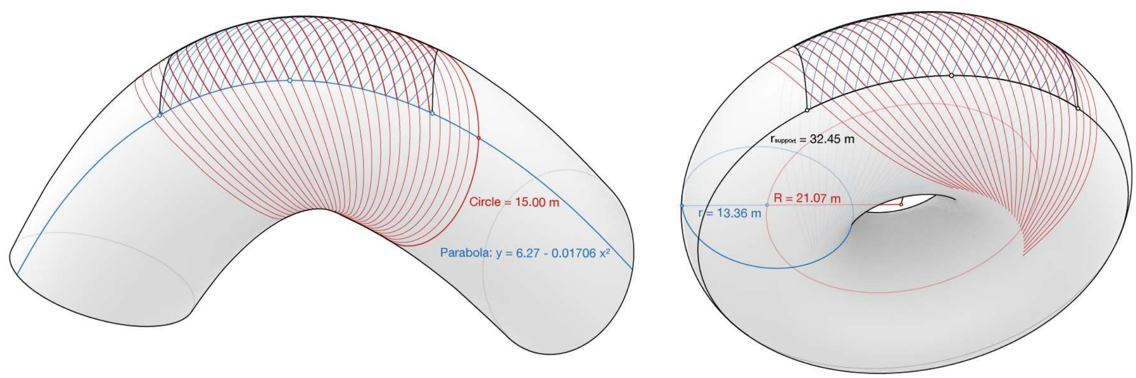


Figure 16: Comparison of geometric approaches: Left: Circular arches along a parabola. Right: Geodesic curves on a torus.

We suggest the most rational layout would be a geodesic network on the surface of a regular torus. If we consider the apex and the lower corner of the given parabolic frame as fixed points, we can define a circle with a radius of 32.45 m as alternative guide curve (directrix). This arc shows a maximum deviation from the parabola of 15 cm. A comparatively small change in geometry (which could have been easily accommodated by the steel connections). This would offer a number of possible simplifications: a repetitive prefabrication of all steel arches, with repetitive spacing of rivet holes (only within an arch the distances are not regular), and lower elastic twisting (0.03 kNm) and bending (0.2 kNm) about the y-axes.

Unfortunately, neither the design drawings nor Šuchov's writings provide any evidence of the exact procedure. A measurement of the rivet distances on site along one arch at the apex and another arch at the lower corner would give clarification.

7. Results

The geometric analysis of Šuchov's steel networks shows that elastic deformations were systematically applied during construction with the goal to simplify prefabrication and allowing the use of simple riveted connections. These deformations have been found to be much higher than current literature suggests and affect not only the local torsion-axis x of profiles, but also both bending about axes y and z, resulting in significant bending moments. The results are summarised in Table 1.

		Tower, Nizhny	Hanging Roof, Nizhny	Lattice, Grosny Planar Arches	Lattice, Grosny Helix	Gridshell, Vyksa Arches on Parabola	Gridshell, Vyksa Geodesics on Torus
M_y	$k_y * E * I_y$	0.000 kNm	0.003 kNm	-0.360 kNm	0.000 kNm	-0.265 kNm	0.316 kNm
M_z	$k_z * E * I_z$	0.000 kNm	0.098 kNm	0.401 kNm	0.000 kNm	1.217 kNm	0.000 kNm
M_T	$k_x * G * I_T$	0.498 kNm	0.007 kNm	0.013 kNm	0.011 kNm	0.034 kNm	0.029 kNm
k_{y0}	k_{n0}	0.000 rad/m	0.000 rad/m	0.100 rad/m	0.101 rad/m	0.067 rad/m	0.068 rad/m
k_{y1}	k_{n1}	0.000 rad/m	0.026 rad/m	0.095 rad/m	0.101 rad/m	0.066 rad/m	0.069 rad/m
k_y	$k_{n1}-k_{n0}$	0.000 rad/m	0.026 rad/m	0.005 rad/m	0.000 rad/m	-0.001 rad/m	0.001 rad/m
k_z	k_g	0.000 rad/m	0.009 rad/m	0.031 rad/m	0.000 rad/m	0.012 rad/m	0.000 rad/m
k_x	τ_g	7.04 °/m	3.05 °/m	2.63 °/m	2.34 °/m	1.29 °/m	1.10 °/m
Profile		L 76 / 76 / 9.5	50.8 x 4.8	L 80 / 40 / 4.5	L 80 / 40 / 4.5	Z 80 / 50 / 7	Z 80 / 50 / 7
G		8,100 kN/cm²	8,100 kN/cm²	8,100 kN/cm²	8,100 kN/cm²	8,100 kN/cm²	8,100 kN/cm²
E		21,000 kN/cm²	21,000 kN/cm²	21,000 kN/cm²	21,000 kN/cm²	21,000 kN/cm²	21,000 kN/cm²
I_T		5.00 cm^4	0.17 cm^4	0.34 cm^4	0.34 cm^4	1.85 cm^4	1.85 cm^4
I_Y		73.08 cm^4	0.05 cm^4	35.00 cm^4	35.00 cm^4	110.30 cm^4	110.30 cm^4
I_Z		73.08 cm^4	5.20 cm^4	6.16 cm^4	6.16 cm^4	47.13 cm^4	47.13 cm^4

Table 1: Data Statement: Table of material, section and curvature values and their calculation.

This analysis offers new insights on the geometric approach Šuchov chose for his designs.

- The torsion in the hyperbolic lattice towers is not constant and proves to be significantly higher than previously assumed, especially in the region of the hyperboloid's waist line. This has the potential to affect his choice of geometry for later designs.
- The steel strips of the hanging roofs in Nizhny reveal not only a twist, but also a change in curvature about both the x and y axes that had to be applied during construction.
- Šuchov's lattice barrel vaults were most likely planned along a helix. With this geometric trick, Šuchov achieved a constant rivet spacing and constant twisting while allowing for prefabrication using L-sections of constant curvature.
- Based on the latest geometric survey of Šuchov's gridshells in Vyksa in 2009, the grid construction demanded a high level of complexity both in punching individual rivet holes during prefabrication and in deforming the Z-section elastically during assembly on site by applying bending moments of up to 1.2 kNm. We consider it likely that Šuchov followed a different, repetitive, geodesic layout, similar to his lattice barrel vaults, along a torus geometry, in order to lower elastic deformations significantly and thus to simplify fabrication.

8. Conclusion

Šuchov's construction planning was extremely precise and anticipated geometric effects down to the millimetre. It is safe to assume that Šuchov was aware of, and intentionally designed for an elastic deformations during construction, carefully considering where the workers would have to apply bending and torsional moments, even if he did not quantify this mathematically, relying instead on practical experience on site.

These findings broaden the understanding of Šuchov's structures. Not only did he use regular, rotationally symmetrical grids to allow the use of identical components, he also skilfully played with the symbiosis of plastic prefabrication and elastic deformation on site to find the most pragmatic hybrid solutions. Šuchov's flexible way of thinking can be seen, not only as a precursor of Frei Otto's strained timber gridshells, but also as a fundamental approach to rationalizing the construction of curved steel construction. Designing for elastic deformation – in the right measure - has the potential to simplify the fabrication process significantly while maintaining a resilient load-bearing structure. Computational methods today allow us to control the curvature of surface and network to a high degree during the design phase and, thus, open up new potential for this elastic approach (Schling 2018).

Future Research

The modelling and calculation of curvatures and bending moments in this paper is based on a theoretical, perfect geometry. It remains unclear how tolerances in the factory affected the elastic behaviour and whether our calculated effects were noticeable to the same extent on site. Furthermore, when elastic torsion and bending is enforced at regular points in slender open sections, a degree of geometrical adaption between these points through warping or buckling might arise. Specifically for open L- and Z-profiles, the influence of the principal axes on the bending behaviour needs to be investigated further using Finite Element Analysis.

Finally, the curvature analysis is used to derive new hypotheses on Šuchov's construction and design decisions. A systematic survey of the layout of rivet holes would provide evidence for our predictions and could finally clarify the design, prefabrication and on-site construction process.

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