Income Risks and Optimal Attention-Consumption Allocation*

Yulei Luo†
University of Hong Kong

Penghui Yin‡
Goethe University Frankfurt

May 4, 2020

Abstract

This paper studies how agents allocate their limited attention between capital income and labor income risks in a two-period consumption-saving model with recursive utility. Specifically, we examine how the optimal attention and consumption-saving decisions are affected by the key model elements including the attention and wealth endowments, the risk and time preferences, and the amount of income risks. We also find that the simple model can have the potential to explain the consumption responses to the income risks and the relative volatility of consumption to income observed in the U.S. economy. Finally, we find that the welfare losses due to limited attention are insignificant.

Keywords: Capital Income and Labor income Risks; Optimal Attention Allocation; Consumption and Saving Decisions.

JEL Classification Numbers: C61; D83; E21.

*We thank Jean Roch Donsimoni and Leyla Jianyu Han for helpful comments and discussions related to this paper. Luo thanks the General Research Fund (GRF, No. HKU17500117 and HKU17500619) in Hong Kong for financial support. All remaining errors are our responsibility.

†Faculty of Business and Economics, The University of Hong Kong, Hong Kong. E-mail: `yulei.luo@gmail.com`.

‡Department of Money and Macroeconomics, Goethe University Frankfurt, Germany, E-mail: `penghui.yin@hof.uni-frankfurt.de`

1 Introduction

A consensus seems to be emerging that attention is a scarce resource that restricts economic and financial decisions, and consequently, ordinary consumers face not only budget constraints but also attention constraints. Given that consumers have limited attention capacity, it is natural to ask how they optimally allocate their limited attention across different aspects in their decision-making problem. Intertemporal consumption-saving choice is a fundamental topic in modern economics and finance. It is therefore important for us to understand how departures from unlimited attention affect an individual's consumption-saving decisions and the policy and welfare implications. The purpose of this paper is to construct a consumption-saving model with recursive utility and both capital income and labor income, and study how a consumer with limited attention –thought of as "inattention" due to information-processing constraints– would make optimal attention and consumption-savings decisions and their policy and welfare implications.

Sims (2003) first introduces the rational inattention hypothesis into economics by arguing that agents only have finite information-processing ability when processing economic-related variables. He also argues that rational inattention is a plausible way for introducing slow adjustments, randomness, and delay into economic models. In his formulation, agents have limited Shannon channel capacity, limiting their ability to process available information when making decisions. In addition, rational inattention introduces endogenous noises due to finite capacity into economic models. As a result, they only react to the shocks gradually and with delay, and face more uncertainty about the states. Another important implication of limited attention is that attention is a scarce resource and agents have to face a related attention allocation problem. For example, Peng and Xiong (2006) find that limited attention leads to category-learning behavior, i.e., investors tend to process more market and sector-wide information than firm-specific information. Maćkowiak and Wiederholt (2009) show how monopolistic competitive firms allocate their limited attention between idiosyncratic and aggregate shocks when setting optimal prices. Mondria (2010) and Van Nieuwerburgh and Veldkamp (2010) discuss how attention allocation affects portfolio choice. Maćkowiak and Wiederholt (2015) find that a business cycles model with inattentive households and firms can generate the observed impulse responses to monetary policy shocks and aggregate technology shocks.[1] A key feature of these studies is that the multiple shocks that limited attention is

[1]Coibion and Gorodnichenko (2008) and Andrade and Le Bihan (1997) use the survey data on the expectations of households and professionals on inflation, unemployment, and real GDP growth to test the degrees of information rigidity and find that the rational inattention theory is supported by the survey data.

applied to are *symmetric*.[2] In addition, Sims (2003), Luo and Young (2014), Miao, Wu, and Young (2020) apply rational inattention theory into linear-quadratic consumption models with *additive* labor income risks but they do not model capital income risk and recursive utility. However, in reality many households face not only labor income risk, but also capital income risk.[3] There are many empirical studies in the literature on how consumption responds to different types of changes in labor income including expected, unexpected, permanent, or transitory, aggregate, or idiosyncratic changes (See Jappelli and Pistferri (2010) for a survey and Deaton (1993) for a textbook treatment on this issue). However, there are relatively few studies on how consumption responds to both capital income and labor income risks.[4] One exception is Christelis, Georgarakos, and Jappelli (2015), and they attempt to estimate the separate impact of three different shocks, shocks to stocks and housing and unemployment shocks, on households' expenditures using recently available microdata. In Section 2, we will use the same data set as in Christelis, Georgarakos, and Jappelli (2015) to estimate how consumers with heterogenous degrees of wealth and expectations on future unemployment react differently to the shocks to financial and housing wealth and the unemployment shocks.[5]

Furthermore, recursive utility is often employed in the macro-finance literature in order to disentangle the effects of elasticity of intertemporal substitution and relative risk aversion on consumption-investment decisions. However, we still do not know so well about how these two attitudes affect attention allocation to different income shocks. In this paper, we seek to make progress in filling this void by solving and inspecting an attention-consumption allocation problem of consumers who have recursive utility and face both capital income and labor income risks. When facing these two risks, how will rationally inattentive consumers allocate their limited attention to these two income risks? How will this optimal attention allocation affect their consumption-saving decisions? In addition, what are the policy and welfare implications of this additional attention allocation problem? Specifically, in this

[2] In Mondria (2010) and Van Nieuwerburgh and Veldkamp (2010) stochastic asset returns enter the budget constraint by multiplying their corresponding portfolio share. In Maćkowiak and Wiederholt (2009) aggregate and idiosyncratic shocks enter the profit-maximizing price in additive terms.

[3] As shown in Bertaut and Starr-McCluer (2002), about 49% of US households invest directly or indirectly in stocks and other risky assets.

[4] There are also some empirical studies on how consumption responds to capital income shocks. The results based on micro-data are mixed, with some papers finding large responses of expenditure to house and stock prices shocks, while others find smaller effects. In addition, Sinai and Souleles (2005), Campbell and Cocco (2007), and Attanasio, Blow, Hamilton, and Leicester (2009) find that the consumption response to changes in capital income is quite heterogeneous across the population.

[5] See Tables 1-3 for the detailed empirical results.

paper, following Sims (2003), we assume that consumers have limited Shannon capacity to process information regarding income shocks, such that they face a trade-off between paying attention to labor income risk and paying attention to capital income risk.

As the first contribution of this paper, we construct and solve a two-period consumption-saving model in which there are: (i) two fundamental risks, labor income risk and capital income risk; (ii) recursive utility; and (iii) limited attention. It is worth noting that the two-period specification is tractable and useful because almost all of the key issues about optimal attention-consumption allocation between the two income risks can be examined within this two-period setting.[6] More specifically, using the two-step backward procedure adopted in Maćkowiak and Wiederholt (2009), we solve for the optimal attention-consumption allocation numerically.

Second, using the optimal solution, we then examine how the optimal attention and consumption-saving decisions are affected by the key model parameters: (i) the amount of exogenous income risks, (ii) the attention and wealth endowments, and (iii) the risk and time preferences. Specifically, holding other parameters unchanged, we find that agents allocate more attention to labor (capital) income risk as its prior variance increases relative to the prior variance of capital (labor) income risk, which is not surprising and is in line with the existing results in the rational inattention literature. What is more interesting is that agents allocate more attention to the *multiplicative* capital income risk than to the *additive* labor income risk even when the prior variance of labor income is higher than that of capital income.[7] Furthermore, we find that the optimal attention devoted to monitoring capital income risk is increasing with the level of initial wealth and the discount factor, i.e. rich and patient households pay more attention to capital income risk. The reason is that the rich and patient individuals hold more risky wealth, which makes future capital income more volatile. We also find that when attention is very scarce, agents pay more attention to capital income risk; as the attention capacity increases, agents pay more and more attention to labor income risk and eventually pay equal amounts of attention to both income shocks when attention capacity is sufficiently large.

Third, to tackle the different effects of the risk and time preferences on optimal attention and consumption allocation, we introduce a Kreps-Porteus-Seldon type recursive utility to fully separate the elasticity of intertemporal substitution (EIS) from the coefficient of relative

[6] Leland (1968), Sandmo (1970), Kimball and Weil (2009), and Seldon and Wei (2018) also adopt the two-period setting to examine the effects of income uncertainty on savings.

[7] Note that a common conclusion in the previous studies on rational inattention is that when prior variances of different risks are the same, agents would pay the same amount of attention to each risk.

risk aversion (CRRA). Angeletos (2007) and Wang, Wang, and Yang (2016) find that these two parameters have different effects on the consumption-saving allocation. In this paper, we find that a reduction in both the EIS and CRRA increase the optimal attention allocated to capital income risk.[8]

Fourth, the optimal solution allows us to examine how the expected saving rate and consumption growth are affected by the key parameters mentioned above. In addition, our model can also generate some important testable implications. For example, we find that our model can have the potential to explain the patterns of the consumption responses to capital income and labor income risks found in the U.S. microdata. In addition, we also find that our model can explain the observed relative consumption inequality in the U.S. economy.[9]

Finally, we examine the taxation policy and welfare implications of attention allocation. When introducing linear tax rates on capital income and labor income, we find that an increase in capital (labor) income tax rate will lead to a reduction in the attention amount allocated to the capital (labor) income risk. The intuition is that the increases in the taxes on one income reduce the disposable amount of capital (labor) income, which makes the agents switch attention to the other risk. In addition, we also find that the expected saving rates are increasing with the tax rates due to the precautionary motive. We find that the welfare losses due to limited attention are insignificant and the agents with low initial levels of wealth and attention can benefit more from increasing the attention capacity.

The remainder of this paper is organized as follows. Section 2 provides an empirical motivation for this paper. Section 3 describes our model setup, introducing key elements step by step. Section 4 presents key results of the expected utility case and discusses the implications of attention allocation for the consumption-saving dynamics. Section 5 presents the results of a recursive utility case. Section 6 examines taxation policies and welfare implications of attention allocation. Section 7 concludes.

[8]Luo, Nie, Wang, and Young (2017) and Yin (2019) also discuss the effects of the EIS and the CRRA on the optimal attention amount. However, since they only consider one-type risk and have no attention allocation problem, their results can not be compared with that obtained in this paper.

[9]In this paper, we use "inequality" and "dispersion" interchangeably to describe the cross-sectional distributions of changes in consumption and income.

2 An Empirical Motivation: Consumption Response to Shocks to Capital and Labor Income

In the previous literature of consumption dynamics, we often see discussions about how consumption responds to permanent and transitory labor income shocks. However, few empirical studies investigate how consumption responds to different types of income shocks, such as labor income and capital income shocks. One recent paper that discusses this issue is Christelis, Georgarakos, and Jappelli (2015), who use data from the 2009 Internet Survey of the Health and Retirement Study (HRS) and show that, on average, for every 10% loss in housing and financial wealth, the estimated drop in household expenditure was about 0.56% and 0.9%, respectively. Those who became unemployed reduced spending by 10%. At first glance, consumers respond to labor income is much stronger than to capital income shocks because according to their results, the estimated drop in households expenditure due to unemployment is larger than that caused by a huge loss in the value of assets. But we are interested in why labor income shocks are so important in household's consumption decisions, and whether there exists heterogeneity in households' consumption responses.

To achieve this, we use the same dataset as that in Christelis, Georgarakos, and Jappelli (2015) and also investigate how reduction in the value of assets and being unemployed affect the elasticity of expenditure. However, we work one step further and check what factors drive these individuals react more strongly to unemployment shocks than to reductions in asset value. We mainly focus on two characteristics, namely prior variance of labor income and wealth. First, we use the information of the likelihood of losing a job in their previous interview as a proxy for the prior variance of labor income shock. Here we argue that individuals who answered in their previous interview (2006 wave) that they are likely to lose their jobs have higher prior volatility in their future income than those who answered they will not lose their jobs. Second, we want to investigate whether people with different wealth react differently to different types of income shocks. In the data we find two measures for wealth, financial asset holding and cash on hand, which is the sum of financial wealth and current income. In the following subsection, we will discuss details of the data and main results.

2.1 Data

The empirical analysis in this section is based on two micro-data surveys. The first data source is the HRS main survey in 2006 and 2008. It is a longitudinal, nationally representative survey interviewing respondents aged fifty and above in the U.S. economy. The survey is conducted on a biannual basis since 1992 and it provides information on households demographic characteristics, income, and asset holding.[10] The second source is the HRS internet Survey, which is conducted from March 2009 to August 2009, and contains $4,415$ respondents belonging to $3,438$ households. To reduce the possibility that estimates are affected by outliers, we delete observations with percentage changes in consumption over 0.85.

For the purpose of this paper, an important feature of the 2006 wave of the HRS main Survey is that respondents are asked about their expectations regarding the likelihood that they will lose their jobs in the future. An important feature of the 2009 Internet survey is that they are asked to report percentage changes in their total spending compared to the previous year, percentage changes in the value of their house and financial assets.[11] Table 1 provides summary statistics on key socioeconomic characteristics. The mean age is about 62. In 2007 and 2008 around 4.74% of these respondents or their partners became unemployed. The average income was about $71,320$ dollars, while the average financial asset is $273,693$ dollars. In subsection 2.2 below, we use cash-on-hand defined as the sum of current income and financial wealth to measure total wealth. In the 2006 HRS survey, about 44% of respondents think that there is no chance of being unemployed in the next year, and on average the likelihood of losing a job is 14.2%.

2.2 Estimation Results

Following Christelis, Georgarakos, and Jappelli (2015), the consumption effects of unemployment and changes in values of house and financial assets can be studied in a linear specification. The estimation model is specified as follows:

$$\frac{\Delta C_{it}}{C_{i,t-1}} = \alpha + \beta\frac{\Delta HW_{it}}{HW_{i,t-1}} + \gamma\frac{\Delta FW_{it}}{FW_{i,t-1}} + \delta\Delta U_{it} + \xi\Delta X_{it} + \epsilon_{it}, \tag{1}$$

where i denotes individual households, the term on the left-hand-side of the equation is percentage change in consumption, the second and third terms on the right-hand-side are percentage changes in the values of housing and financial wealth, ΔU indicates whether an

[10] The details about the survey can be found in Hauser and Willis (2005).

[11] Respondents also report the amount of change in the value of their house compared to its value in the summer of 2006. For each assets owners of employer retirement saving plans (incl. 401k's), individual retirement accounts (IRAs) or Keogh plans, investment trusts, mutual funds, directly held stocks, they are asked to report the percentage decline of the asset value since September 2008, which was the month in which Lehman Brothers collapsed. The discussion regarding biased estimation due to measurement error in Christelis, Georgarakos, and Jappelli (2015) also holds here in our analysis.

individual becomes unemployed, other changes over time in a vector of demographic and economic variables X, and ϵ_{it} is an error term.

Table 2 reports the elasticities derived from linking the percentage changes in the values of the two assets and the semi-elasticity of consumption to being unemployed. Columns 1 and 3 show results for respondents who have low prior variance about their labor income, i.e. those who believe that there is no chance they will lose their jobs in the next year. Columns 2 and 4 report results for respondents who have high prior variance about their labor income, i.e. those who think there is a chance of losing their job.

From the table, we can see heterogeneous responses of consumption to changes in values of wealth and employment status. When comparing these two groups of observations, those who have low prior volatility in labor income react more strongly to being unemployed, whereas those who have high prior volatility in labor income react more strongly to losses in financial wealth. To understand this heterogeneity, we first recall one of the core prediction of the life-cycle theory of consumption is that changes in consumption react more strongly to unexpected shocks in wealth and income. Therefore, one explanation for the above results is that unemployment shocks are more unexpected for individuals who ex ante believe that there is no chance to lose their job. Oppositely, reductions in financial wealth are better predictable for individuals who believe that there is no chance to lose their job. We can link these more/less unexpected shocks to rational inattention theory: for shocks A and B with the same prior variance, if an agent pays more attention to shock A than to B, then she has lower posterior uncertainty in A than in B. In Section 3 below, we will show how to use a rational inattention model with both labor and capital income shocks to explain these empirical facts. When the agent has a sufficiently large prior variance in labor income shock relative to that of capital income shock, she will pay more (less) attention to the labor (capital) income shock, which means that labor (capital) income shock is more (less) predictable. Table 2 shows that changes in the values of their house have no significant effect on consumption changes. However, in Christelis, Georgarakos, and Jappelli (2015) the elasticity of consumption with respect to the value of their house is roughly equal to 0.056. We also notice that controlling income and asset holding does not change our main results.

Table 3 reports another interesting heterogeneity in the consumption response to changes in the values of their house and financial asset and unemployment status. From these results we can see that rich people (both high cash-on-hand and financial wealth) tend to react more strongly to labor income shock (being unemployed) and less strongly to capital income shock. We can again link this empirical fact to individual's attention allocation to different income shocks because they have always paid more attention to capital income risk. For example, from previous studies, such as Dynan, Skinner, and Zeldes (2004), we know that US data shows a positive association between lifetime income and the saving rate. This means that wealthier individuals invest a larger share of their resources, and financial income tend to attract more attention. As a result, they pay less attention to their labor income shocks and react more strongly to such unexpected shocks when they become unemployed.

As standard life-cycle consumption models did not discuss these two empirical facts simultaneously, in the following sections we try to explain these facts by introducing rational inattention theory into an otherwise standard consumption-saving model.

3 A Consumption-Saving Model with Recursive Utility and Attention Allocation

In this section, we first describe households' preferences, budget constraints, and two fundamental shocks they face: shocks to capital income and labor income. We then discuss how to incorporate rational inattention due to information-processing constraints into the otherwise standard two-period consumption-saving model. The two-period model specification is tractable and useful because almost all of the key issues about attention allocation between two income risks can be examined in the two-period setting.

3.1 Households' Preferences and Budget Constraints

To address how risk aversion and intertemporal substitution affect the optimal attention-consumption allocation, we follow Kimball and Weil (2009), Bommier and Le Grand (2019) and Seldon and Wei (2018) in assuming a general KPS (Kreps and Porteus (1978) and Seldon (1978)) preference structure as well as the two-period specification. Specifically, in the model economy, households live for two periods: $t \in \{0, 1\}$, and have the following recursive utility:

$$U = u(C_0) + \beta u \left(V^{-1}(\mathbb{E}V(C_1)) \right), \tag{2}$$

where $\beta \in (0, 1)$ denotes the households' subjective discount factor, C_0 and C_1 are consumption in periods 0 and 1, respectively. It is worth noting that (2) is equivalent to the following recursions:

$$U = u(C_0) + \beta W^{-1}(\mathbb{E}[W(U_1)]), \text{ or} \tag{3}$$

$$\widetilde{U} = u^{-1}\left[u(C_0) + \beta u\left(V^{-1}\left(\mathbb{E}\left[V\left(\widetilde{U}_1\right)\right]\right)\right)\right] \tag{4}$$

where $W = V \circ u^{-1}$, U_1 is future uncertainty utility, and $U = u\left(\widetilde{U}\right)$. (Using (3) or (4) is just a matter of normalization.) The functions, $u(\cdot)$ and $V(\cdot)$, that govern the preferences for intertemporal substitution and risk aversion are characterized by the CES certainty and CRRA risk preferences functional forms, respectively. Specifically, we assume that:

$$u(x) = \frac{x^{1-1/\psi}}{1-1/\psi} \text{ and } V(x) = \frac{x^{1-\gamma}}{1-\gamma}, \tag{5}$$

where γ is the coefficient of relative risk aversion (CRRA), whereas ψ is the elasticity of intertemporal substitution (EIS) (i.e., $1/\psi$ is the relative resistance to intertemporal substitution.) This recursive utility specification rules out any possible time inconsistency problem. When $1/\psi = \gamma$, this specification reduces to the standard expected utility case. In this specification, U represents the time preference over certain $(C_0, \widehat{C}_1)$ pairs, where $\widehat{C}_1$ is the period-2 certainty equivalent associated with the random period-2 consumption, C_1: $\widehat{C}_1 = V^{-1}\left(\mathbb{E}V(C_1)\right)$.

We assume that households make consumption and saving decisions for a given initial endowment in period 0, and receive both capital income from this risky saving behavior and a risky labor income in period 1. Specifically, the households' budget constraints in period 0 and 1 can be written as:

$$C_0 + K_1 = Y_0, \tag{6}$$

$$C_1 = A_1 K_1 + Y_1, \tag{7}$$

respectively, where Y_0 is initial wealth which is strictly positive, and $K_1 > 0$ is the total savings/investment in period 0.

3.2 Shocks and Information Structure

The capital return (A_1) and labor income (Y_1) processes are assumed as follows:

$$A_1 = \exp(\epsilon_a) \text{ and } Y_1 = \exp(\epsilon_y), \tag{8}$$

where ϵ_a and ϵ_y are exogenously iid shocks. Households are endowed with prior beliefs about the distributions from which these shocks are drawn: $\epsilon_a \sim N(\mu_a, \sigma_a^2)$ and $\epsilon_y \sim N(-0.5\sigma_y^2, \sigma_y^2)$.[12] However, the realizations of these two shocks are unobservable in period 0.

[12]This implies that $\bar{Y}_1 = \mathbb{E}\left[Y_1\right] = 1$.

We assume that households learn exogenous income shocks by observing the following signals:[13]

$$S_0 = \begin{bmatrix} S_a \\ S_y \end{bmatrix} = \begin{bmatrix} \epsilon_a + \zeta_a \\ \epsilon_y + \zeta_y \end{bmatrix}, \tag{9}$$

where the signals are noisy but unbiased. $\zeta_a \sim N(0, \eta_a^2)$ and $\zeta_y \sim N(0, \eta_y^2)$ are the endogenous noises induced by limited-information processing capacity. The variance of signal regarding capital income shock is $\sigma^2 + \eta_a^2$, and therefore, the precision of the signal is defined $1/\left(\sigma^2 + \eta_a^2\right)$. Similarly, the variance of signal regarding labor income shock is $\sigma^2 + \eta_y^2$, and therefore, the precision of the signal is defined $1/\left(\sigma^2 + \eta_y^2\right)$.

Households now use Bayes' Law to combine their prior beliefs and the observed noisy signals in (9) to update their beliefs about the shocks such that $\epsilon_a|S_a \sim N(\hat{\epsilon}_a, \hat{\sigma}_a^2)$ and $\epsilon_y|S_y \sim N(\hat{\epsilon}_y, \hat{\sigma}_y^2)$, where $\hat{\epsilon}_a$ and $\hat{\epsilon}_y$ are the posterior means and $\hat{\sigma}_a^2$ and $\hat{\sigma}_y^2$ are the posterior variances determined by the following updating rules:

$$\hat{\epsilon}_a \equiv \mathbb{E}[\epsilon_a|S_a = s_a] = \frac{\mu_a \eta_a^2 + \sigma_a^2 s_a}{\sigma_a^2 + \eta_a^2}, \tag{10}$$

$$\hat{\sigma}_a^2 \equiv \mathrm{var}[\epsilon_a|S_a = s_a] = \frac{\sigma_a^2 \eta_a^2}{\sigma_a^2 + \eta_a^2}, \tag{11}$$

$$\hat{\epsilon}_y \equiv \mathbb{E}[\epsilon_y|S_y = s_y] = \frac{(-0.5\sigma_y^2)\eta_y^2 + \sigma_y^2 s_y}{\sigma_y^2 + \eta_y^2}, \tag{12}$$

$$\hat{\sigma}_y^2 \equiv \mathrm{var}[\epsilon_y|S_y = s_y] = \frac{\sigma_y^2 \eta_y^2}{\sigma_y^2 + \eta_y^2}. \tag{13}$$

Given the prior beliefs, (11) and (13) imply that the signal precision can be uniquely determined by the posterior variance. We can now define information sets before and after observing the signals, which are called Stages 1 and 2 of period 0.

Definition. $\mathbb{I}^1$ *and* $\mathbb{I}^2$ *are the information sets in Stages 1 and 2, respectively:*

$$\mathbb{I}^1 = \left\{ Y_0, \epsilon_a \sim N\left(\mu_a, \sigma_a^2\right), \epsilon_y \sim N\left(-\frac{\sigma_y^2}{2}, \sigma_y^2\right) \right\}$$
$$\mathbb{I}^2 = \mathbb{I}^1 \cup \{S_a, S_y\}$$

Following Sims (2003, 2010), we assume that households face a limited information-processing capacity, κ:

$$\kappa_a + \kappa_y \leq \kappa, \tag{14}$$

[13] Sims (2010) provides two ways to solve models with limited information-processing capacity. The first way is to solve the optimal joint distribution of the control variable and the unobservable state variable. The second way is to assume a signal structure, and then solve for the optimal policy as a function of signal. However, as argued by Sims (2010), the optimal joint distribution can be characterized by many different combinations of signal structure and policy function.

where $0 < \kappa < \infty$, κ_a and κ_y are capacity levels devoted to monitoring capital and labor income shocks, respectively. For simplicity, following Maćkowiak and Wiederholt (2009), we assume that the two signals are independent such that:

$$\mathbb{I}(\epsilon_a, S_a) = \mathbb{H}(\epsilon_a) - \mathbb{H}(\epsilon_a|S_a) = \frac{1}{2}\log\left(\frac{\sigma_a^2}{\hat{\sigma}_a^2}\right) = \kappa_a, \tag{15}$$

$$\mathbb{I}(\epsilon_y, S_y) = \mathbb{H}(\epsilon_y) - \mathbb{H}(\epsilon_y|S_y) = \frac{1}{2}\log\left(\frac{\sigma_y^2}{\hat{\sigma}_y^2}\right) = \kappa_y, \tag{16}$$

where κ_a and κ_y are measured in nats[14]; $\mathbb{H}(\cdot)$ is the entropy of productivity shock and $H(\cdot|\cdot)$ is the conditional entropy of productivity shock given signal observation; $I(\cdot,\cdot)$ is called the mutual information between the fundamental shock and signal observation and can be interpreted as how much information about the fundamental shock is contained in the corresponding noisy signal.

3.3 Households' Optimization Problem

In this model, households need not only solve an optimal consumption-saving problem but also solve an optimal attention allocation problem. The whole optimization problem can be formalized as follows:

$$U = \max_{\{\kappa_a,\kappa_y\}} \mathbb{E}_{\mathbb{I}^1}\left[u(C_0^*) + \beta u\left(V^{-1}(\mathbb{E}[V(C_1^*)|S_0])\right)\right] \tag{17}$$

$$\text{Subject to: } C_0^* = \arg\max_{C_0} U(C_0, C_1) = u(C_0) + \beta u\left(V^{-1}(\mathbb{E}_{\mathbb{I}^2}[V(C_1)])\right), \tag{18}$$

$$C_1^* = A_1(Y_0 - C_0^*) + Y_1, \tag{19}$$

$$\kappa_a + \kappa_y \leq \kappa, \tag{20}$$

where Equation (17) is the objective function for the household; $\mathbb{E}_{\mathbb{I}^2}[\cdot]$ is the expectation operator conditional on the information set $\mathbb{I}^2$; $\mathbb{E}_{\mathbb{I}^1}[\cdot]$ is the expectation over all possible signals; the budget constraints are incorporated into Equation (18) and (19); and Equation (20) is the information constraint.

[14] Sims (2003) states that the logarithm in the formula can be to any base, because the base only determines a scale factor for the information measure, but conventionally it takes the logarithm to base 2, and as a result the entropy of a discrete distribution with equal weight on two points is 1 or $(-0.5\log(0.5) - 0.5\log(0.5))$, which is the unit of information called a "bit". When the base is e, the unit of information is a "nat".

3.4 Solution Method

As illustrated in Figure 1, we decompose the optimization problem proposed above into two stages: (i) attention allocation and (ii) consumption-saving choice. In the first stage, before observing the noisy signals about capital return and labor income, households decide how much attention to allocate to learning capital return and labor income respectively. This procedure determines how precise these two signals are. In the second stage, after observing the signals, households then decide how much to consume and how much to save out of the initial endowment. Following Maćkowiak and Wiederholt (2009), we solve these two sub-problems backward.

First, for *any* attention allocation strategy, we solve the following consumption-saving problem:

$$U = \frac{(Y_0 - K_1)^{1-1/\psi}}{1-1/\psi} + \beta \frac{\left(\mathbb{E}\left[(A_1 K_1 + Y_1)^{1-\gamma} | S_0\right]\right)^{\frac{1-1/\psi}{1-\gamma}}}{1-1/\psi}. \tag{21}$$

The first order condition for K_1 is:

$$\frac{\partial U}{\partial K_1} = -(Y_0 - K_1)^{-1/\psi} + \beta \left(\mathbb{E}\left[(A_1 K_1 + Y_1)^{1-\gamma} | S_0\right]\right)^{\frac{\gamma-1/\psi}{1-\gamma}} \mathbb{E}\left[(A_1 K_1 + Y_1)^{-\gamma} A_1 | S_0\right] = 0. \tag{22}$$

It is straightforward that the first order condition determines a unique solution to the consumption-saving problem. Solving the condition yields the optimal choice of K_1 in period 0. Plugging $K_1^*(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2)$ back to the utility function gives us the indirect utility. Taking the unconditional expectations by evaluating over S_a and S_y allows us to solve the first-stage attention allocation problem. The detailed procedure is provided in Appendix 8.

4 Main Results

In this section, we will first solve the model numerically and then examine the model's implications for the consumption and saving behavior of households with limited attention. To solve the model numerically, we choose the following parameter values as their baseline values: $\gamma = 3$, $\psi = 1/3$, $\beta = 0.7$, $Y_0 = 3$, $\kappa = 1$.[15] It is a consensus in macroeconomic studies that the value of γ is between 1 to 5 and $\gamma = 3$ is widely adopted in the literature on consumption and savings. However, there is no concesus on the magnitude of the EIS (ψ) and the evidence is still mixed as the literature has found a very wide range of values. For

[15]The value of initial wealth (Y_0) may vary largely for different individuals, from 2 to 20 in the Survey of Consumer Finances (SCF) given that the mean income is 1. For robustness check, we also use $\gamma = 2, 4$, $\beta = 0.5, 0.9$, $Y_0 = 2$, 4, and $\kappa = 2, 3$.

example, Visising-Jorgensen and Attanasio (2003) estimate the EIS to be well in excess of 1, while Campbell and Cocco (2007) estimate a value well below 1 (and possibly 0). Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1). Havránek (2015) surveys the vast literature and suggests that a range around 0.3-0.4 is appropriate after correcting for selective reporting bias. Best, Cloyne, Ilzetzki, and Kleven (2020) use U.K. mortgage data and find the EIS is close to 0.1. Luo (2008) shows that when $\kappa = 0.5$, the otherwise standard permanent income model can generate the observed aggregate consumption smoothness. In a recent study, Coibion and Gorodnichenko (2015) use the SPF forecast survey data to test the degree of information rigidities governed by the degree of inattention and find that their model can fit the data well when κ is close to 0.5. Since our model consider two types of risks, we set the baseline value of κ to be 1. Our main results do not rely on the choice of these parameter values.

When we do the comparative statics analysis, we vary one parameter while holding other parameters fixed at their baseline values. In addition, following Campbell (2003), we set the prior variance of capital income risk σ_a^2 to be 0.02, and assume that $\sigma_y^2/\sigma_a^2 \in [1, 3, 5, 7]$. We set the expected gross return of risky asset is 1.04, and the expected labor income in the second period is 1.

4.1 Optimal Attention Allocation

In this subsection, we consider the effects on optimal attention allocation of the relative prior volatility of labor income risk to capital income risk, the endowments of attention and wealth, and the risk and time preferences.

The effect of relative prior variance $\left(\sigma_y^2/\sigma_a^2\right)$. From Figure 2, it is clear that agents allocate more attention to the labor income shock when the prior variance of labor income risk becomes larger relative to that of capital income risk. This is in line with many previous studies on optimal attention allocation, such as Maćkowiak and Wiederholt (2009). The intuition for this result is straightforward. When labor income risk becomes more volatile, its relative importance to capital income also increases, as a result agents pay more and more attention to labor income risk relative to capital income risk. However, as capital income risk takes multiplicative form and labor income risk takes an additive form, they enter the budget constraint asymmetrically and the resulting attention allocation strategy is significantly different from that obtained in the previous literature. More precisely, in previous studies with symmetric risks, we have often seen that when prior variances of two risks are the same, the agent allocates equal amounts of attention to each risk. However, as shown in

Figure 2, agents pay the same amount of attention to two risks when the variance of labor income shock is about 4 times that of the capital income shock. Intuitively, this asymmetry itself affects agents' attention allocation. Now the question is why the multiplicative capital income risk attracts more attention than the additive labor income risk. One potential explanation is that when the prior variance of labor income shock increases, prudent agents will save more following a precautionary motive and the resulting larger amount of savings makes her care more about the capital income shock. More precisely, there are two opposite effects of increasing the prior variance of labor income shocks: (i) the direct effect makes agents pay more attention to labor income risk and (ii) the indirect effect motivates agents to pay more attention to capital income risk through the saving decision. We can see from Figure 7 that the expected saving rate, $\mathbb{E}[s]$, is increasing along the prior variance of labor income risk. As shown in Figure 2, when σ_y^2/σ_a^2 is below approximately 4, the indirect effect dominates, meaning that agents pay more and more attention to labor income risk and less and less attention to capital income risk, but the amount of attention to capital income risk is always larger ($\kappa_a > \kappa_y$). When the ratio becomes sufficiently large, the direct effect dominates.

The effect of initial wealth (Y_0). In this section, we examine how initial wealth affects optimal attention allocation. From Figure 3, the optimal amount of attention devoted to capital income risk is increasing with the initial wealth level. Intuitively, this result shows that rich individuals pay more attention to capital income risk compared to poor individuals. One potential explanation is that rich people have more risky assets in absolute amount than poor people and therefore have incentives to pay more attention to capital income risk.

To the best of our knowledge, the first paper that studies the impact of wealth inequality on attention choice is Yin (2019). In Yin (2019), there is only one income shock from stochastic capital returns and the information-processing cost is fixed, whereas the present paper investigates attention allocation in a more general model with both labor income and capital income shocks. Yin (2019) finds that the marginal benefit of paying attention is decreasing with initial wealth, such that wealthier individuals are willing to pay less attention than the poor. In this paper, we fix the amount of attention endowment so that agents have equal amount of attention but adopt different attention allocation strategies. We find that wealthier agents pay more attention to capital income risk because they hold more risky wealth, which make future capital income more volatile relative to labor income.

The effect of attention capacity (κ). From the right panel of Figure 3, we can see that the amounts of attention allocated to both capital and labor income shocks increase with the total amount of attention capacity. From these parallel lines in this graph, we argue that the pattern of absolute attention allocation does not change. However, it is more interesting to see how the share of attention to each shock over total attention capacity changes. As can be seen in Figure 4, since these lines do not overlap, we can conclude that these relative patterns change when the attention capacity changes. More precisely, we first notice that when $\sigma_y^2 = \sigma_a^2 = 0.02$, agents allocate almost all of their attention to capital income shocks if the total amount is only 0.5 nats. However, κ_a/κ and κ_y/κ become flatter and flatter as the total amount of attention increases. This implies that individuals with more capacity have more balanced allocation between two shocks and when κ is sufficiently large, agents will pay equal amounts of attention to both shocks.

The effect of the discount factor (β). As shown in Figure 5, the discount factor has significant effects on attention allocation. More patient agents (higher β) allocate more attention to capital income shock compared to that to labor income shock. The intuition for this result is that patient agents save more and the larger amount of savings makes paying attention to capital income risk more valuable.

The effect of the CRRA (γ). From the right panel of Figure 5, the results discussed above hold for different values of the coefficient of relative risk aversion: κ_y increases with the prior variance of labor income shock, and κ_a decreases with the prior variance of labor income shock. Second, we can also see that the optimal amount of attention devoted to capital income risk is decreasing with the risk aversion degree, meaning more risk averse agents pay more attention to labor income shock and less attention to capital income shock. To explain this result, we follow the argument in Maćkowiak and Wiederholt (2015) that increasing the coefficient of relative risk aversion always reduces a household's incentive to pay attention to the macroeconomy given the steady state consumption is larger than unity.[16] In our model, capital income shock is the aggregate shock, whereas labor income shock is a pure idiosyncratic shock. Therefore, increasing the coefficient of relative risk aversion decreases κ_a but increases κ_y.

The effect of the EIS (ψ). To examine the effect of EIS on attention allocation, we vary the value of the EIS, fix $\gamma = 3$, and use the same prior variances as above. The right panel of Figure 6 shows how a reduction in the EIS leads to larger amount attention to capital income shock, and smaller amount of attention to labor income shock. For example, when reducing the EIS from 0.5 to 0.2, we observe an increase (decrease) of 0.3 nats in κ_a (κ_y). The EIS governs the willingness to substitute consumption across periods, and the lower the

[16] Maćkowiak and Wiederholt (2015) study an attention-allocation problem with approximated linear-quadratic setup, and therefore, they can discuss the effect of increasing relative risk aversion degree on attention allocation analytically.

EIS, the more reluctant agents are willing to substitute consumption intertemporally. They would like to pay more attention to capital income shocks because savings in period 0 is the only device in smoothing consumption between the two periods.

4.2 Consumption-Saving Allocation

We use the unconditional mean of the ratio of savings in period 0 over initial wealth $\mathbb{E}[s] = \mathbb{E}[K_1/Y_0]$ to denote the expected saving rate. Figure 7 illustrates how the expected saving rates vary with the prior variance of labor income shock, holding the prior variance of capital income risk fixed. The intuition is that as σ_y^2 increases, agents choose to save a larger share of their initial endowment due to the precautionary motive against potential loss in future income.

Figure 7 also shows how the average saving rate changes with other model parameters. First, we can see that wealthier people save at higher rates. This is in line with many empirical studies that show heterogeneous saving behavior across different wealth groups (see Dynan, Skinner, and Zeldes (2004).). For example, increasing Y_0 from 2 to 3 leads to a rise of the expected saving rate by approximately 40%. The explanation in our paper is that rich people pay more attention to the capital income risk, which makes their posterior variance in labor income smaller and saving in this asset becomes more attractive. However, as shown in Figure 3, the increase in κ_y is decreasing with initial wealth (Y_0), so it is not surprising to see that the growth rate of the expected saving rate is decreasing. More precisely, when increasing initial wealth from 3 to 4, the increase in the expected saving rate is only by about 15%. Second, the average saving rate decreases with total attention capacity. To understand this, let us take an extreme case as an example. When attention is zero, i.e. agents pay no attention to the income shocks, they face greater posterior uncertainty in future income than those who pay some attention, and consequently, they choose to save at a higher rate due to the precautionary motive. We also notice that the pattern of the expected saving rate becomes flatter and flatter when increasing the total attention capacity. This is intuitive because when agents have more capacity to process information, the difference in their posterior variance is not big no matter how large the prior variance is. In another extreme case, when $\kappa \to \infty$, our model becomes completely deterministic and the saving rate becomes flat. Third, the average saving rate increases with the discount factor. As this parameter governs the degree of the agents' patience, a higher value of β leads agents to save at a higher rate. Finally, more risk averse agents save at higher rates. This is due to the fact that more risk averse agents have a stronger precautionary saving motive.

From the middle-right panel of Figure 7, it is clear that increasing the CRRA raises the expected saving rate. In contrast, when fixing γ and varying ψ, we can see from the lower-left panel of Figure 7 that for each value of the EIS, agents, on average, save at higher rates for higher prior variance of labor income shock. In addition, we can also see that EIS has significant effects on the expected saving rate. Agents who are more reluctant to substitute consumption intertemporally (smaller ψ) have higher expected saving rates. For example, a decrease of the EIS from 0.8 to 0.3 leads to an increase of the expected saving rate by approximately 23% (from 0.244 to 0.299). It is worth noting that although increasing CRRA and reducing EIS have a similar effect on the expected saving rate, their economic intuitions are totally different. The former is due to the precautionary saving motive and the latter is due to the smoothness motive. Finally, the lower-right panel presents the negative effect of increasing the prior variance of capital income risk on the expected saving rate. However, comparing to the effects of changing other parameters, the effect of an increase in σ_a^2 is very small. The reason is that increasing σ_a^2 has two opposite effects on saving behavior: on the one hand, it makes saving in this asset less attractive due to a higher volatility in the return, on the other hand, it leads to more attention to capital income risk, and less attention to labor income risk, which drives agents to save more due to precautionary motive.

5 Testable Implications

In this section, we will discuss the testable implications of our model and compare our model's predictions with the empirical counterparts in the U.S. microdata.

5.1 The Consumption Responses to Income Shocks

An important question we try to answer in this paper is how the change in consumption responds to attention allocation. As we set the unconditional mean of the second period labor income to be 1, we can see that the second period consumption is smaller than that of the first period. From Figure 8, we first notice that the expected decrease is smaller for agents with larger prior variance of labor income. This model's prediction is consistent with the first empirical fact reported in Table 2. Comparing with individuals who have lower prior volatility in labor income, individuals with higher prior volatility in labor income pay more (less) attention to labor (capital) income for given prior variance of capital (labor) income shock, meaning that they can observe more precise signals regarding their labor income shocks, and then being unemployed is more predictable. By contrast, those with lower prior variance pay less attention to labor income shock, such that unemployment is less predictable, and is more like an unexpected shock. As a result, individuals with a lower prior variance react more strongly to being unemployed.

In Figure 8, we can also see that wealthier agents have larger decreases in their consumption than those with less initial wealth. This prediction is also consistent with the second empirical fact reported in Table 3. When comparing the left two panels in Figure 3, we can see that wealthier individuals pay more attention to capital income shocks than poor individuals. This can again be explained by the endogenous saving decision, meaning that wealthier individuals hold larger amounts of risky assets, and they would like to pay larger (smaller) amount of attention to the capital (labor) income shock. Consequently, when wealthier individuals lose their jobs, they react more strongly to labor income shock. The lower-left panel of Figure 8 clearly shows that the change in consumption is smaller if the value of the EIS is lower. For example, when the EIS takes values of 0.3, 0.5 and 0.8 respectively, the corresponding decreases in the expected growth rate of consumption are -8%, -14%, and -22%, respectively. The intuition is straightforward: Agents with smaller EIS pay more attention to capital income shocks and save more (as discussed above) to avoid larger fluctuations, and therefore they experience smaller changes in their consumption. Finally, as argued above the effect of changing the prior variance of capital income risk on expected saving rate is small, and as a result its effect on the consumption growth is also negligible.

5.2 The Relative Dispersion of Changes in Consumption to Changes in Income

Given our two-period specification, our model is not suitable to discuss how attention allocation affects the wealth inequality. We therefore focus on the model's prediction on how attention allocation affects the relative dispersion of changes in consumption to income.[17][18] The upper panel of Figure 9 shows that the relative dispersion of changes in consumption and income ($\mu \equiv \mathrm{sd}\,(\Delta C)\,/\,\mathrm{sd}\,(\Delta Y)$) decreases with the prior volatility of labor income risk (σ_y^2), for different values of attention (κ). In addition, the relative dispersion is increasing with κ for given values of σ_y^2. The upper panel of Figure 9 shows how the relative dispersion varies with the EIS. It clearly shows that the relative volatility is increasing with the EIS. The

[17] In this paper the relative consumption dispersion/inequality is measured as the ratio of the standard deviation of the change in consumption to the standard deviation of the change in income.

[18] See Luo et al. (2017) and Lei (2019) for the issue on how information friction affects the wealth inequality in infinite-horizon settings.

intuition is that a lower EIS leads to more attention to capital income shock and a higher saving rate, which makes consumption smoother. As we mentioned in Section , although theoretically it is reasonable to assume that the EIS is greater than 1, in this paper we follow the empirical studies in economics and choose the EIS to be less than 1.

To examine how the model's predictions are matched with the empirical evidence, we use the same panel data set that contains both consumption and income at the household level as in Luo, Nie, and Young (2020). The upper panel of Figure 9 shows the evolutions of consumption and income dispersions as well as the relative dispersion of changes in consumption to income between 1980 and 2010.[19] From the figure, the average empirical value of the relative dispersion (μ) is 0.4 for the 1980-1996 period and 0.34 for the 1980-2010 period. The minimum and maximum values of the empirical relative dispersion from 1980 to 2010 are 0.20 (year 2006) and 0.53 (year 1983), respectively. Comparing two lower panels in Figure 9, we can see that the model's predicted relative volatility can match the empirical counterpart well when the EIS is relatively low. For example, when $\gamma = 3$, $\psi = 0.4$, and $\kappa = 2$, the model predicts that $\mu = 0.34$, which equals the empirical counterpart for the sample from 1980 to 2010.[20]

6 Policy and Welfare Implications

In this section, we will examine the tax policy and welfare implications of our attention allocation model.

6.1 Implications on Tax Policies

In this subsection, we want to discuss how different types of taxation policy affect the saving rate and consumption dynamics via the optimal attention allocation channel. Here we study a partial equilibrium case, where there exists a government who takes taxes from the household sector exogenously. Accordingly, the budget constraints can be written as:

$$C_0 + K_1 = Y_0, \tag{23}$$

$$C_1 = (1 - \tau_k)\, A_1 K_1 + (1 - \tau_y)\, Y_1, \tag{24}$$

[19]See the Online Appendix A in Luo, Nie, and Young (2020) for more details on how the panel is constructed.

[20]In the lower-right panel, we set $\kappa = 2$, $\gamma = 3$, and vary ψ from 0.3 to 0.5.

where τ_k and τ_y are linear tax rates on capital income and labor income, respectively. The first order condition for K_1 in the second stage problem is then:

$$\beta \left(\mathbb{E}\left[\left(1-\tau_k\right) A_1 K_1 + \left(1-\tau_y\right) Y_1\right)^{1-\gamma} | S_0\right]\right)^{\frac{\gamma-1/\psi}{1-\gamma}} \mathbb{E}\left[\left(\left(1-\tau_k\right) A_1 K_1 + \left(1-\tau_y\right) Y_1\right)^{-\gamma} (1-\tau_k) A_1 | S_0\right]$$
$$= (Y_0 - K_1)^{-1/\psi}. \tag{25}$$

Solving this problem again with the two-step approach shown above, we can see how different tax schemes affect attention allocation and consumption-saving behavior as shown in Figures 10 and 11.

Let us start with the saving behavior because the effects of issuing tax on consumption-saving decisions are straightforward. The saving rate increases with both the capital income tax rate (τ_a) and the labor income tax rate (τ_y). This is intuitive because a higher tax rate reduces the household's saving motive. As can be seen from Figure 10 given that $\tau_y = 30\%$, increasing τ_a by 100% (i.e. from 10% to 20%) leads to the expected saving rate increase by about 3%. We can also see that given $\tau_a = 30\%$, increasing τ_y by 100$ (also from 10% to 20%) leads to an increase of 7% in the expected saving rate. Therefore, we can conclude that changing tax rates of labor income has larger effects on the household's consumption-saving behavior than changing that of capital income.

Figure 11 presents how changing the tax rate affects optimal attention allocation. First, we can see that an increase in the marginal tax rate on labor (capital) income leads to a decrease in the amount of attention allocated to labor (capital) income shock. To understand the attention allocation behavior, we need to link changes in the tax rate and changes in income volatility. Elmendorf and Kimball (2000) argue that increases in the marginal tax rate on labor income can cause large enough reductions in the after-tax labor income risk, which leads to a lower amount of attention to labor income shock. Similarly, it also holds for the change in the capital income tax. Second, we find that increasing τ_y by 100% has much larger effects on attention allocation between income shocks than increasing τ_a by the same amount.

The discussions above provide us with useful policy implications on the effects of different types of tax under rational inattention. As increasing labor income tax rate has larger effects on both attention allocation and consumption-saving allocation, we can argue that labor income taxes can help government achieve its goals of, for example, adjusting the household's savings more efficiently than capital income taxes.

6.2 Welfare Implications of Limited Attention

In this section, we compute the welfare gains if the inattentive agents are allowed to increase their channel capacity. Specifically, we follow Luo (2008) and also conduct a welfare analysis.[21] As shown in Table 4, we calculate the utility losses for three different values of κ and four different values of σ_y^2, Y_0, γ, and ψ. Here is the procedure to conduct the welfare analysis. Our main purpose for this exercise is to investigate We first choose $\kappa = 1$, 2, and 3 as the starting values, and calculate the corresponding unconditional expected lifetime utility. Then we increase each starting value of attention capacity κ by 100% and compute the corresponding unconditional expected lifetime utility for each κ. Finally, we can compute the percentage increase of expected lifetime utility using this formula:

$$\left| \frac{E[U(\kappa_{\text{new}})] - E[U(\kappa_{\text{baseline}})]}{E[U(\kappa_{\text{baseline}})]} \right|. \tag{26}$$

First we find from all four panels that the utility gains are increasing with the amount of attention capacity. This result is intuitive and in line with the findings in Luo (2008): with higher attention capacity agents can better predict their future income and in the extreme case when they have infinite capacity it converges to a full information scenario. Second, if we compare vertically for each panel, it shows that the change in the expected utility is decreasing. More precisely, increasing κ from 1 to 2 has a larger effect on welfare gains than increasing κ from 2 to 4; similarly, increasing κ from 2 to 4 has a larger effect on welfare gains than increasing κ from 3 to 6. These results suggest a heterogeneity in the welfare gains for agents with different levels of attention capacity. Third, we also see another significant heterogeneity in the effects of changing other parameters on welfare gains. Panel A, C and E show that for a given attention capacity, the expected utility is increasing with prior variance of labor income shock, CRRA and the capital income tax rate, whereas Panel B, D and F show that welfare gains are decreasing with initial wealth, the EIS, and the labor income tax rate. These results imply that if an increase in the parameter value leads to more attention to labor income risk, agents can experience larger increases in welfare gains. On the contrary, if an increase in the parameter value that leads to a rise in the amount of attention to capital income risk, the increase of welfare gains is decreasing.

[21] Different from our two-period consumption model with two income shocks, Luo (2008) studies an infinite horizon permanent income model with a single labor income shock. He examines the welfare effects of income shocks under rational inattention by calculating how much utility agents will lose if the actual consumption path under rational inattention deviates from the first-best consumption path under full information.

7 Conclusion

We have constructed and solved a two-period consumption-saving model with recursive utility, capital income and labor income risks, and limited attention in this paper. The key feature of this paper is to allow agents with limited attention to choose optimal attention allocation. We have examined how the optimal attention-consumption allocation is affected by the key model parameters including the relative prior variance of the two exogenous income risks (capital income and labor income risks), the endowments of wealth and attention, and the risk and time preferences. We also found that the simple model can capture some key aspects of the consumption behavior we observed in the U.S. microdata. Finally, we found that taxes on capital income and labor income can have different welfare implication of limited attention and the welfare gains from increasing attention capacity are insignificant.

8 Appendix: Solving the Recursive Utility Model

We adopt the outer optimization approach to solve the optimal attention allocation problem in the recursive utility case. For any given amount of attention to capital income shock κ_a, we can obtain the distribution of the signal on capital income shock, S_a. As the total amount of attention is fixed, we can also obtain the amount of attention to labor income shock, κ_y, and the distribution of the signal on capital income shock, S_y. Then, we can solve the optimal savings K_1^* for a combination of (s_a, s_y) by using the same approach as in the previous section.

Here we maximize the unconditional expected utility (evaluating over possible signals) by choosing the optimal attention allocation:

$$\begin{aligned} U(K_1) &= \frac{(Y_0 - K_1)^{1-1/\psi}}{1-1/\psi} + \beta \frac{\left(\mathbb{E}\left[(A_1 K_1 + Y_1)^{1-\gamma} | S_0\right]\right)^{\frac{1-1/\psi}{1-\gamma}}}{1-1/\psi} \\ &= U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2, K_1). \end{aligned}$$

Define $V(\hat{\sigma}_a^2, \hat{\sigma}_y^2) = \mathbb{E}_{\mathbb{I}}[U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2, K_1)]$, the attention allocation is to choose κ_a:

$$\max_{\kappa_a} V(\kappa_a), \tag{27}$$

subject to

$$\frac{1}{2}\log\left(\frac{\sigma_a^2}{\hat{\sigma}_a^2}\right) = \kappa_a \tag{28}$$

$$\frac{1}{2}\log\left(\frac{\sigma_y^2}{\hat{\sigma}_y^2}\right) = \kappa_y \tag{29}$$

$$\kappa_a + \kappa_y = \kappa. \tag{30}$$

Here we can easily show the mean and variance of the signals can be written as:

$$\mathbb{E}[S_a] = \mu_a,\ \mathrm{var}(S_a) = \frac{(\sigma_a^2)^2}{\sigma_a^2 - \hat{\sigma}_a^2};\ \mathbb{E}[S_y] = \mu_y,\ \mathrm{var}(S_y) = \frac{(\sigma_y^2)^2}{\sigma_y^2 - \hat{\sigma}_y^2}.$$

Their corresponding density functions are :

$$f_{S_a} = \frac{1}{\sqrt{2\pi\,\mathrm{var}(S_a)}} \exp\left(-\frac{(s_a-\mu_a)^2}{2\,\mathrm{var}(S_a)}\right) \text{ and } f_{S_y} = \frac{1}{\sqrt{2\pi\,\mathrm{var}(S_y)}} \exp\left(-\frac{(s_y-\mu_y)^2}{2\,\mathrm{var}(S_y)}\right).$$

Define $t_a = \frac{s_a - \mu_a}{\sqrt{2\,\mathrm{var}(S_a)}}$ and $t_y = \frac{s_y - \mu_y}{\sqrt{2\,\mathrm{var}(S_y)}}$, we have

$$s_a = \mu_a + \sqrt{2\,\mathrm{var}(S_a)}t_a = \mu_a + \frac{\sqrt{2}\sigma_a^2}{\sqrt{\sigma_a^2 - \hat{\sigma}_a^2}}t_a,$$
$$s_y = \mu_y + \sqrt{2\,\mathrm{var}(S_y)}t_y = \mu_a + \frac{\sqrt{2}\sigma_y^2}{\sqrt{\sigma_y^2 - \hat{\sigma}_y^2}}t_y.$$

We apply the Gaussian quadrature approach to approximate the unconditional expectation of the utility and obtain the value for some κ_a (κ_y). In the second step, we adopt the inner optimization approach to solve the corresponding optimal consumption-saving problem. Specifically, the RHS of the Euler equation (22) can be written as:

$$\beta \left(\mathbb{E}\left[(A_1K_1 + Y_1)^{1-\gamma}|S_0\right]\right)^{\frac{\gamma-1/\psi}{1-\gamma}} \mathbb{E}\left[(A_1K_1 + Y_1)^{-\gamma}A_1|S_0\right].$$

The conditional distributions of $\epsilon_a|S_a = s_a$ and $\epsilon_y|S_y = s_y$ can be written as:

$$f_1(\epsilon_a, \epsilon_y) = \left(\exp\left(\sqrt{2}\hat{\sigma}_a x_a + \frac{\hat{\sigma}_a^2}{\sigma_a^2}\mu_a + \left(1 - \frac{\hat{\sigma}_a^2}{\sigma_a^2}\right)s_a\right)K_1 + \exp\left(\sqrt{2}\hat{\sigma}_y x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y\right)\right)^{1-\gamma}$$

$$f_2(\epsilon_a, \epsilon_y) = \begin{pmatrix} \exp\left(\sqrt{2}\hat{\sigma}_a x_a + \frac{\hat{\sigma}_a^2}{\sigma_a^2}\mu_a + \left(1 - \frac{\hat{\sigma}_a^2}{\sigma_a^2}\right)s_a\right)K_1 \\ +\exp\left(\sqrt{2}\hat{\sigma}_y x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y\right) \end{pmatrix}^{-\gamma} \exp\left(\sqrt{2}\hat{\sigma}_y x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y\right).$$

Define

$$x_a = \frac{\epsilon_a - \left(\mu_a + \sigma_a\rho_a\frac{s_a - \mathbb{E}[S_a]}{\sqrt{\mathrm{var}(S_a)}}\right)}{\sigma_a\sqrt{1-\rho_a^2}\sqrt{2}} \text{ and } x_y = \frac{\epsilon_y - \left(\mu_y + \sigma_y\rho_y\frac{s_y - \mathbb{E}[S_y]}{\sqrt{\mathrm{var}(S_y)}}\right)}{\sigma_y\sqrt{1-\rho_y^2}\sqrt{2}},$$

we have

$$\epsilon_a = \sigma_a\sqrt{1-\rho_a^2}\sqrt{2}x_a + \mu_a + \sigma_a\rho_a\frac{s_a - \mathbb{E}[S_a]}{\sqrt{\mathrm{var}(S_a)}},$$
$$\epsilon_y = \sigma_y\sqrt{1-\rho_y^2}\sqrt{2}x_y + \mu_y + \sigma_y\rho_y\frac{s_y - \mathbb{E}[S_y]}{\sqrt{\mathrm{var}(S_y)}},$$

where $\rho_a^2 = 1 - \hat{\sigma}_a^2/\sigma_a^2$, $\sqrt{1-\rho_a^2} = \hat{\sigma}_a/\sigma_a$, $\mathbb{E}[S_a] = \mu_a$, $\mathrm{var}(S_a) = (\sigma_a^2)^2/\left(\sigma_a^2 - \hat{\sigma}_a^2\right)$, $\rho_y^2 = 1 - \hat{\sigma}_y^2/\sigma_y^2$, $\sqrt{1-\rho_y^2} = \hat{\sigma}_y/\sigma_y$, $\mathbb{E}[S_y] = \mu_y$, and $\mathrm{var}(S_y) = (\sigma_y^2)^2/\left(\sigma_y^2 - \hat{\sigma}_y^2\right)$. Finally, we have

$$\epsilon_a = \hat{\sigma}_a\sqrt{2}x_a + \frac{\hat{\sigma}_a^2}{\sigma_a^2}\mu_a + \left(1 - \frac{\hat{\sigma}_a^2}{\sigma_a^2}\right)s_a,$$
$$\epsilon_y = \hat{\sigma}_y\sqrt{2}x_y + \frac{\hat{\sigma}_y^2}{\sigma_y^2}\mu_y + \left(1 - \frac{\hat{\sigma}_y^2}{\sigma_y^2}\right)s_y.$$

Applying the Gaussian quadrature approach, we can approximate the RHS as follows:

$$\mathbb{E}[f_1(\epsilon_a,\epsilon_y)|S_a,S_y] = \int\int f_1(x_a,x_y)e^{-x_a^2}e^{-x_y^2}dx_a dx_y \cong \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\pi}\omega_{a,i}^{GH}\omega_{y,j}^{GH}f_1^*(\xi_{a,i}^{GH},\xi_{y,i}^{GH}),$$

$$\mathbb{E}[f_2(\epsilon_a,\epsilon_y)|S_a,S_y] = \int\int f_2(x_a,x_y)e^{-x_a^2}e^{-x_y^2}dx_a dx_y \cong \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\pi}\omega_{a,i}^{GH}\omega_{y,j}^{GH}f_2^*(\xi_{a,i}^{GH},\xi_{y,i}^{GH}),$$

where ξ_a and ξ_y are nodes and ω_a and ω_y are weights.

Next, we solve for the optimal attention allocation:

$$\max_{\kappa_a,\kappa_y} V = \mathbb{E}[U(S_a,S_y,\hat{\sigma}_a^2,\hat{\sigma}_y^2)],$$

subject to (28)-(30). We then use the Gaussian-quadrature approach to approximate the indirect utility:

$$\mathbb{E}[U(S_a,S_y,\hat{\sigma}_a^2,\hat{\sigma}_y^2)] = \int\int U(S_a,S_y,\hat{\sigma}_a^2,\hat{\sigma}_y^2)dt_a dt_y \cong \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\pi}\omega_{sa,i}^{GH}\omega_{sy,j}^{GH}U(\xi_{sa,i}^{GH},\xi_{sy,j}^{GH}),$$

where ξ_{sa} and ξ_{sy} are nodes and ω_a and ω_y are weights.

In summary, we solve the model backwards. First, Solving $F(K_1) = -(Y_0 - K_1)^{-1/\psi} + \beta\left(\mathbb{E}[f_1(\epsilon_a,\epsilon_y)|S_a,S_y]\right)^{\frac{\gamma-1/\psi}{1-\gamma}}\mathbb{E}[f_2(\epsilon_a,\epsilon_y)|S_a,S_y] = 0$ yields the optimal savings, K_1^*. Then plugging this result back into the utility function yields the indirect utility, $U(S_a,S_y,\kappa_a,\kappa_y)$. We can then compute the unconditional expected utility evaluated over signal observations and solve for the optimal attention allocation, κ_a^* and κ_y^*, by maximizing the unconditional expected utility. The following is the detailed procedure of solving the model:

1. Set $\kappa_a^{min} = 0.0001$ and $\kappa_a^{max} = \kappa - 0.0001$, such that $\kappa_y^{max} = \kappa - 0.0001$ and $\kappa_y^{min} = 0.0001$.

2. For κ_a^{min}, use the Legendre-Gauss approach compute 7 nodes for S_a and their corresponding weights. Similarly S_y for κ_y^{max}. For each combination (s_a,s_y), compute the optimal savings K_1^*, and then compute the value of $V(\kappa_a^{min})$.

3. For κ_a^{max}, use the Legendre-Gauss approach compute 8 nodes for S_a and their corresponding weights. Similarly S_y for κ_y^{min}. For each combination (s_a,s_y), compute the optimal savings K_1^*, and then compute the value of $V(\kappa_a^{max})$.

4. Compute the slope $\left(V(\kappa_a^{max}) - V(\kappa_a^{min})\right)/\left(\kappa_a^{max} - \kappa_a^{min}\right)$. If the slope is positive, set $\kappa_a^{min} = \left(\kappa_a^{min} + \kappa_a^{max}\right)/2$; if the slope is negative, set $\kappa_a^{max} = \left(\kappa_a^{min} + \kappa_a^{max}\right)/2$.

5. Iterate the steps above till the slope is close to zero, and we have $\kappa_a = \kappa_a^{max} = \kappa_a^{min}$.

Bibliography

Andrade, P., and H. Le Bihan (1997): "Inattentive Professional Forecasters," *Journal of Monetary Economics*, 55(1).

Angeletos, G.-M. (2007): "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," *Review of Economic Dynamics*, 10, 1–30.

Attanasio, O., L. Blow, R. Hamilton, and A. Leicester (2009): "Booms and Busts: Bonsumption, House Prices and Expectations," *Economica*, 76(5).

Bertaut, C., and M. Starr-McCluer (2002): "Household Portfolios in the United States," *In L. Guiso, M. Haliassos, and T. Jappelli, Editor, Household Portfolios MIT Press, Cambridge, MA*.

Best, M., J. Cloyne, E. Ilzetzki, and H. Kleven (2020): "Estimating the Elasticity of Intertemporal Substitution Using Mortgage Notches," *Review of Economic Studies, forthcoming*.

Bommier, A., and F. Le Grand (2019): "Risk Aversion and Precautionary Savings in Dynamic Settings," *Management Science*, 65(3), 1386–1397.

Campbell, J. (2003): "Consumption-Based Asset Pricing," *In Handbook of the Economics and Finance Vol. 1B, edited by George Constantinides, Milton Harris, and Rene Stulz*.

Campbell, J., and J. Cocco (2007): "How Do House Price Affect Consumption? Evidence from Micro Data.," *Journal of Monetary Economics*, 54(3).

Christelis, D., D. Georgarakos, and T. Jappelli (2015): "Wealth Shocks, Unemployment Shocks and Consumption in the Wake of the Great Recession," *Journal of Monetary Economics*, 72.

Coibion, O., and Y. Gorodnichenko (2008): "Information Rigidity and Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, 105(8).

Deaton, A. (1993): "Understanding Consumption," *Oxford University Press*.

Dynan, K. E., J. Skinner, and S. Zeldes (2004): "Do the Rich Save More?," *Journal of Political Economy*, 112(2), 397–444.

Elmendorf, D., and M. Kimball (2000): "Taxation of Labor Income and the Demand for Risky Assets," *International Economic Review*, 41(3).

Guvenen, F. (2006): "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective," *Journal of Monetary Economics*, 53(7), 339–357.

Hauser, R., and R. Willis (2005): "Survey design and methodology in Health and Retirement Study and the Wisconsin Longitudinal Study," *In: Waite, L. (Ed.), Aging, Health, and Public Policy: Demographic and Economic Perspectives, The Population Council, Inc., New York*, pp. 209–235.

Havránek, T. (2015): "Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting," *Journal of the European Economic Association*, 13(6).

Jappelli, T., and L. Pistferri (2010): "The Consumption Response to Income Changes," *Annual Review of Economics*, 2(1).

Kimball, M., and P. Weil (2009): "Precautionary Saving and Consumption Smoothing across Time and Possibilities," *Journal of Money, Credit, and Banking*, 41(2-3), 254–284.

Kreps, D., and E. Porteus (1978): "Temporal Resolution of Uncertainty and Dynamic Theory," *Econometrica*, 46, 185–200.

Leland, H. (1968): "Saving and Uncertainty: The Precautionary Demand for Saving," *The Quarterly Journal of Economics*, 82(3).

Luo, Y. (2008): "Consumption Dynamics under Information Processing Constraint," *Review of Economic Dynamics*, 11(2), 366–385.

Luo, Y., J. Nie, G. Wang, and E. Young (2017): "Rational Inattention and the Dynamics of Consumption and Wealth in General Equilibrium," *Journal of Economic Theory*, 172, 55–87.

Luo, Y., J. Nie, and E. Young (2020): "Ambiguity, Low Risk-Free Rates, and Consumption Inequality," *Economic Journal, forthcoming.*

Luo, Y., and E. Young (2014): "Signal Extraction and Rational Inattention," *Economic Inquiry*, 52(2), 811–829.

Maćkowiak, B., and M. Wiederholt (2009): "Optimal Sticky Prices under Rational Inattention," *American Economic Review*, 99(3), 769–803.

——— (2015): "Business Cycle Dynamics under Rational Inattention," *Review of Economic Studies*, 82(4), 1502–1532.

Miao, J., J. Wu, and E. Young (2020): "Multivariate Rational Inattention," *Working paper*.

Mondria, J. (2010): "Portfolio Choice, Attention Allocation, and Price Comovement," *Journal of Economic Theory*, 145(5), 1837–1864.

Peng, L., and W. Xiong (2006): "Investor Attention, Overconfidence and Category Learning," *Journal of Financial Economics*, 80.

Sandmo, A. (1970): "The Effect of Uncertainty on Saving Decisions. Review of Economic Studies," *Review of Economic Studies*, 37(3).

Seldon, L. (1978): "A New Representation of Preferences over 'Certaint x Uncertainty' Consumption Pairs: the 'Ordinal Certainty Equivalent' Hypothesis," *Econometrica*, 46(5), 1045–1060.

Seldon, L., and X. Wei (2018): "Capital Risk: Precautionary and Excess Saving," *Working Paper*.

Sims, C. (2003): "Implications of Rational Inattention," *Journal of Monetary Economics*, 50, 665–690.

——— (2010): "Rational Inattention and Monetary Economics," *Benjamin J. Friedman, Michael Woodford (Eds.), Handbook of Monetary Economics*, pp. 155–181.

Sinai, T., and N. Souleles (2005): "Owner-occupied Housing as a Hedge Against Rent Risk," *Quarterly Journal of Economics*, 120(2).

Van Nieuwerburgh, S., and L. Veldkamp (2010): "Information Acquisition and Under-Diversification," *Review of Economic Studies*, 77, 779–805.

Visising-Jorgensen, A., and O. Attanasio (2003): "Stock-Market Participation, Intertemporal Substitution, and Risk-Aversion," *American Economic Review*, 93(2), 383–391.

Wang, C., C. Wang, and J. Yang (2016): "Optimal Consumption and Savings with Stochastic Income and Recursive Utility," *Journal of Economic Theory*, 165.

Yin, P. (2019): "The Optimal Amount of Attention to Capital Income Risk," *CESifo Working Paper No. 7413*.

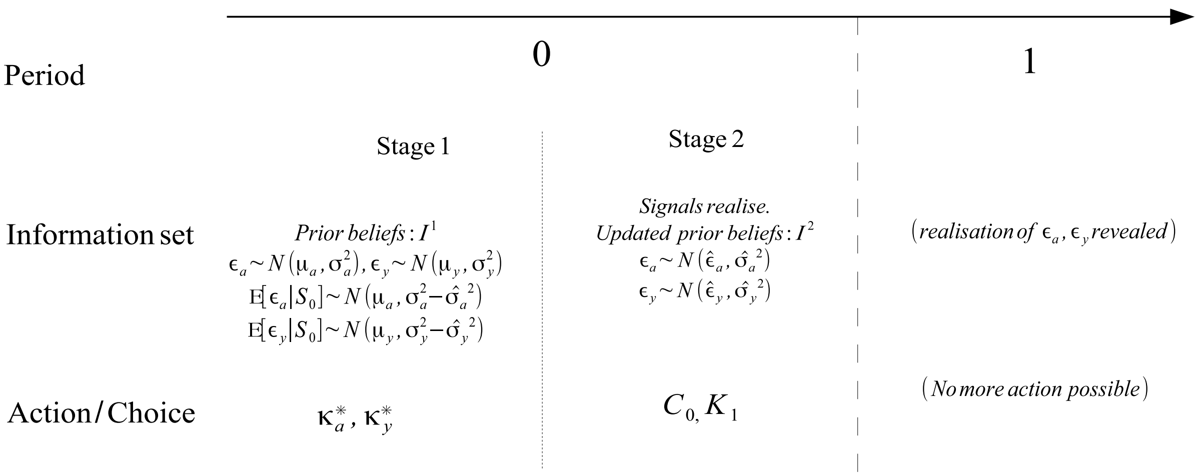

Figure 1: Timeline

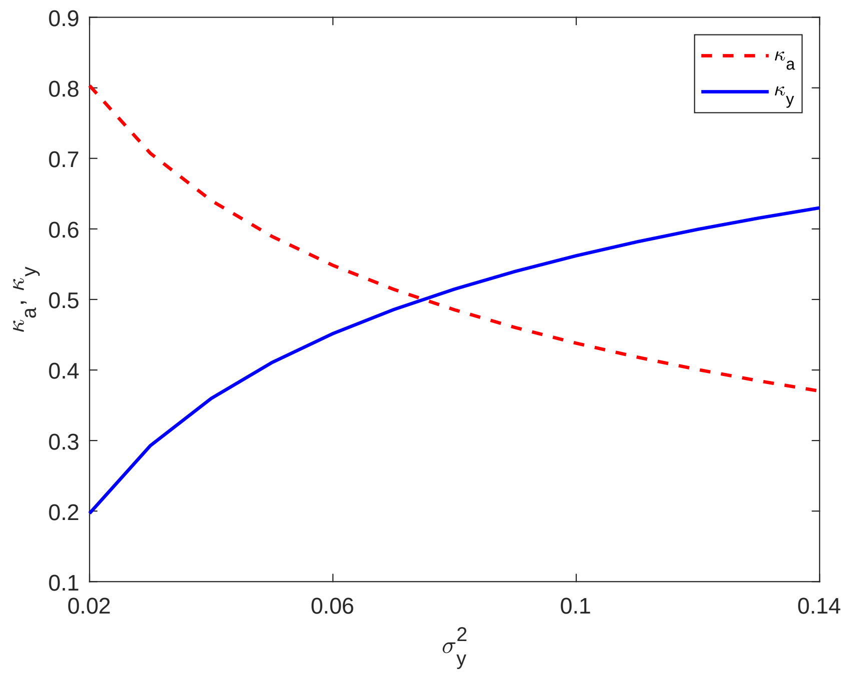

Figure 2: Attention allocation and relative prior variance: $\gamma = 3$

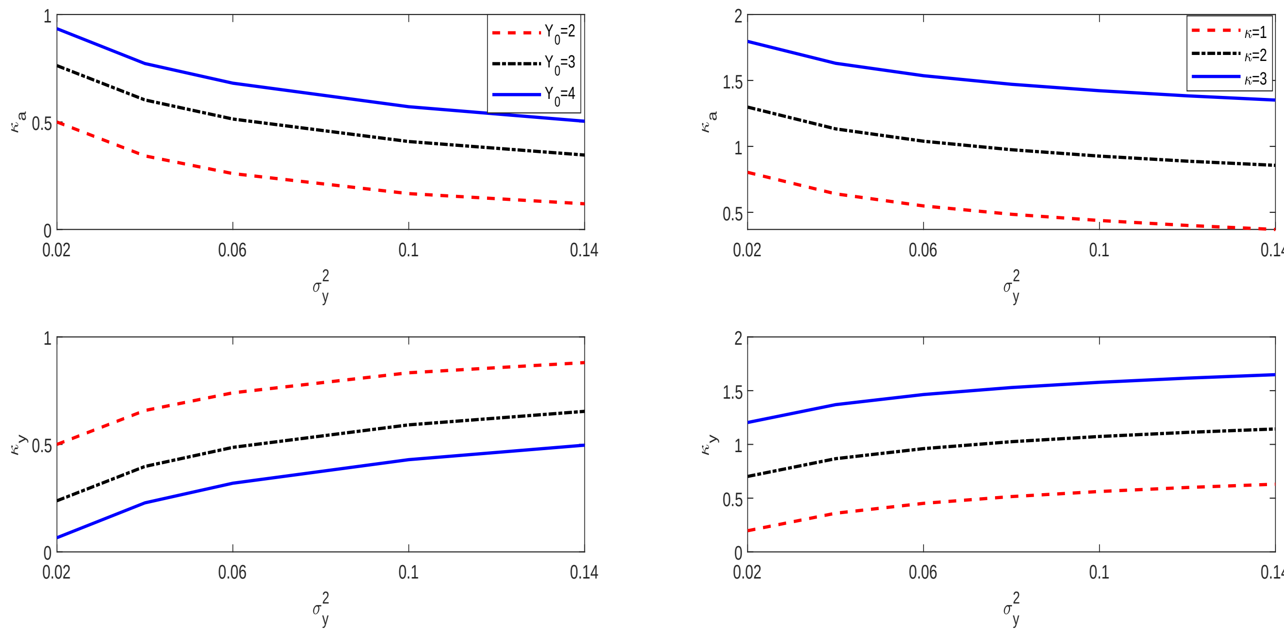

Figure 3: Initial wealth, attention capacity, and attention allocation

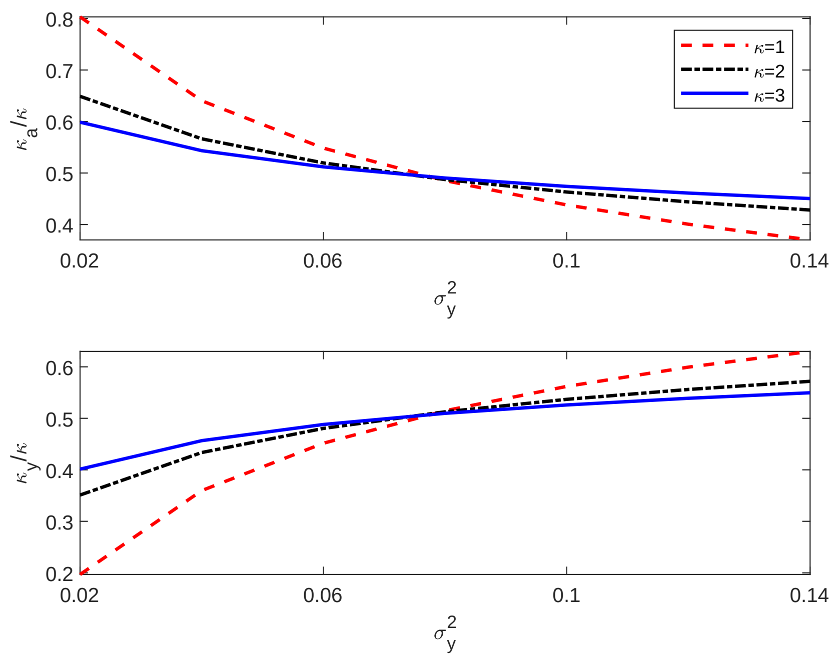

Figure 4: Attention allocation and attention capacity (2)

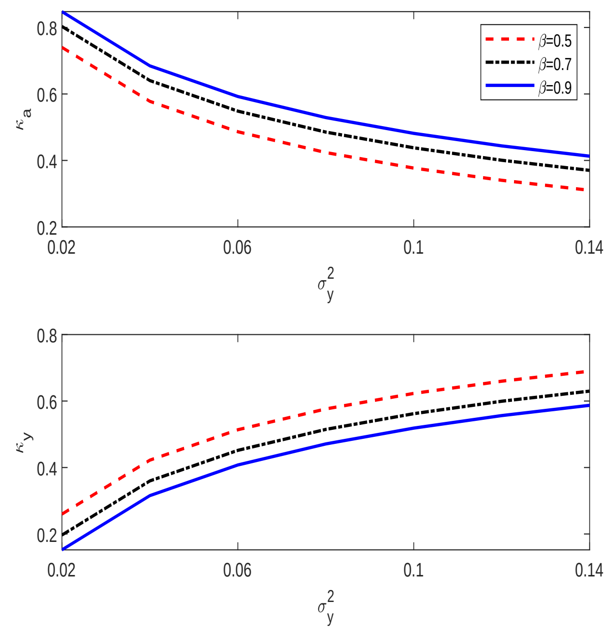

Figure 5: Discount factor, risk aversion, and attention allocation

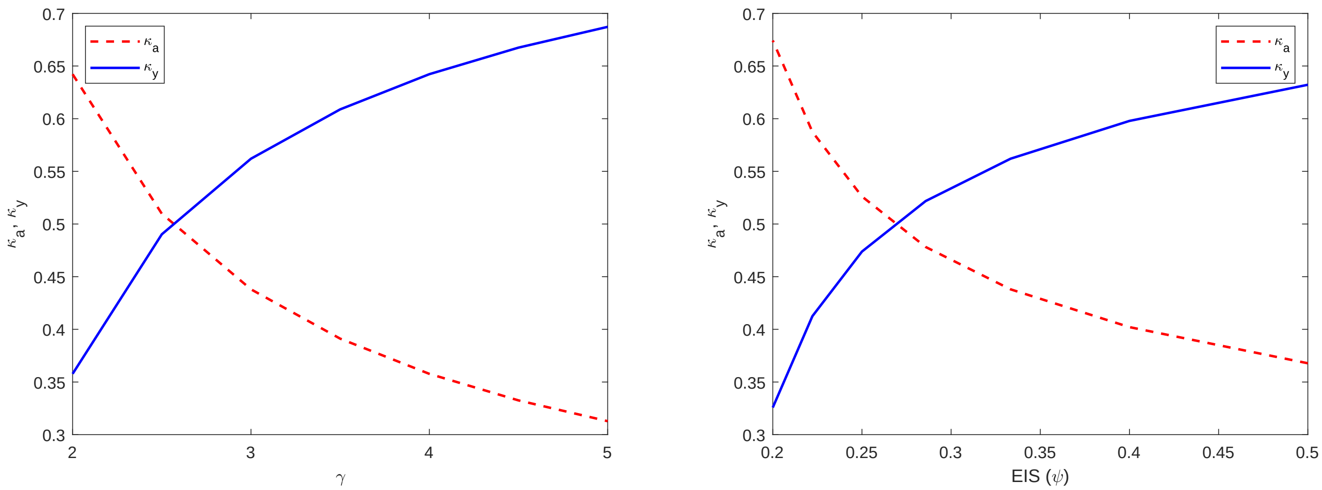

Figure 6: Initial wealth, attention capacity, and consumption growth

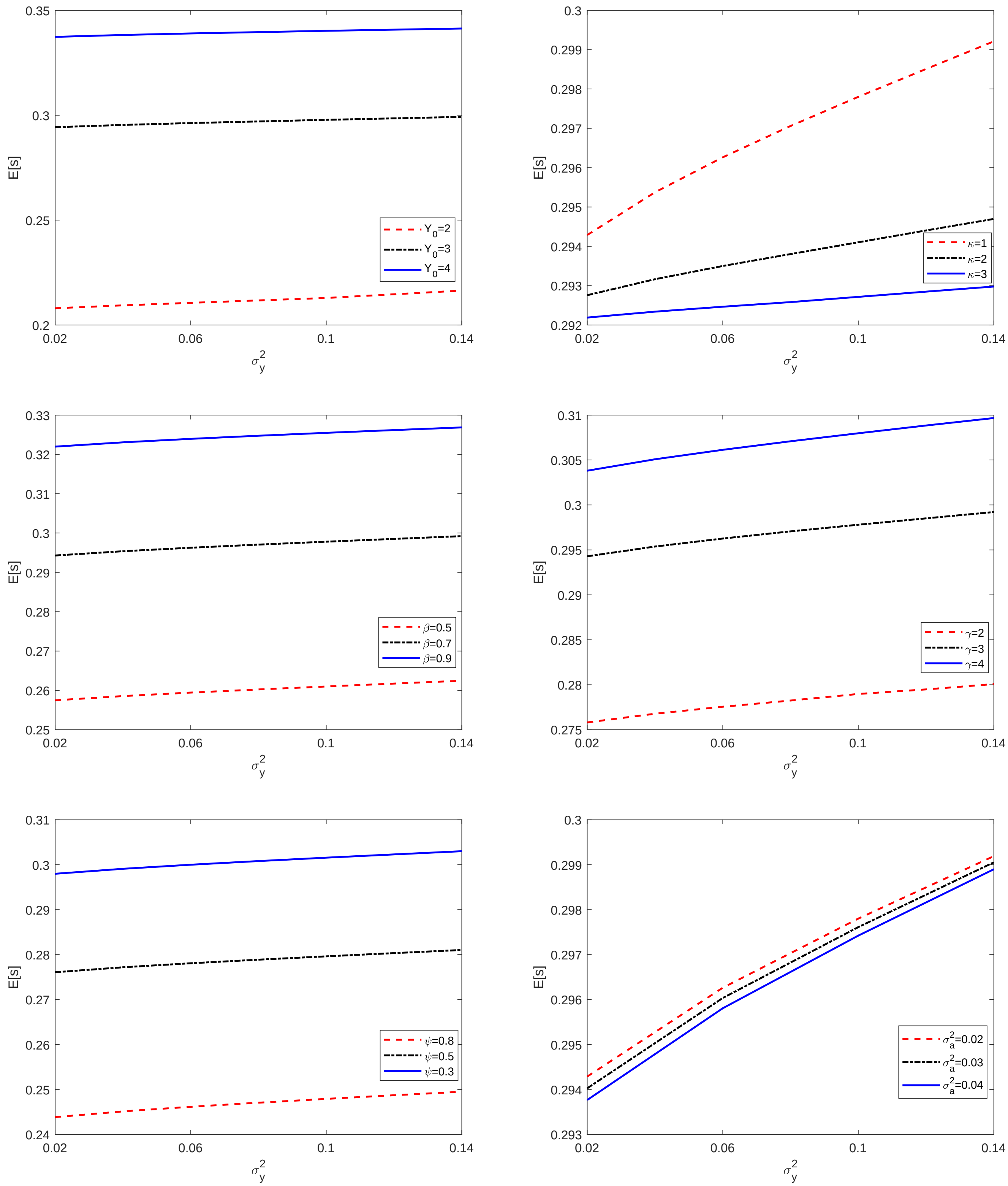

Figure 7: Comparative analysis: average saving rate

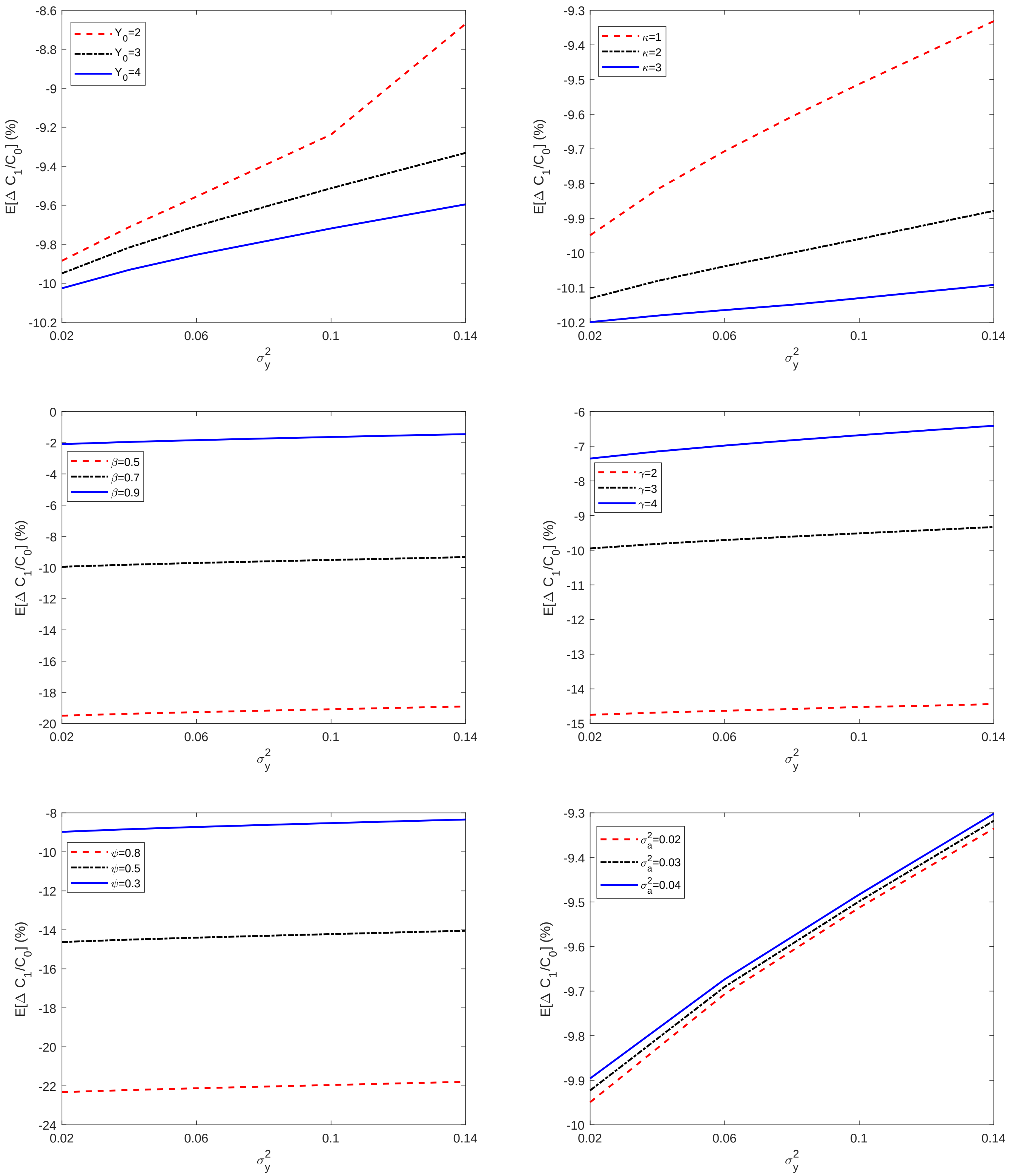

Figure 8: Initial wealth, attention capacity, and consumption growth

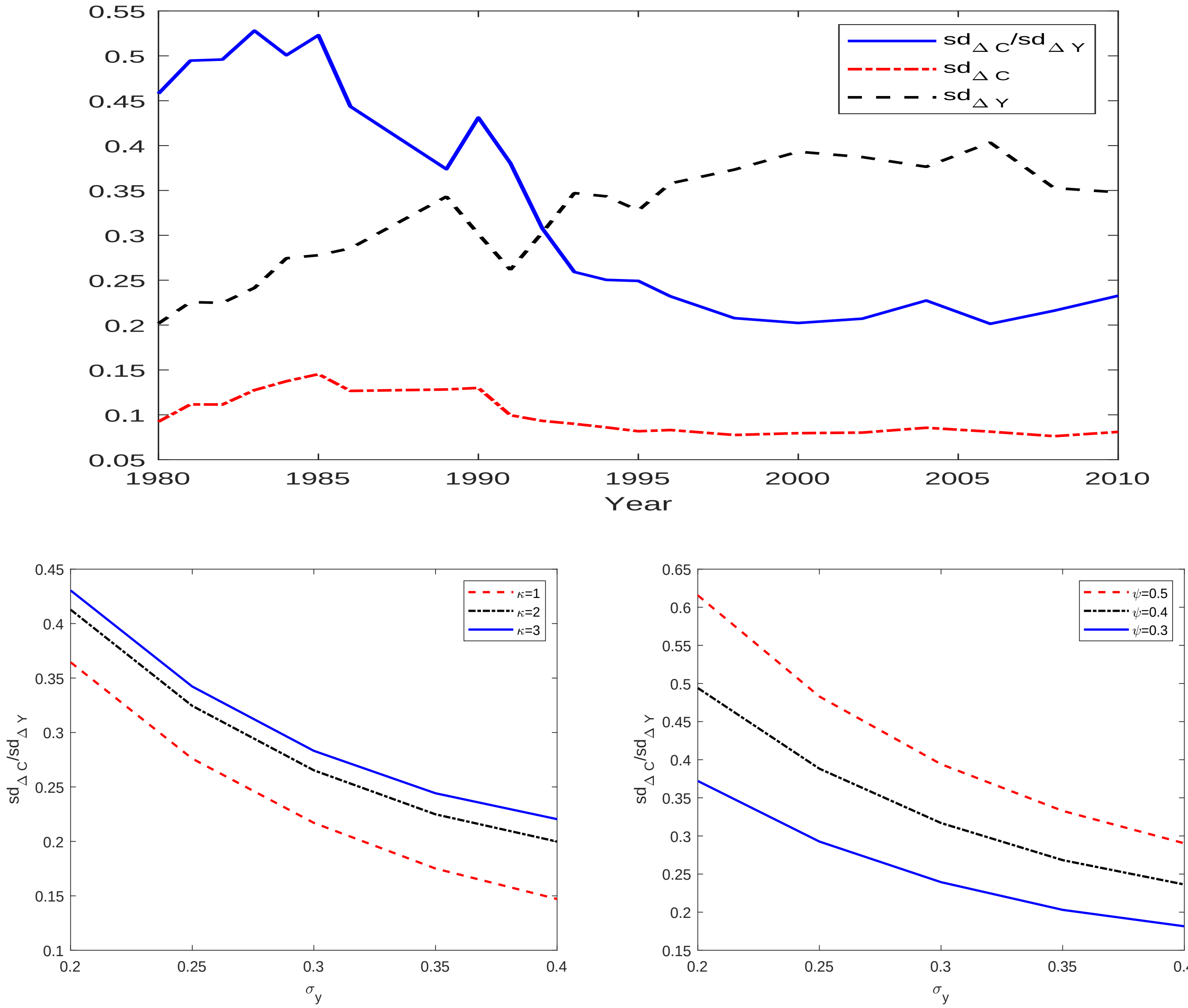

Figure 9: Initial wealth, attention capacity, and consumption growth

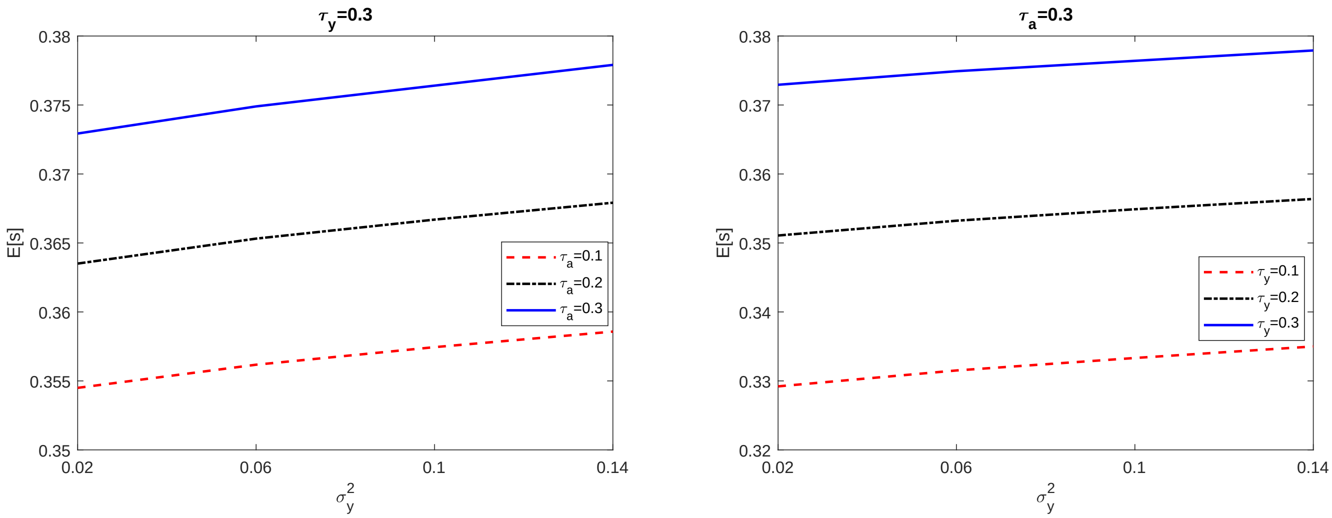

Figure 10: Expected saving rate and income taxes

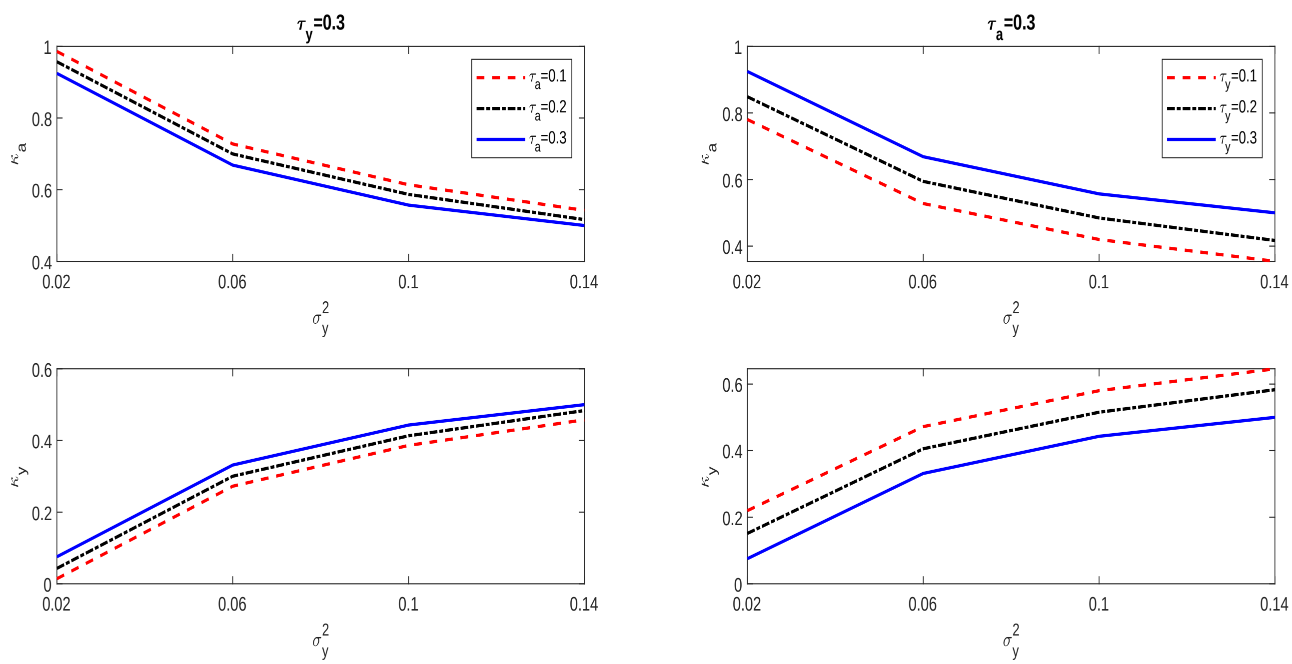

Figure 11: Attention allocation and income taxes

Table 1: Summary table

Demographic and economic characteristics in the sample. Source: 2006 and 2009 HRS Internet Survey, 2008 HRS main Survey.

	Mean
Age	62
Education	14.3
Male	0.474
Couple	0.8
Spending change	-0.042
House value change	-0.183
Financial asset value change	-0.239
Being unemployed in 2007 or 2008	0.042
Household size	1.84
Income	71320
Financial asset	273693
Likelihood of being unemployed (2006 survey)	0.142

Table 2: Elasticities of consumption w.r.t. values of assets and unemployment

Dependent variable is percentage change in consumption. Main explanatory variables are percentage change in financial assets and in the values of the house, and whether respondent became unemployed in 2007 and 2008. Low prior means that in the 2006 survey, respondent answered that he or she has no chance to lose job next year, whereas high prior means that respondents answered that there is a positive chance of losing job in next year. Control variables include education, marriage status, gender. Standard errors are shown in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)
	Low prior	High prior	Low prior	High prior
Percentage change in value of the main residences	0.0475	0.0952**	0.0474	0.0976**
	(0.0643)	(0.0460)	(0.0648)	(0.0461)
Percentage change in value of financial asset	0.105	0.148***	0.107	0.148***
	(0.0649)	(0.0495)	(0.0654)	(0.0497)
Becomes unemployed	-0.187***	-0.0673*	-0.187***	-0.0678**
	(0.0553)	(0.0343)	(0.0555)	(0.0337)
Becomes retired	-0.0566*	-0.0116	-0.0568*	-0.0130
	(0.0320)	(0.0266)	(0.0321)	(0.0266)
Age/100	0.311***	0.299***	0.315***	0.314***
	(0.118)	(0.105)	(0.121)	(0.106)
Household size	0.0261	-0.0246	0.0249	-0.0246
	(0.0350)	(0.0562)	(0.0352)	(0.0563)
Income (million \$)			0.0311	-0.0115
			(0.0830)	(0.0380)
Financial asset (million \$)			0.00505	-0.0164**
			(0.00713)	(0.00758)
Constant	-0.326***	-0.113	-0.322***	-0.138
	(0.113)	(0.116)	(0.115)	(0.115)
R^2	8.6%	5.2%	8.7%	5.5%
N	415	530	415	530

Table 3: Elasticities of consumption w.r.t. values of assets and unemployment

Dependent variable is percentage change in consumption. Main explanatory variables are percentage change in financial assets and in the values of the house, and whether respondent became unemployed in 2007 and 2008. Low prior means that in the 2006 survey, respondent answered that he or she has no chance to lose job next year, whereas high prior means that respondents answered that there is a positive chance of losing job in next year. Control variables include education, marriage status, gender. Cash-on-hand is defined as the sum of financial asset and current income. Standard errors are shown in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)
Percentage change in value of the main residences	0.100**	0.0992**
	(0.0428)	(0.0421)
Percentage change in value of financial asset	0.142***	0.134***
	(0.0487)	(0.0465)
Becomes unemployed	-0.0982**	-0.103***
	(0.0412)	(0.0392)
Cash-on-hand (million $)	-0.0257*	
	(0.0149)	
Cash-on-hand*percentage change in house value	-0.0786**	
	(0.0399)	
Cash-on-hand*percentage change in financial asset	-0.0457	
	(0.0582)	
Cash-on-hand*unemployment	-0.0480*	
	(0.0276)	
Financial asset (million $)		-0.0221
		(0.0150)
Financial asset*percentage change in house value		-0.0970**
		(0.0422)
Financial asset*percentage change in financial asset		-0.0196
		(0.0616)
Financial asset*unemployment		-0.0443*
		(0.0257)
Constant	-0.218***	-0.220***
	(0.0824)	(0.0823)
R^2	8.6%	5.2%
N	945	945

Table 4: Welfare gains in the expected utility case.

Panel A	$\sigma_y^2 = 0.02$	$\sigma_y^2 = 0.06$	$\sigma_y^2 = 0.1$
$\kappa = 1$	0.1776%	0.3117%	0.4068%
$\kappa = 2$	0.0893%	0.1568%	0.2047%
$\kappa = 3$	0.0361%	0.0634%	0.0828%

Panel B	$Y_0 = 2$	$Y_0 = 3$	$Y_0 = 4$
$\kappa = 1$	0.4228%	0.4066%	0.3730%
$\kappa = 2$	0.2135%	0.2047%	0.1878%
$\kappa = 3$	0.0864%	0.0828%	0.0760%

Panel C	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$
$\kappa = 1$	0.1600%	0.4066%	0.7626%
$\kappa = 2$	0.0834%	0.2045%	0.3841%
$\kappa = 3$	0.0341%	0.0827%	0.1555%

Panel D	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.8$
$\kappa = 1$	0.4250%	0.2722%	0.0843%
$\kappa = 2$	0.2138%	0.1373%	0.0426%
$\kappa = 3$	0.0864%	0.0556%	0.0173%

Table 5: Welfare gains analyses (continued).

Panel E	$\tau_y = 0.1$	$\tau_y = 0.2$	$\tau_y = 0.3$
$\kappa = 1$	0.4349%	0.4241%	0.4088%
$\kappa = 2$	0.2188%	0.2134%	0.2057%
$\kappa = 3$	0.0885%	0.0863%	0.0832%

Panel F	$\tau_a = 0.1$	$\tau_a = 0.2$	$\tau_a = 0.3$
$\kappa = 1$	0.3775%	0.3927%	0.4088%
$\kappa = 2$	0.1901%	0.1977%	0.2057%
$\kappa = 3$	0.0769%	0.0800%	0.0832%