

Energy Efficiency of Rate-Splitting Multiple Access, and Performance Benefits over SDMA and NOMA

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Abstract—Rate-Splitting Multiple Access (RSMA) is a general and powerful multiple access framework for downlink multi-antenna systems, and contains Space-Division Multiple Access (SDMA) and Non-Orthogonal Multiple Access (NOMA) as special cases. RSMA relies on linearly precoded rate-splitting with Successive Interference Cancellation (SIC) to decode part of the interference and treat the remaining part of the interference as noise. Recently, RSMA has been shown to outperform both SDMA and NOMA rate-wise in a wide range of network loads (underloaded and overloaded regimes) and user deployments (with a diversity of channel directions, channel strengths and qualities of channel state information at the transmitter). Moreover, RSMA was shown to provide spectral efficiency and QoS enhancements over NOMA at a lower computational complexity for the transmit scheduler and the receivers. In this paper, we build upon those results and investigate the energy efficiency of RSMA compared to SDMA and NOMA. Considering a multiple-input single-output broadcast channel, we show that RSMA is more energy-efficient than SDMA and NOMA in a wide range of user deployments (with a diversity of channel directions and channel strengths). We conclude that RSMA is more spectrally and energy-efficient than SDMA and NOMA.

Index Terms—rate-splitting multiple access, energy efficiency, NOMA, SDMA

I. INTRODUCTION

The cellular network is envisioned to be ultra-dense with the proliferation of the Mobile Internet and the Internet of Things (IoT). The corresponding energy cost is increasing rapidly and becomes a major threat for sustainable development. Much efforts have been spent to solve the problem of Energy Efficiency (EE) maximization so as to keep the optimal trade-off between the Weighted Sum Rate (WSR) and the total power consumption [1]. In [2], the EE maximization problem in Multiple-Input Single-Output Broadcast Channel (MISO BC) subject to sum power and individual Signal-to-Interference plus Noise Ratio (SINR) constraints is investigated. The optimal EE beamformer is obtained by using the Successive Convex Approximation (SCA)-based approach and it is further extended to multi-cell in [3]. A detailed comparison of the related work on the EE problem in multi-antenna systems is illustrated in [4]. The authors solve the EE maximization problem in Multiple-Input Multiple-Output (MIMO) BC by using the successive pseudoconvex approximation approach. All of the above works consider Space Division Multiple Access (SDMA) based on Multi-User Linear Precoding (MU-LP)

beamforming. Each receiver only decodes its intended message by fully treating any residual interference from other users as noise. However, SDMA based on MU-LP is only suited to the underloaded regime and the scenarios where the user channels are sufficiently orthogonal.

Power-domain Non-orthogonal Multiple Access (NOMA) (simply referred to as NOMA in the sequel) based on Superposition Coding (SC) at the transmitter and Successive Interference Cancellation (SIC) at the receivers has been recognized as a promising multiple access scheme for future mobile networks [5]. Different from SDMA, some users are forced to fully decode and cancel interference from other users. Though NOMA has the capability of serving users in an overloaded regime, it is only suited to the deployments where the user channels are sufficiently aligned and exhibit a large disparity in channel strengths. Efforts have been paid to the precoder design so as to achieve the best EE in the NOMA-based MIMO BC. In [6], the EE maximization problem in the two-user NOMA-based MIMO BC is investigated under the assumption of only statistical Channel State Information (CSI) to be known at the transmitter. The K -user NOMA-based MIMO BC is investigated in [7]. However, the precoder is designed based on interference alignment and only power is optimized.

To overcome the shortcomings of SDMA and NOMA, a novel multiple access scheme called Rate-Splitting Multiple Access (RSMA) is proposed in [8]. RSMA, based on linear precoded Rate-Splitting (RS), has the ability to partially decode interference and partially treat interference as noise. As a consequence, RSMA bridges and unifies the two extremes of SDMA and NOMA. To partially decode interference, various messages of users are split into common and private parts in RS. The common parts are jointly encoded and decoded by multiple users while the private parts are decoded by the corresponding users only. RSMA has been shown in [8] to be more spectrally efficient than SDMA and NOMA in a wide range of user deployments (with a diversity of channel directions and channel strengths), and in the presence of perfect and imperfect CSI at the Transmitter (CSIT). However, the EE performance of RSMA has never been studied. In the literature, the two-user RS-assisted EE maximization problem in MIMO Interference Channel (IC) has been investigated in [9] by allowing one of the two users to use RS. Different from [9], we initiate the investigation of RS-assisted EE maximization problem in MISO BC.

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Building upon the results in [8], we study the EE of RSMA in this work and compare with SDMA and NOMA. To investigate the EE region achieved by different multiple access schemes, we consider a EE metric defined as the WSR divided by the total power consumption. A SCA-based beamforming algorithm is proposed to solve the RSMA EE maximization problem. SDMA and NOMA EE maximization problems are obtained as a special case of the RSMA EE maximization framework. The performance of the proposed SCA-based algorithm and the resulting EE regions achieved by different multiple access schemes are compared in the numerical results. The convergence rate of the proposed algorithm is shown to be high and the EE region of RSMA is shown to be equal to or larger than that of SDMA and NOMA in any user deployments. RSMA is therefore more energy-efficient than SDMA and NOMA.

The rest of the paper is organized as follows. Section II specifies the system model and the formulated EE problem of SDMA and NOMA. The proposed EE problem of RSMA is discussed in Section III followed by the proposed algorithm based on SCA in Section IV. Section V shows numerical results and Section VI concludes the paper.

Notations: \mathbb{C} refers to the complex space and $\mathbb{E}\{\cdot\}$ refers to the statistical expectation. $\text{tr}(\cdot)$ is the trace. The boldface uppercase and lowercase letters represent matrices and vectors, respectively. $\|\cdot\|$ is the Euclidean norm. The superscripts $(\cdot)^T$ and $(\cdot)^H$ are transpose and conjugate-transpose operators.

II. SYSTEM MODEL AND EXISTING MULTIPLE ACCESS

A. System Model

Consider the downlink transmission of a two-user¹ MISO system where one Base Station (BS) equipped with N_t transmit antennas serves two single-antenna users. The BS wants to transmit the messages W_1, W_2 respectively to user-1 and user-2 in each time frame. The messages are encoded based on different multiple access schemes and form the transmit signal $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$. The total transmit power of the BS is subject to a power constraint P_t as $\mathbb{E}\{\|\mathbf{x}\|^2\} \leq P_t$.

The received signal at user- $k, \forall k \in \{1, 2\}$ is

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \quad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$ is the channel from user- k to BS. It is perfectly known at BS. $n_k \sim \mathcal{CN}(0, \sigma_{n,k}^2)$ is the Additive White Gaussian Noise (AWGN) at user- k with zero mean and variance $\sigma_{n,k}^2$.

B. Power Consumption Model

The power consumption at BS contains not only the transmit power, but also the circuit power due to the electronic operations. The linear power model specified in [10] is adopted in this work. The total power consumption is

$$P_{\text{total}} = \frac{1}{\eta} P_{\text{tran}} + P_{\text{cir}}, \quad (2)$$

where $P_{\text{tran}} \triangleq \mathbb{E}\{\|\mathbf{x}\|^2\}$ is the transmit power. $\eta \in [0, 1]$ is the power amplifier efficiency. $P_{\text{cir}} = N_t P_{\text{dyn}} + P_{\text{sta}}$ is the circuit power. P_{dyn} denotes the dynamic power consumption. It is the power consumption of one active Radio Frequency

chain. P_{sta} denotes the static power consumption, which is the power consumption of cooling systems, power supply and so on. The power consumption at user sides is omitted since the power consumption of users is negligible compared with the power consumption of BS [10].

C. Existing Multiple Access

We briefly review two baseline multiple access schemes, namely, SDMA and NOMA and the corresponding EE maximization problem. At the BS, the messages W_1 and W_2 are independently encoded into the data streams s_1 and s_2 . The data streams are respectively multiplied by the beamforming vectors $\mathbf{p}_1, \mathbf{p}_2 \in \mathbb{C}^{N_t \times 1}$ and superposed as

$$\mathbf{x} = \mathbf{P}\mathbf{s} = \mathbf{p}_1 s_1 + \mathbf{p}_2 s_2, \quad (3)$$

where $\mathbf{s} \triangleq [s_1, s_2]^T$ and $\mathbf{P} \triangleq [\mathbf{p}_1, \mathbf{p}_2]$. Assuming that $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$, the transmit power becomes $P_{\text{tran}} = \text{tr}(\mathbf{P}\mathbf{P}^H)$. It is constrained by $\text{tr}(\mathbf{P}\mathbf{P}^H) \leq P_t$.

1) *SDMA:* In the well-known MU-LP based SDMA scheme, each user only decodes its desired message by treating any residual interference as noise. The SINR at user- $k, \forall k \in \{1, 2\}$ is given by $\gamma_k(\mathbf{P}) = |\mathbf{h}_k^H \mathbf{p}_k|^2 / (|\mathbf{h}_k^H \mathbf{p}_j|^2 + N_{0,k})$, where $j \neq k, j \in \{1, 2\}$. $N_{0,k} = W\sigma_{n,k}^2$ is the noise power at user- k over the transmission bandwidth W . The corresponding achievable rate of user- k is $R_k(\mathbf{P}) = W \log_2(1 + \gamma_k(\mathbf{P}))$.

For a given weight vector $\mathbf{u} = [u_1, u_2]$, the SDMA-based EE maximization problem is given by

$$\text{EE}_{\text{SDMA}} \begin{cases} \max_{\mathbf{P}} & \frac{\sum_{k \in \{1,2\}} u_k R_k(\mathbf{P})}{\frac{1}{\eta} \text{tr}(\mathbf{P}\mathbf{P}^H) + P_{\text{cir}}} \\ \text{s.t.} & \text{tr}(\mathbf{P}\mathbf{P}^H) \leq P_t \end{cases} \quad (4a)$$

$$(4b)$$

2) *NOMA:* Contrary to SDMA where each user only decodes its desired message, linearly precoded superposition coding is used in NOMA and one of the two users is required to fully decode the interfering message before decoding the desired message. SIC is deployed at user sides. The decoding order is required to be optimized together with the precoder. π denotes one of the decoding orders. The message of user- $\pi(1)$ is decoded before user- $\pi(2)$. At user- $\pi(1)$, the desired message is decoded directly by treating any interference as noise. The SINR at user- $\pi(1)$ is given by $\gamma_{\pi(1)}(\mathbf{P}) = |\mathbf{h}_{\pi(1)}^H \mathbf{p}_{\pi(1)}|^2 / (|\mathbf{h}_{\pi(1)}^H \mathbf{p}_{\pi(2)}|^2 + N_{0,\pi(1)})$. At user- $\pi(2)$, the interference from user- $\pi(1)$ is decoded before decoding the desired message. The SINR at user- $\pi(2)$ to decode the message of user- $\pi(1)$ is given by $\gamma_{\pi(2) \rightarrow \pi(1)}(\mathbf{P}) = |\mathbf{h}_{\pi(2)}^H \mathbf{p}_{\pi(1)}|^2 / (|\mathbf{h}_{\pi(2)}^H \mathbf{p}_{\pi(2)}|^2 + N_{0,\pi(2)})$. Once the message of user- $\pi(1)$ is decoded and removed from the retrieved signal via SIC, user- $\pi(2)$ decodes its intended message. The SINR experienced at user- $\pi(2)$ to decode the desired message is $\gamma_{\pi(2)}(\mathbf{P}) = |\mathbf{h}_{\pi(2)}^H \mathbf{p}_{\pi(2)}|^2 / N_{0,\pi(2)}$. The corresponding achievable rates of user- $\pi(1)$ and user- $\pi(2)$ are $R_{\pi(1)}(\mathbf{P}) = \min\{W \log_2(1 + \gamma_{\pi(1)}(\mathbf{P})), W \log_2(1 + \gamma_{\pi(2) \rightarrow \pi(1)}(\mathbf{P}))\}$ and $R_{\pi(2)}(\mathbf{P}) = W \log_2(1 + \gamma_{\pi(2)}(\mathbf{P}))$.

For a given weight vector \mathbf{u} , the NOMA-based EE maximization problem is given by

$$\text{EE}_{\text{NOMA}} \begin{cases} \max_{\pi, \mathbf{P}} & \frac{\sum_{k \in \{1,2\}} u_{\pi(k)} R_{\pi(k)}(\mathbf{P})}{\frac{1}{\eta} \text{tr}(\mathbf{P}\mathbf{P}^H) + P_{\text{cir}}} \\ \text{s.t.} & \text{tr}(\mathbf{P}\mathbf{P}^H) \leq P_t \end{cases} \quad (5a)$$

$$(5b)$$

¹We consider a two-user scenario for readability and page constraint reasons, though the system model can be extended to the general K -user. The extension will be treated in the journal version of this paper.

To maximize EE, the decoding order is required to be jointly optimized with the beamforming vectors.

III. PROBLEM FORMULATION OF RSMA

In this section, we introduce RSMA and the formulated RSMA-based EE maximization problem. When there are two users in the system, the generalized RSMA proposed in [8] reduces to the 1-layer RS investigated in [11]. It is easy to extend the following formulated problem to the K -user 1-layer RS, 2-layer Hierarchical RS (HRS) as well as the generalized RS of [8].

RSMA differs from the existing multiple access schemes mainly due to the generation of the transmit signal \mathbf{x} . The messages of both SDMA and NOMA are encoded into independent streams directly. In contrast, the message $W_k, \forall k \in \{1, 2\}$ of RSMA is split into a common part $W_{k,c}$ and a private part $W_{k,p}$. The common parts of both users $W_{1,c}, W_{2,c}$ are jointly encoded into a common stream s_c using a codebook shared by both users. s_c is intended for both users. The private parts are encoded into s_1 and s_2 for user-1 and user-2, respectively. The stream vector $\mathbf{s} = [s_c, s_1, s_2]^T$ is linearly precoded using the beamformer $\mathbf{P} = [\mathbf{p}_c, \mathbf{p}_1, \mathbf{p}_2]$. The resulting transmit signal is

$$\mathbf{x} = \mathbf{P}\mathbf{s} = \mathbf{p}_c s_c + \mathbf{p}_1 s_1 + \mathbf{p}_2 s_2, \quad (6)$$

The transmit power $\text{tr}(\mathbf{P}\mathbf{P}^H)$ is constrained by P_t as well.

The common stream s_c is decoded first at both users by treating the interference from the private streams s_1 and s_2 as noise. As s_c contains part of the intended message as well as part of the message of the interfering user, it enables the capability of partially decoding interference and partially treating interference as noise. The SINR of decoding the common stream s_c at user- $k, \forall k \in \{1, 2\}$ is

$$\gamma_{c,k}(\mathbf{P}) = \frac{|\mathbf{h}_k^H \mathbf{p}_c|^2}{|\mathbf{h}_k^H \mathbf{p}_1|^2 + |\mathbf{h}_k^H \mathbf{p}_2|^2 + N_{0,k}}. \quad (7)$$

The achievable rate of decoding s_c at user- k is $R_{c,k}(\mathbf{P}) = W \log_2(1 + \gamma_{c,k}(\mathbf{P}))$. To guarantee that s_c is decoded by both users, the common rate shall not exceed

$$R_c(\mathbf{P}) = \min\{R_{c,1}(\mathbf{P}), R_{c,2}(\mathbf{P})\}. \quad (8)$$

Note that $R_c(\mathbf{P})$ is shared by both users. Denote C_k as the k th user's portion of the common rate. We have

$$C_1 + C_2 = R_c(\mathbf{P}). \quad (9)$$

Once s_c is decoded and removed from the received signal via SIC, user- k decodes its desired private stream s_k by treating the interference of user- j ($j \neq k$) as noise. The SINR of decoding the private stream s_k at user- $k, \forall k \in \{1, 2\}$ is

$$\gamma_k(\mathbf{P}) = \frac{|\mathbf{h}_k^H \mathbf{p}_k|^2}{|\mathbf{h}_k^H \mathbf{p}_j|^2 + N_{0,k}}. \quad (10)$$

The achievable rate of decoding s_k at user- k is $R_k(\mathbf{P}) = W \log_2(1 + \gamma_k(\mathbf{P}))$. The achievable rate of user- k is $R_{k,tot}(\mathbf{P}) = C_k + R_k(\mathbf{P})$. Recall from [8], SDMA and NOMA are both sub-schemes of RSMA.

Based on the above model, the RSMA-based EE maximization problem for a given weight vector \mathbf{u} is

$$\text{EE}_{\text{RSMA}} \begin{cases} \max_{\mathbf{c}, \mathbf{P}} \frac{\sum_{k \in \{1,2\}} u_k (C_k + R_k(\mathbf{P}))}{\frac{1}{\eta} \text{tr}(\mathbf{P}\mathbf{P}^H) + P_{\text{cir}}} & (11a) \\ \text{s.t.} & C_1 + C_2 \leq R_{c,k}(\mathbf{P}), \forall k \in \{1, 2\} & (11b) \\ & \text{tr}(\mathbf{P}\mathbf{P}^H) \leq P_t & (11c) \\ & \mathbf{c} \geq \mathbf{0} & (11d) \end{cases}$$

where $\mathbf{c} = [C_1, C_2]$ is the common rate vector required to be optimized with the beamforming vectors. Constraint (11b) ensures that the common stream can be successfully decoded at both users.

IV. SCA-BASED OPTIMIZATION FRAMEWORK

The EE maximization problems described above are non-convex fractional programs. Motivated by the SCA algorithms adopted in the literature of EE maximization [2], [3], [12], [13], we propose a SCA-based beamforming algorithm to solve the formulated EE maximization problems. To achieve a high-performance approximation, auxiliary variables are introduced to first transform the original EE problem into its equivalent problem and approximations are applied to the transformed problem iteratively. The procedure to solve the EE_{RSMA} problem will be explained next and it can be easily applied to solve the $\text{EE}_{\text{MU-LP}}$ and EE_{NOMA} problems.

First of all, by introducing scalar variables ω^2 , z and t , respectively representing the weighted sum rate, total power consumption and EE metric, problem (11) is equivalently transformed into

$$\max_{\mathbf{c}, \mathbf{P}, \omega, z, t} t \quad (12a)$$

$$\text{s.t.} \quad \frac{\omega^2}{z} \geq t \quad (12b)$$

$$\sum_{k \in \{1,2\}} u_k (C_k + R_k(\mathbf{P})) \geq \omega^2 \quad (12c)$$

$$z \geq \frac{1}{\eta} \text{tr}(\mathbf{P}\mathbf{P}^H) + P_{\text{cir}} \quad (12d)$$

$$(11b), (11c), (11d) \quad (12e)$$

The equivalence between (12) and (11) is established based on the fact that constraints (12b), (12c) and (12d) must hold with equality at optimum.

The difficulty of solving (12) lies in the non-convexity of the constraints (11b), (12b) and (12c). We further introduce variables $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]^T$ representing the set of the private rates. Constraints (12c) is equivalent to

$$(12c) \Leftrightarrow \begin{cases} \sum_{k \in \{1,2\}} u_k (C_k + \alpha_k) \geq \omega^2 & (13a) \\ R_k(\mathbf{P}) \geq \alpha_k, \forall k \in \{1, 2\} & (13b) \end{cases}$$

To deal with the non-convex constraint (13b), we add variables $\boldsymbol{\vartheta} = [\vartheta_1, \vartheta_2]^T$ representing 1 plus the SINR of each private stream. The rate constraint becomes

$$(13b) \Leftrightarrow \begin{cases} \vartheta_k \geq 2^{\frac{\alpha_k}{W}}, \forall k \in \{1, 2\} & (14a) \\ 1 + \gamma_k(\mathbf{P}) \geq \vartheta_k, \forall k \in \{1, 2\} & (14b) \end{cases}$$

where (14a) is transformed from $W \log_2 \vartheta_k \geq \alpha_k$. $\gamma_k(\mathbf{P})$ is calculated based on (10). (14b) is further transformed into

$$(14b) \Leftrightarrow \begin{cases} \frac{|\mathbf{h}_k^H \mathbf{p}_k|^2}{\beta_k} \geq \vartheta_k - 1, \forall k \in \{1, 2\} & (15a) \\ \beta_k \geq N_{0,k} + \sum_{j \neq k} |\mathbf{h}_k^H \mathbf{p}_j|^2, \forall k \in \{1, 2\} & (15b) \end{cases}$$

where $\boldsymbol{\beta} = [\beta_1, \beta_2]^T$ are new variables representing the interference plus noise at each user to decode its private steam. Therefore, constraint (12c) can be replaced by

$$(12c) \Leftrightarrow (13a), (14a), (15)$$

Similarly, we introduce variables $\boldsymbol{\alpha}_c = [\alpha_{c,1}, \alpha_{c,2}]^H$ representing the common rate at user sides, $\boldsymbol{\vartheta}_c = [\vartheta_{c,1}, \vartheta_{c,2}]^T$ representing 1 plus the SINR of the common stream as well as $\boldsymbol{\beta}_c = [\beta_{c,1}, \beta_{c,2}]^T$ representing the interference plus noise at each user to decode the common steam, constraint (11b) is equivalent to

$$(11b) \Leftrightarrow \begin{cases} C_1 + C_2 \leq \alpha_{c,k}, \forall k \in \{1, 2\} & (16a) \\ \vartheta_{c,k} \geq 2 \frac{\alpha_{c,k}}{W}, \forall k \in \{1, 2\} & (16b) \\ \frac{|\mathbf{h}_k^H \mathbf{p}_c|^2}{\beta_{c,k}} \geq \vartheta_{c,k} - 1, \forall k \in \{1, 2\} & (16c) \\ \beta_{c,k} \geq N_{0,k} + |\mathbf{h}_k^H \mathbf{p}_1|^2 + |\mathbf{h}_k^H \mathbf{p}_2|^2, \forall k & (16d) \end{cases}$$

Therefore, problem (11) is equivalently transformed into

$$\begin{aligned} & \max_{\mathbf{c}, \mathbf{P}, \omega, z, t, \boldsymbol{\alpha}_c, \boldsymbol{\alpha}, \boldsymbol{\vartheta}_c, \boldsymbol{\vartheta}, \boldsymbol{\beta}_c, \boldsymbol{\beta}} t \\ & \text{s.t.} \quad (11c), (11d), (12b), (12d) \\ & \quad (13a), (14a), (15), (16) \end{aligned}$$

The constraints of the transformed problem are convex except (12b), (15a) and (16c). So we use the linear approximation to approximate the non-convex part of the constraints in each iteration. The left side of (12b) is approximated using the first-order lower approximation, which is given by

$$\frac{\omega^2}{z} \geq \frac{2\omega^{[n]}}{z^{[n]}}\omega - \left(\frac{\omega^{[n]}}{z^{[n]}}\right)^2 z \triangleq \Omega^{[n]}(\omega, z), \quad (17)$$

where $(\omega^{[n]}, z^{[n]})$ are the values of the variables (ω, z) at the output of the n th iteration. The left side of (15a) and (16c) are written using the linear lower bound approximation at the point $(\mathbf{p}_k^{[n]}, \beta_k^{[n]})$ and $(\mathbf{p}_c^{[n]}, \beta_{c,k}^{[n]})$ respectively as

$$\begin{aligned} \frac{|\mathbf{h}_k^H \mathbf{p}_k|^2}{\beta_k} & \geq 2\text{Re} \left((\mathbf{p}_k^{[n]})^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{p}_k \right) / \beta_k^{[n]} \\ & - \left(\left| \mathbf{h}_k^H \mathbf{p}_k^{[n]} \right| / \beta_k^{[n]} \right)^2 \beta_k \triangleq \Psi_k^{[n]}(\mathbf{p}_k, \beta_k), \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{|\mathbf{h}_k^H \mathbf{p}_c|^2}{\beta_{c,k}} & \geq 2\text{Re} \left((\mathbf{p}_c^{[n]})^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{p}_c \right) / \beta_{c,k}^{[n]} \\ & - \left(\left| \mathbf{h}_k^H \mathbf{p}_c^{[n]} \right| / \beta_{c,k}^{[n]} \right)^2 \beta_{c,k} \triangleq \Psi_{c,k}^{[n]}(\mathbf{p}_c, \beta_{c,k}). \end{aligned} \quad (19)$$

Based on the approximations (17)–(19), problem (11) is approximated at iteration n as

Algorithm 1: SCA-based beamforming algorithm with RS

- 1 **Initialize:** $n \leftarrow 0, t^{[n]}, \omega^{[n]}, z^{[n]}, \mathbf{P}^{[n]}, \boldsymbol{\beta}_c^{[n]}, \boldsymbol{\beta}^{[n]}$;
- 2 **repeat**
- 3 $n \leftarrow n + 1$;
- 4 Solve problem (20) using $\omega^{[n-1]}, z^{[n-1]}, \mathbf{P}^{[n-1]}, \boldsymbol{\beta}_c^{[n-1]}, \boldsymbol{\beta}^{[n-1]}$ and denote the optimal objective as t^* and the optimal variables as $\omega^*, z^*, \mathbf{P}^*, \boldsymbol{\beta}_c^*, \boldsymbol{\beta}^*$;
- 5 Update $t^{[n]} \leftarrow t^*, \omega^{[n]} \leftarrow \omega^*, z^{[n]} \leftarrow z^*, \mathbf{P}^{[n]} \leftarrow \mathbf{P}^*, \boldsymbol{\beta}_c^{[n]} \leftarrow \boldsymbol{\beta}_c^*, \boldsymbol{\beta}^{[n]} \leftarrow \boldsymbol{\beta}^*$;
- 6 **until** $|t^{[n]} - t^{[n-1]}| < \epsilon$;

$$\begin{aligned} & \max_{\mathbf{c}, \mathbf{P}, \omega, z, t, \boldsymbol{\alpha}_c, \boldsymbol{\alpha}, \boldsymbol{\vartheta}_c, \boldsymbol{\vartheta}, \boldsymbol{\beta}_c, \boldsymbol{\beta}} t \\ & \text{s.t.} \quad \Omega^{[n]}(\omega, z) \geq t \\ & \quad \Psi_k^{[n]}(\mathbf{p}_k, \beta_k) \geq \vartheta_k - 1, \forall k \in \{1, 2\} \\ & \quad \Psi_{c,k}^{[n]}(\mathbf{p}_c, \beta_{c,k}) \geq \vartheta_{c,k} - 1, \forall k \in \{1, 2\} \\ & \quad (11c), (11d), (12d), (13a), (14a), \\ & \quad (15b), (16a), (16b), (16d) \end{aligned} \quad (20)$$

The problem (20) is convex and can be solved using CVX, a package for solving disciplined convex programs in Matlab [14]. The SCA-based beamforming algorithm with RS is outlined in Algorithm 1. In each iteration, problem (20) is solved and $\omega^{[n]}, z^{[n]}, \mathbf{P}^{[n]}, \boldsymbol{\beta}_c^{[n]}, \boldsymbol{\beta}^{[n]}$ are updated using the corresponding optimized variables. $t^{[n]}$ is the maximized EE at the output of the n th iteration. ϵ is the tolerance of the algorithm.

Initialization: The beamformer $\mathbf{P}^{[0]}$ is initialized by finding the feasible beamformer satisfying the transmit power constraint (11c). The common rate vector $\mathbf{c}^{[0]}$ is initialized by assuming the common rate $R_{c,k}(\mathbf{P}^{[0]})$ is uniformly allocated to user-1 and user-2. $\omega^{[0]}, z^{[0]}, \beta_k^{[0]}$ and $\beta_{c,k}^{[0]}$ are initialized by replacing the inequalities of (12c), (12d), (15b) and (16d) with equalities, respectively.

Convergence Analysis: As (12b), (15a) and (16c) are relaxed by the first-order lower bounds (17)–(19), the solution of problem (20) at iteration $[n]$ is also a feasible solution at iteration $[n + 1]$. Therefore, the optimized objective is non-decreasing as iteration increases, $t^{[n+1]} \geq t^{[n]}$ always holds. As the EE t is bounded above by the transmit power constraint (11c), the proposed algorithm is guaranteed to converge. Due to the linear approximation of the constraints (12b), (15a) and (16c), the global optimality of the achieved solution can not be guaranteed.

V. NUMERICAL RESULTS

In this section, we evaluate the performance of RSMA by comparing the EE region of RSMA with that of SDMA and NOMA. The two-user EE region consists of all achievable individual EE-pairs $(\text{EE}_1, \text{EE}_2)$. The individual EE is defined as the individual achievable rate divided by the sum power. For example, the individual EE of user- k in RSMA is

$$\text{EE}_k = \frac{C_k + R_k(\mathbf{P})}{\frac{1}{\eta} \text{tr}(\mathbf{P}\mathbf{P}^H) + P_{\text{cir}}}, \forall k \in \{1, 2\}. \quad (21)$$

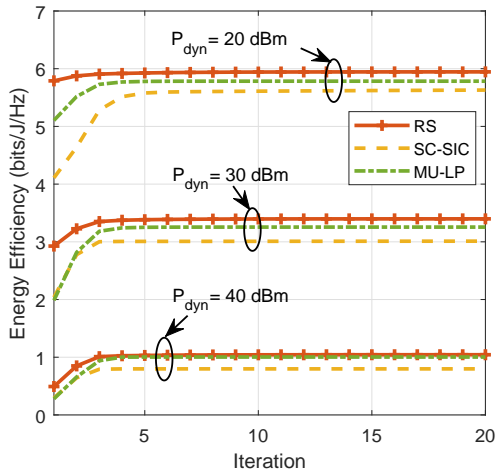


Fig. 1: Convergence of the proposed SCA-based beamforming algorithm with different schemes, $u_1 = 1, u_2 = 1, \gamma = 1, \theta = \frac{2\pi}{9}, P_{\text{dyn}} = 20, 30$ and 40 dBm.

The boundary of the EE region is calculated by varying the weights assigned to users. Following the rate region simulation in [8], the weight of user-1 in this work is fixed to $u_1 = 1$ and that of user-2 is changed as $u_2 = 10[-3, -1, -0.95, \dots, 0.95, 1, 3]$. The BS is assumed to have four transmit antennas ($N_t = 4$). Without loss of generality, unit noise variance ($\sigma_{n,k}^2 = 1$) and unit bandwidth ($W = 1$ Hz) is considered. The transmit power constraint is $P_t = 40$ dBm.² The static power consumption is $P_{\text{sta}} = 30$ dBm and the power amplifier efficiency is $\eta = 0.35$. We follow the channel model in [8] to investigate the effect of channel angle and channel gain disparity on EE region shape. The channels are given by $\mathbf{h}_1 = [1, 1, 1, 1]^H$, $\mathbf{h}_2 = \gamma \times [1, e^{j\theta}, e^{j2\theta}, e^{j3\theta}]^H$. γ controls the channel gain disparity while θ controls the channel angle. To simplify the notations in the results, SC-SIC is used to represent the transmission scheme of NOMA based on SC-SIC. MU-LP and RS are used to represent the transmission scheme of SDMA and RSMA, respectively.

The convergence of the proposed SCA-based beamforming algorithm with RS, SC-SIC and MU-LP using one specific channel realization ($\gamma = 1, \theta = \frac{2\pi}{9}$) are compared in Fig. 1. The weights of the users are fixed to 1 ($u_1 = u_2 = 1$). As mentioned in Section II-C2, the decoding order π of NOMA is required to be optimized with the beamformer. For each decoding order, the SCA-based algorithm is used to solve the EE maximization problem. Only the convergence result of the decoding order that achieves the maximal EE is illustrated in Fig. 1. For various dynamic power values P_{dyn} , the convergence rate of the algorithm with the three schemes are fast. All of them converge within a few iterations. However, as the SCA-based beamforming algorithm is used twice to solve the EE_{NOMA} problem, the transmitter complexity of NOMA is increased comparing with MULP and RS. The convergence rates of all the schemes are slightly increasing as P_{dyn} decreases. This is due to the fact that the overall optimization space is enlarged as P_{dyn} decreases.

Fig. 2 and Fig. 3 show the EE region comparison of different schemes when $P_{\text{dyn}} = 27$ dBm. γ is equal to 1 and

²As the noise power is normalized, $P_t = 40$ dBm also implies the transmit SNR is 10 dB.

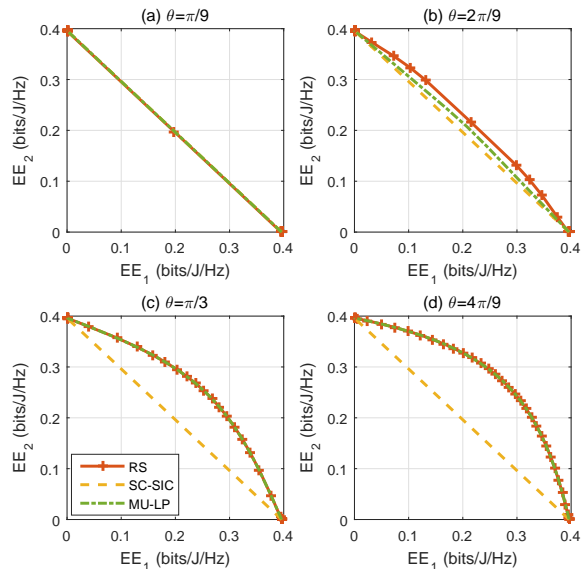


Fig. 2: Achievable energy efficiency region comparison of different schemes, $\gamma = 1, P_{\text{dyn}} = 27$ dBm.

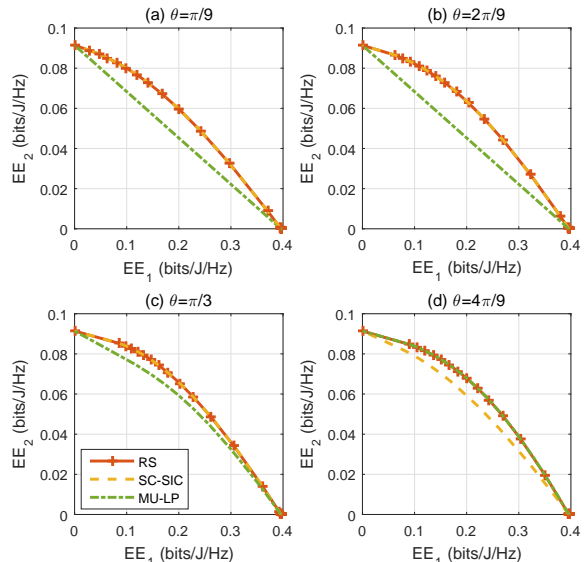


Fig. 3: Achievable energy efficiency region comparison of different schemes, $\gamma = 0.3, P_{\text{dyn}} = 27$ dBm.

0.3, respectively. In all subfigures, the EE region achieved by RS is equal to or larger than that of SC-SIC and MU-LP. In subfigure (b) of Fig. 2, RS outperforms SC-SIC and MU-LP. RS achieves a better EE region especially when the user channels are neither orthogonal nor aligned. As θ increases, the gap between RS and MU-LP decreases because MU-LP works well when the user channels are sufficiently orthogonal. The performance of SC-SIC becomes better when there is a 5 dB channel gain difference between users in Fig. 3. The EE region of SC-SIC is almost overlapped with RSMA in subfigures (a)–(c) of Fig. 3. However, when the user channels become sufficiently orthogonal, there is an obvious EE region improvement of RS over SC-SIC.

Fig. 4 and Fig. 5 show the EE region comparison of different schemes when $P_{\text{dyn}} = 40$ dBm. γ is equal to 1 and 0.3, respectively. As P_{dyn} increases, the EE regions of

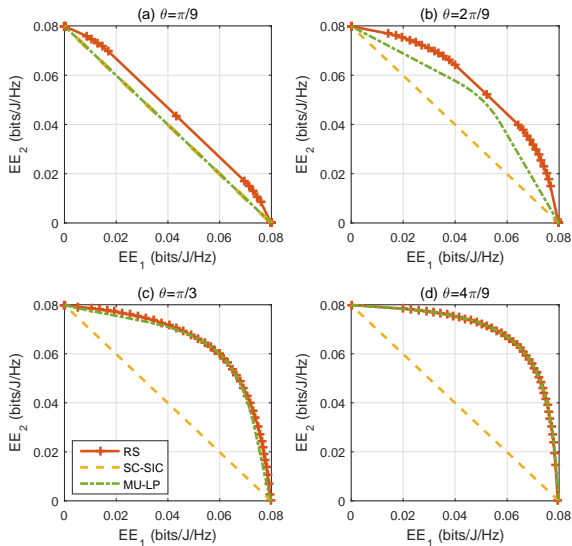


Fig. 4: Achievable energy efficiency region comparison of different schemes, $\gamma = 1$, $P_{\text{dyn}} = 40$ dBm.

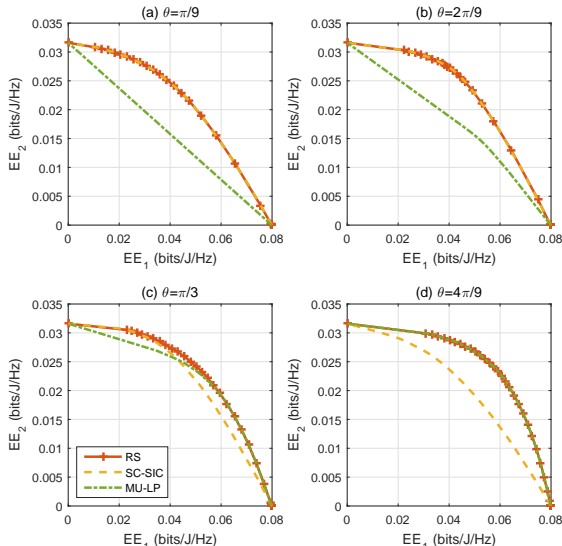


Fig. 5: Achievable energy efficiency region comparison of different schemes, $\gamma = 0.3$, $P_{\text{dyn}} = 40$ dBm.

all multiple access schemes decrease since the denominators of individual EE increase. Comparing the corresponding subfigures of Fig. 4 and Fig. 2 (or Fig. 5 and Fig. 3), RS exhibits a more prominent EE region improvement over MU-LP and SC-SIC. The EE region gaps among RS, MU-LP and SC-SIC are increasing with P_{dyn} . When P_{dyn} is large, the circuit power dominates the total power consumption. EE is not vulnerable to the change of transmit power compared with when P_{dyn} is small. Hence, the results of EE maximization resemble that of the WSR maximization illustrated in [8] for large P_{dyn} . In contrast, when P_{dyn} is small, the power of data transmission dominates the total power consumption. EE is larger when little power is used for transmission. Interestingly, the EE regions of SC-SIC and MU-LP outperform each other at one part of the rate region while the EE region of RS is in general larger than the convex hull of SC-SIC and MU-LP regions. It clearly shows that RS softly bridges and outperforms SC-SIC and MU-LP.

VI. CONCLUSIONS

To conclude, the Energy Efficiency (EE) maximization problem of RSMA in the MISO BC is investigated. An SCA-based algorithm is proposed to solve the problem. As a novel multiple access scheme, RSMA allows common symbols decoded by multiple users to be transmitted together with the private symbols decoded by the corresponding users only. It has the capability of partially decoding interference and partially treating interference as noise. Numerical results show that RSMA softly bridges and outperforms SDMA based on MU-LP and NOMA based on SC-SIC in the realm of EE. The EE region achieved by RSMA is always equal to or larger than that achieved by SDMA and NOMA in a wide range of user deployments (with a diversity of channel directions and channel strengths). Therefore, we conclude that RSMA is not only more spectrally efficient, but also more energy efficient than SDMA and NOMA.

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