Interference Cancellation at Receivers in Cache-Enabled Wireless Networks

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Abstract—In this paper, we propose to exploit the limited cache packets as side information to cancel incoming interference at the receiver side. We consider a stochastic network where the random locations of base stations and users are modeled using Poisson point processes. Caching schemes to reap both the local caching gain and the interference cancellation gain for the users are developed based on two factors: the density of different user subsets and the packets cached in the corresponding subsets. The packet loss rate (PLR) is analyzed, which depends on both the cached packets and the channel state information (CSI) available at the receiver. Theoretical results reveal the tradeoff between caching resource and wireless resource. The performance for different caching schemes are analyzed and the minimum achievable PLR for the distributed caching is derived.

Index Terms—Caching, interference cancellation, packet loss rate, caching scheme, Poisson point process.

I. INTRODUCTION

Both the wireless network topology and the information transmission mode are changing with the advancement of information and communication technologies. In regard to the wireless network topology, nodes are densely and randomly located, yielding serious interference [1]. Interference is a key limitation on future networks. In regard to information transmission, content-centric services (e.g., multimedia transmission) instead of connection-centric services (e.g., voice communication) account for most of mobile traffic [2], [3]. Caching exploiting content-centric traffic has been proposed to unleash the ultimate potential of the network [4], [5].

There are some works on interference management in cache-enabled networks, which are in terms of the degrees of freedom (DoF) from the information-theoretic point of view. In [6], an interference channel with three transmitters and three receivers is considered. It is shown that caching split files at transmitters can increase the throughput via interference management. In [7], the DoF gain is proved to be achievable via caching parts of files at the transmitters for a three-user interference channel. Caching at base stations (BSs) for opportunistic multiple-input multiple-output cooperation is proposed in [8] to achieve the DoF gain without requiring high-speed fronthaul links. In [3], physical layer (PHY)-caching at BSs is proposed to mitigate interference and improve the number of DoF in wireless networks. The storage-latency tradeoff is analyzed in [9] for the network with cache-enabled transmitters, and the transmission rate is specified by the DoF. The standard DoF is adopted in [10] as the performance metric for the network with interference channels. Transmitters and receivers are with caching strategy of equal file splitting. [11] studies the benefit of caching for the system with two cache-enabled transmitters and two cache-enabled receivers in the interference channel. The layered architecture is proposed and the DoF for the optimal strategy is computed. The concept of fractional delivery time (FDT) is proposed in [12] to reflect the DoF enhancement due to transmitter caching and the load reduction due to receiver caching. A complete constant-factor approximation of the DoF is proposed in [13] for the network with caching at both transmitters and receivers.

However, previous works with information-theoretic framework assume that the global channel state information (CSI) is available. The performance for the network with only partial CSI available should be investigated. The randomness and complexity of node locations due to the stochastic topology of the network need to be addressed. And interference management is performed based on the caching at the transmitter side in previous works, where extra payload of fronthaul/backhaul is needed for the cooperation among transmitters. Moreover, caching schemes for cache-enabled networks to exploit both the local caching gain and the interference cancellation gain at the receiver side need to be elaborated further. Different from the information-theoretic framework, in this paper, we focus on the stochastic network where random numbers of BSs and users are spatially located in the two-dimensional plane. The effects of CSI are addressed and the caching schemes are analyzed. Our main contributions are summarized as follows,

- We propose to cancel the incoming interference with partial CSI and cached packets at users. Random numbers of BSs and users are considered in the stochastic network.
- The effects of the CSI are analyzed on the network performance in terms of packet loss rate (PLR), specifically, when partial, global and none of CSI are available.
- The effects of the caching scheme on the PLR are further elaborated. And the optimal caching scheme for the users with distributed caching are provided to reap both the local caching gain and the interference cancellation gain.

II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

A. Network Architecture

Consider the wireless network where BSs and users are independently located according to Poisson Point Processes
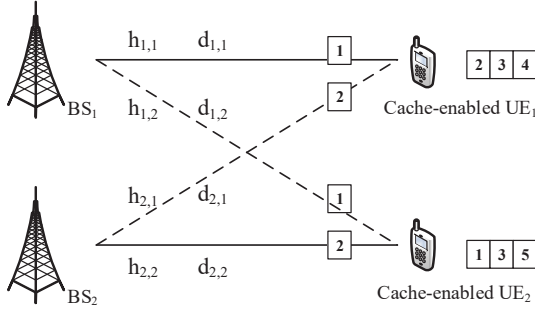
Fig. 1. Interference cancellation in the cache-enabled wireless network.

(PPPs) Φ_b and Φ_u. The intensities of Φ_b and Φ_u are λ_b and λ_u, respectively. The system is slotted and the duration of each slot is τ (seconds). Each user is assumed to randomly request a packet of the fixed length T (Mb) in a slot from the packet library $\mathcal{L} \triangleq \{l_1, l_2, \cdots, l_N\}$ [12]. The packet $l_i, i \in \{1, 2, \cdots, N\}$ is requested by the user with the probability $f_i \in [0, 1]$ in the slot. Define $\mathcal{F} \triangleq \{f_1, f_2, \cdots, f_N\}$ as the packet access probability set. Note that $\sum_{i=1}^{N} f_i = 1$ and without loss of generality (w.l.o.g.), $f_1 \geq f_2, \cdots, \geq f_N$.

Each user has a limited caching storage with size of $M \times T$ (Mb), and M of the N packets in the packet library $\mathcal{L}$ are pre-cached at the user. Therefore, there are totally $\binom{N}{M}$ kinds of caching schemes for different users. Classify all the users into $\binom{N}{M}$ subsets according to their caching schemes. Denote the density of the users in the i-th subset as $p_i \lambda_u$ for $p_i \in [0, 1]$, $i = 1, 2, \cdots, \binom{N}{M}$ and $\sum_{i=1}^{\binom{N}{M}} p_i = 1$. W.l.o.g., assume $p_1 \geq p_2, \cdots, \geq p_{\binom{N}{M}}$. Let $q_{i,j} \in \{0, 1\}$ denote whether users in the i-th subset has cached packet l_j ($j = 1, 2, \cdots, N$), where $q_{i,j} = 1$ indicates that packet l_j has been cached in the users of the i-th subset, and $q_{i,j} = 0$ otherwise. Then the caching scheme for the network depends on matrices $\mathbf{P} \triangleq [p_i]_{N \times 1}$ and $\mathbf{Q} \triangleq [q_{i,j}]_{\binom{N}{M} \times N}$. Denote the set of the cached packets in the users of the i-th subsect as $\mathcal{L}_i \triangleq \{l_j : q_{i,j} = 1, j = 1, 2, \cdots, N\}$. When the requested packet has been cached at the user, the user reads it out immediately from its local caching; otherwise, the user should obtain the requested packet from its nearest BS. Requests are waiting to be served in the infinite buffer of the BS, and each BS is assumed to transmit a packet in each slot on the FIFO-basis (first-in, first-out). Consider the service discipline that a request is dropped out of the buffer at the end of the slot assigned to it, no matter whether the BS has transmitted the requested packet successfully or not. The packet loss rate will be elaborated in Section III.

B. Interference Cancellation with Cached Packets

All BSs share the same wireless channel with bandwidth of B (MHz) to transmit packets to users in the downlink. Consider the CSI in the downlink is available at the user, if the distance from the BS to the user is smaller than R_b. As a simplified prototype of the network, Fig. 1 illustrates two BSs (BS$_1$ and BS$_2$) and two cache-enabled users. The first cache-enabled user has stored packets l_2, l_3, l_4 and the second cache-enabled user has stored packets l_1, l_3, l_5. The two users

are covered by BS$_1$ and BS$_2$, respectively. When BS$_1$ and BS$_2$ transmit packet l_1 and l_2, respectively, to the first and second users at the same slot, the received signal of the i-th user is

$$y_i = \sqrt{P} d_{i,i}^{-\frac{\beta}{2}} h_{i,i} x_i + \sqrt{P} d_{j,i}^{-\frac{\beta}{2}} h_{j,i} x_j + n_0, \qquad (1)$$

for $i \neq j \in \{1, 2\}$. P is the transmit power, $d_{i,i}$ and $d_{j,i}$ are respectively the distance from the serving BS$_i$ and the interfering BS$_j$ to the i-th user, $h_{i,i}$ and $h_{j,i}$ are the corresponding channel fading, $\beta \geq 2$ denotes the path-loss exponent, x_i and x_j are the transmit signal with unit power, $n_0 \sim \mathcal{CN}(0, \sigma^2)$ is the zero-mean additive white Gaussian noise (AWGN) with power σ^2. When the distance $d_{j,i}$ from BS$_j$ to the i-th user is smaller than R_b, the CSI $\sqrt{P} d_{j,i}^{-\frac{\beta}{2}} h_{j,i}$ is known to the i-th user. Consider there is a specific index table of the packets and it is known to all BSs and users. Before the interfering BS$_j$ transmits a packet (e.g., packet l_j) which has been cached by the users around, BS$_j$ broadcasts the packet index via extra interactive signals to let the users around know the incoming interference signal x_j in the slot. Therefore, based on the CSI knowledge (i.e., $\sqrt{P} h_{j,i} d_{j,i}^{-\frac{\beta}{2}}$) and the side information (i.e., the cached packet l_j: x_j), the i-th user can cancel the interference (i.e., the term "$\sqrt{P} d_{j,i}^{-\frac{\beta}{2}} h_{j,i} x_j$" in equation (1)). Accordingly, the signal-to-interference-plus-noise ratio (SINR) of the i-th user is SINR$_i = P |h_{i,i}|^2 d_{i,i}^{-\beta} \sigma^{-2}$.

III. THE PACKET LOSS RATE

In each slot, $(1 - q_{i,j}) f_j$ of users in the i-th subset sent their requests to BSs to obtain packet l_j. Therefore, in the plane, totally $\sum_{i=1}^{\binom{N}{M}} p_i (1 - q_{i,j}) f_j$ of users sent their requests to the BSs to obtain packet l_j. Then based on the aforementioned transmission scheme, in each slot the fraction of BSs transmitting packet l_j is α_j, which can be calculated by

$$\alpha_j = \frac{\sum_{i=1}^{\binom{N}{M}} p_i (1 - q_{i,j}) f_j}{\sum_{j=1}^{N} \sum_{i=1}^{\binom{N}{M}} p_i (1 - q_{i,j}) f_j} \triangleq \frac{f_j - q_j f_j}{1 - \sum_{j=1}^{N} q_j f_j}, \qquad (2)$$

where $q_j \triangleq \sum_{i=1}^{\binom{N}{M}} p_i q_{i,j}$ for $j = 1, 2, \cdots, N$. It is the average caching probability of packet l_j over the whole network.

Based on Slivnyak's theorem, we conduct the analysis with considering that there is a typical user u_0 at the origin of the Euclidean area [1]. Denote R as the distance between the typical user and its serving BS. Due to the randomness of the BSs and users, the distance R between the typical user and its serving BS is variable. The probability density function (PDF) of R is $f_R(r) = 2\pi \lambda_b r e^{-\pi \lambda_b r^2}$ [1]. When the typical user is in the i-th subset and it requests packet $l_l \in \mathcal{L} \backslash \mathcal{L}_i$ which has not been cached in the local caching, the received signal of the typical user from its serving BS is given by

$$y_{0,i} = \sqrt{P} d_{0,0}^{-\frac{\beta}{2}} h_{0,0} x_0 + \sum_{j \in \Phi_{b1} \odot \max\{R, R_b\}} \sqrt{P} d_{j,0}^{-\frac{\beta}{2}} h_{j,0} x_j$$

$$+ \sum_{k \in \Phi_{b2} \odot R} \sqrt{P} d_{k,0}^{-\frac{\beta}{2}} h_{k,0} x_k + n_0, \qquad (3)$$

where x_0 is the desired signal of the typical user u_0 for packet l_l. x_j and x_k are interfering signals. $d_{0,0}$ is the distance

between the typical user and its serving BS. $d_{j,0}$ and $d_{k,0}$ are the distances from interfering BSs to the typical user. $h_{0,0}$, $h_{j,0}$ and $h_{k,0}$ are the corresponding channel fading. Consider Rayleigh fading channel with average unit power in this paper. Φ_{b1} and Φ_{b2} are independently thinning PPPs with parameter $\lambda_b \sum_{j=1}^{N} \alpha_j q_{i,j}$ and $\lambda_b(1 - \sum_{j=1}^{N} \alpha_j q_{i,j})$, respectively. Φ_{b1} is the distribution of interfering BSs transmitting packets included in the cached packet set $\mathcal{L}_i$, and Φ_{b2} is that of interfering BSs transmitting packets included in the complementary set $\mathcal{L}\backslash\mathcal{L}_i$. Note that the interference from the interfering BSs (distributed with Φ_{b1}) inside the circle (centered at the origin with radius R_b) can be cancelled by the typical user with the knowledge of the CSI and the cached packets. Therefore, the residual interference of the typical user comes from i): the BSs distributed with Φ_{b1} outside the circle centered at the origin with radius $\max\{R, R_b\}$ (denoted by $\Phi_{b1} \odot \max\{R, R_b\}$), and ii): the BSs distributed with Φ_{b2} outside the circle centered at the origin with radius R (denoted by $\Phi_{b2} \odot R$). It can be observed that $\lambda_b \sum_{j=1}^{N} \alpha_j q_{i,j} = 0$, if all the users are with the same caching scheme (i.e., $q_{i,j} \equiv q_{k,j} \in \{0,1\}, \forall i \neq k \in \{1, 2, \cdots, \binom{N}{M}\}$). It implies the network cannot reap any interference cancellation gain when all users are with the same caching scheme. The impacts of caching schemes on the network performance will be investigated in Section IV.

Therefore, the SINR of the typical user u_0 is given by

$$\frac{P|h_{0,0}|^2 d_{0,0}^{-\beta}}{\sum\limits_{j \in \Phi_{b1}\odot\max\{R,R_b\}} P|h_{j,0}|^2 d_{j,0}^{-\beta} + \sum\limits_{k \in \Phi_{b2}\odot R} P|h_{k,0}|^2 d_{k,0}^{-\beta} + \sigma^2}$$

$$\triangleq \frac{P|h_{0,0}|^2 d_{0,0}^{-\beta}}{I_c + I_u + \sigma^2} \triangleq \frac{P|h_{0,0}|^2 d_{0,0}^{-\beta}}{\bar{I}_c}, \quad (4)$$

where $I_c + I_u$ are the residual interference after the interference cancellation. A packet will be lost if it cannot be transmitted completely over the slot assigned to it. The PLR $\mathcal{P}_l$ can be calculated by

$$\mathcal{P}_l \triangleq \mathbb{E}\left[\mathbb{P}\left[\tau B \log_2(1 + \text{SINR}) < T\right]\right]$$
$$= \mathbb{E}\left[\mathbb{P}\left(\text{SINR} < 2^{\frac{T}{\tau B}} - 1\right)\right]. \quad (5)$$

Denote $\bar{T} \triangleq 2^{\frac{T}{\tau B}} - 1$. The average is taken over both the channel fading distribution and the spatial PPP. We then have,

Theorem 1: The PLR for users in the i-th subset to obtain the un-cached packets with partial CSI can be calculated with equation (6) at the bottom of the next page.

Proof: Please refer to the Appendix. ∎

Remark 1: The PLR for the un-cached packet deceases with the increase of R_b. It can be proved that $\frac{\partial \mathcal{P}}{\partial R_b} < 0$ when $R_b < +\infty$. Increasing R_b (more CSI) is helpful to reap the interference cancellation gain and to reduce the PLR.

Moreover, the PLR of cached packets is zero because cached packets can be read out immediately from local caching. Accordingly, when the user in the i-th subset requests packet l_j which has or not been cached, the PLR is

$$\mathcal{P}_{i,j} = (1 - q_{i,j})\mathcal{P}_l. \quad (7)$$

Remark 2: For $R_b \to 0$, the system degenerates to the cache-enabled network without any CSI, making interference cancellation infeasible. The PLR is

$$\mathcal{P}_{i,j} = (1 - q_{i,j})\Big\{1-$$
$$2\pi\lambda_b \int_0^\infty r\exp\Big\{-r^\beta P^{-1}\bar{T}\sigma^2 - \pi r^2\lambda_b\left[Z_1(\bar{T})+1\right]\Big\}\mathrm{d}r\Big\}. \quad (8)$$

In comparison, the reduction of PLR for the network with interference cancellation is $\mathcal{P}_{i,j|R_b} - \mathcal{P}_{i,j|R_b\to0}$. Furthermore, if the noise is relative small than the interference, i.e., in the interference-limited network ($\sigma^2 \to 0$), the PLR in (8) turns to

$$\mathcal{P}_{i,j} = \frac{1 - q_{i,j}}{Z_1^{-1}(\bar{T}) + 1}. \quad (9)$$

Remark 3: For the interference-limited network ($\sigma^2 \to 0$) and $R_b \to \infty$, i.e., when the global interfering CSI are available at the cache-enabled user (yielding the complete interference cancellation), the PLR is

$$\mathcal{P}_{i,j} = (1 - q_{i,j})\Big(1 - 2\pi\lambda_b \int_0^\infty r\exp\Big\{-\pi r^2\left[\lambda_{b2}\mathcal{Z}_1(\bar{T}) + \lambda_b\right]\Big\}\mathrm{d}r\Big)$$
$$= \frac{1 - q_{i,j}}{1 + (1 - \sum_{j=1}^{N} \alpha_j q_{i,j})^{-1}\mathcal{Z}_1^{-1}(\bar{T})}. \quad (10)$$

Compared with the PLR for the un-cached packet in (9), the PLR for the un-cached packet in (10) decreases owing to the performance gain from the interference cancellation.

IV. THE IMPACTS OF CACHING SCHEMES

In the previous section, we have derived the PLR $\mathcal{P}_{i,j}$ for the user in the i-th subset to obtain packet l_j. Then the average PLR in terms of all users over the whole network is

$$\mathcal{P} = \sum_{i=1}^{\binom{N}{M}} p_i \sum_{j=1}^{N} \mathcal{P}_{i,j} f_j. \quad (11)$$

In this section, we analyze the impact of caching schemes on the average PLR. To gain further insight, we focus on the interference-limited network ($\sigma^2 \to 0$) with global CSI for complete interference cancellation ($R_b \to \infty$). Consider users are with distributed caching schemes, i.e., users are without centralized controller and they have no knowledge of the popularity of the packet in advance. So different packets are cached by users in the network randomly with equal probability (i.e., $q_j = \frac{M}{N}$). We have the following proposition,

Proposition 1: For $R_b \to \infty$, $\sigma^2 \to 0$ and $q_j = \frac{M}{N}$, when all the packets have the same access popularity (i.e., $f_j = f_k, \forall j \neq k \in \{1, 2, ..., N\}$), the average PLR is

$$\mathcal{P} = \frac{(1 - \frac{M}{N})^2}{[1 + \mathcal{Z}_1^{-1}(\bar{T})] - \frac{M}{N}}. \quad (12)$$

Proof: Substituting $q_j = \frac{M}{N}$ and $f_j = \frac{1}{N}$ to (2), we have $\alpha_j = \frac{1}{N}$. Base on equations (10) and (11), we then have $\mathcal{P} = \sum_{i=1}^{\binom{N}{M}} p_i \frac{\sum_{j=1}^{N}(1-q_{i,j})f_j}{1+(1-\sum_{j=1}^{N}\alpha_j q_{i,j})^{-1}\mathcal{Z}_1^{-1}(\bar{T})} \overset{(a)}{=} \frac{(N-M)^2}{N^2-NM+N^2\mathcal{Z}_1^{-1}(\bar{T})}$.

Algorithm 1: Programming for caching scheme
1: **Input.** $N, M, B, \tau, T, f_j, \forall j \in \{1, 2, \cdots, N\}$.
2: **Initialize.** $\mathbf{P} \leftarrow (0)_{N \times 1}$, $\mathbf{Q} \leftarrow (0)_{\binom{N}{M} \times N}$.
3: $\binom{N}{M}$ schemes to select M elements out of $\{l_1, l_2, \cdots, l_N\}$.
4: **if** l_j has been selected in the i-th scheme, $\forall i \in \{1, 2, \cdots, \binom{N}{M}\}$
5:      $q_{i,j} \leftarrow 1$
6: **end if**
7: Substitute $\mathbf{Q}$ into the problem and solve the linear optimization problem regarding $\mathbf{P}$ without the constrain $p_i \leq p_j, \forall j < i$.
8: Rearrange the order of rows for $\mathbf{P}, \mathbf{Q}$ to satisfy $p_i \leq p_j, \forall j < i$.
9: **Output.** Caching scheme $\mathbf{P}, \mathbf{Q}$.

Fig. 2. The impact of R_b on the PLR for the un-cached packets in the cache-enabled network.

Step (a) is obtained by noting that $\sum_{i=1}^{\binom{N}{M}} q_{i,j} p_i = q_j = \frac{M}{N}$ and $\sum_{j=1}^{N} q_{i,j} = M$. Then the proof is finished. ∎

The average packet in (12) decreases with the increase of caching storage M, the wireless bandwidth B and the slot duration τ. Increasing caching storage can reduce the burden of the wireless spectrum to meet the demand of a given PLR target. Caching storage can be exchanged for scarce spectrum and time resources. Furthermore, when the packets are with different access popularity, we have the following proposition.

Proposition 2: For $R_b \rightarrow \infty$, $\sigma^2 \rightarrow 0$ and $q_j = \frac{M}{N}$, when the packets have different access popularity, the minimum achievable PLR can be obtained with the caching scheme developed via the following optimization problem.

$$\min_{\mathbf{P},\mathbf{Q}} \quad \mathcal{P} = \sum_{i=1}^{\binom{N}{M}} p_i \frac{1 - \sum_{j=1}^{N} q_{i,j} f_j}{1 + (1 - \sum_{j=1}^{N} f_j q_{i,j})^{-1} \mathcal{Z}_1^{-1}(\bar{T})} \quad (13)$$

$$s.t. \quad \begin{cases} q_{i,j} \in \{0,1\}; \\ 0 \leq p_i \leq 1; \\ \sum_{j=1}^{N} q_{i,j} = M; \\ \sum_{i=1}^{\binom{N}{M}} p_i = 1; \\ \sum_{i=1}^{\binom{N}{M}} q_{i,j} p_i = \frac{M}{N}; \\ p_i \leq p_j, \forall j < i \in \{1, 2, ..., \binom{N}{M}\}. \end{cases} \quad (14)$$

The objective function in (13) is derived from (10) and (11) by noting $\alpha_j = f_j$ when $q_j = \frac{M}{N}$. The programming to get the optimal caching scheme can be simply described in Algorithm 1. We then have the minimum achievable PLR via substituting the optimal $\mathbf{P}$ and $\mathbf{Q}$ into the objective function in (13).

V. NUMERICAL RESULTS AND DISCUSSIONS

We evaluate the performance of the proposed system in this section. The BSs and users are located based on PPPs with intensities of $\{\lambda_b, \lambda_u\} = \{\frac{100}{\pi 500^2}, \frac{2000}{\pi 500^2}\}$ nodes/m². The wireless bandwidth $B = 20$ MHz, the slot duration $\tau = 0.5$ second and the transmit power $P = 33$ dBm. We consider the interference-limited network ($\sigma^2 = 0$) and the path-loss $\beta = 4$. Set the packet access popularity $f_i = \frac{1/i^\gamma}{\sum_{j=1}^{C} 1/j^\gamma}$ according to the Zipf distribution and $\gamma = 0.8$. The packet library size
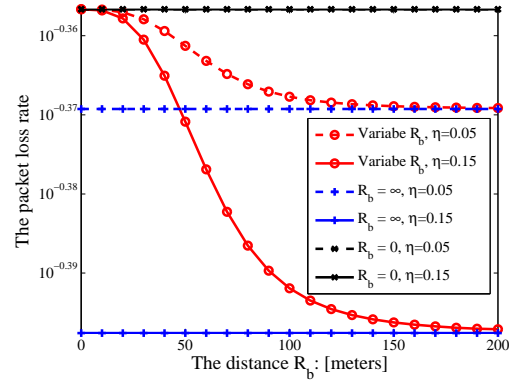
$N = 100$, the packet size $T = 10$ Mbits, and the caching storage of each user $M = 3$.

Fig. 2 demonstrates the impact of R_b on the loss rate of the un-cached packets, larger R_b implies more CSI are available for the cache-enabled users. Denote $\eta = \sum_{j=1}^{N} \alpha_j q_{i,j}$ for users in the i-th subset. Larger η implies more interference can be cancelled with the cached packets for the users. It can be observed that the packet loss rate with complete interference cancellation ($R_b = \infty$) is much smaller that that without interference cancellation ($R_b = 0$). It verifies the necessary and the efficiency of the interference cancellation proposed in this paper for the cache-enabled network. The packet loss rate decrease with the increase of R_b. The performance is approximately equal to that of complete interference cancellation ($R_b = \infty$) when $R_b = 120$ (180) m for $\eta = 0.05$ ($\eta = 0.15$). Higher η needs larger area of CSI (larger R_b) to achieve the approximate performance of the complete interference cancellation. On the other hand, lower packet loss rate can be achieved with larger η because more interference is cancelled. So smaller area of CSI (smaller R_b) is needed to meet the given packet loss rate target when η is larger.

Fig. 3 illustrates the optimal caching scheme which is obtained based on Algorithm 1. The probability (p_i) of users in the i-th subset and the packets cached in the corresponding users are illustrated in the figure. The left and right ordinate are respectively the index of the packets and the probability of users in each subsect. The red circle located at (i, j) means the j-th packet is cached by the users in the i-th subset. It can be observed that users in different subsets are with different densities. The users caching both the popular and unpopular packets are with higher density, yielding both the local caching gain and the interference cancellation gain.

$$\mathcal{P}_l \triangleq 1 - \mathcal{P}_{s,i} - \mathcal{P}_{b,i} = 1 - 2\pi\lambda_b \int_0^{R_b} r \exp\left\{ -r^\beta P^{-1}\bar{T}\sigma^2 - \pi r^2 \left\{ \lambda_{b1} \mathcal{Z}_1 \left[\left(\frac{r}{R_b} \right)^\beta \bar{T} \right] + \lambda_{b2} \mathcal{Z}_1(\bar{T}) + \lambda_b \right\} \right\} \mathrm{d}r$$

$$- 2\pi\lambda_b \int_{R_b}^\infty r \exp\left\{ -r^\beta P^{-1}\bar{T}\sigma^2 - \pi r^2 \lambda_b \left[\mathcal{Z}_1(\bar{T}) + 1 \right] \right\} \mathrm{d}r. \quad (6)$$
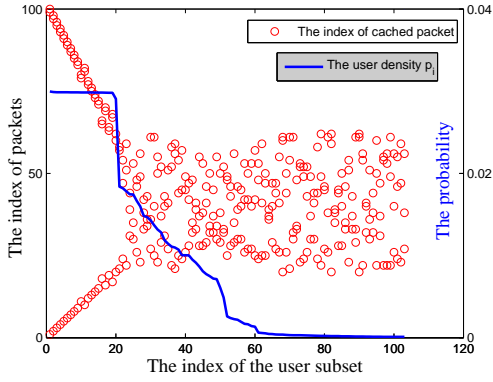
Fig. 3.   The caching scheme based on Algorithm 1.

APPENDIX

Based on (4) and (5), we have

$$
\mathcal{P}_l = 1 - \mathbb{E}\Big[\mathbb{P}\Big(\text{SINR} \geq \bar{T}\Big)\Big] = 1 - \mathbb{E}_r\left[\mathbb{P}\left(h_{0,0} > r^\beta P^{-1}\bar{I}_c\bar{T}\right)\right]
$$
$$
= 1 - \int_0^\infty \mathbb{P}\left[h_{0,0} > r^\beta P^{-1}\bar{I}_c\bar{T}\right] f_R(r)\mathrm{d}r
$$
$$
= 1 - \Big(\int_0^{R_b} + \int_{R_b}^\infty\Big)\mathbb{P}\left[h_{0,0} > r^\beta P^{-1}\bar{I}_c\bar{T}\right] f_R(r)\mathrm{d}r
$$
$$
\triangleq 1 - \mathcal{P}_{s,i} - \mathcal{P}_{b,i}. \tag{15}
$$

Next, we will get $\mathcal{P}_{s,i}$ and $\mathcal{P}_{b,i}$ separately. Firstly, for $\mathcal{P}_{s,i}$, when $R \leq R_b$, the SINR in (4) can be written as follows,

$$
\frac{P|h_{0,0}|^2 d_{0,0}^{-\beta}}{\sum_{j\in\Phi_{b1}\odot R_b} P|h_{j,0}|^2 d_{j,0}^{-\beta} + \sum_{k\in\Phi_{b2}\odot R} P|h_{k,0}|^2 d_{k,0}^{-\beta} + \sigma^2}
$$
$$
\triangleq \frac{P|h_{0,0}|^2 d_{0,0}^{-\beta}}{I_{c,s} + I_{u,s} + \sigma^2} \triangleq \frac{P|h_{0,0}|^2 d_{0,0}^{-\beta}}{\bar{I}_{c,s}}. \tag{16}
$$

Therefore, for $\mathcal{P}_{s,i}$, we have the follows,

$$
\mathcal{P}_{s,i} = \int_0^{R_b} \mathbb{P}\left[h_{0,0} > r^\beta P^{-1}\bar{I}_{c,s}\bar{T}\right] f_R(r)\mathrm{d}r, \tag{17}
$$

where

$$
\mathbb{P}\left[h_{0,0} > r^\beta P^{-1}\bar{I}_{c,s}\bar{T}\right] = \mathbb{E}_{\bar{I}_{c,s}}[e^{-r^\beta P^{-1}\bar{I}_{c,s}\bar{T}}]
$$
$$
= e^{-r^\beta P^{-1}\bar{T}\sigma^2}\mathbb{E}_{I_{c,s}}[e^{-r^\beta P^{-1}\bar{I}_{c,s}\bar{T}}] \times \mathbb{E}_{I_{u,s}}[e^{-r^\beta P^{-1}I_{u,s}\bar{T}}]
$$
$$
= e^{-r^\beta P^{-1}\bar{T}\sigma^2}\mathcal{L}_{I_{c,s}}\left[r^\beta P^{-1}\bar{T}\right] \times \mathcal{L}_{I_{u,s}}\left[r^\beta P^{-1}\bar{T}\right]. \tag{18}
$$

Interference $I_{c,s}$ is from BSs which are spatially distributed as PPP Φ_{b1} and outside the circle centered at the origin with

radius R_b. So the Laplace transform $\mathcal{L}_{I_{c,s}}[r^\beta P^{-1}\bar{T}]$ is

$$
\mathcal{L}_{I_{c,s}}\left[r^\beta P^{-1}\bar{T}\right] = \mathbb{E}_{I_{c,s}}\left[\exp\left(-r^\beta P^{-1}\bar{T}I_c\right)\right]
$$
$$
= \mathbb{E}_{\Phi_{b1},\{|h_{j,0}|^2\}}\Big[\exp\Big(-r^\beta P^{-1}\bar{T}\sum_{j\in\Phi_{b1}\odot R_b} P|h_{j,0}|^2 d_{j,0}^{-\beta}\Big)\Big]
$$
$$
= \mathbb{E}_{\Phi_{b1},\{|h_{j,0}|^2\}}\Big[\prod_{j\in\Phi_{b1}\odot R_b}\exp\Big(-r^\beta\bar{T}|h_{j,0}|^2 d_{j,0}^{-\beta}\Big)\Big]
$$
$$
= \mathbb{E}_{\Phi_{b1}}\Big[\prod_{j\in\Phi_{b1}\odot R_b}\mathbb{E}_{\{|h_{j,0}|^2\}}\Big[\exp\Big(-r^\beta\bar{T}|h_{j,0}|^2 d_{j,0}^{-\beta}\Big)\Big]\Big]
$$
$$
\stackrel{(a)}{=} \mathbb{E}_{\Phi_{b1}}\Big[\prod_{j\in\Phi_{b1}\odot R_b}\frac{1}{1+r^\beta\bar{T}d_{j,0}^{-\beta}}\Big]
$$
$$
= \exp\Big[-2\pi\lambda_{b1}\int_{R_b}^\infty\Big(1-\frac{1}{1+r^\beta\bar{T}v^{-\beta}}\Big)v\mathrm{d}v\Big]
$$
$$
= \exp\Big[-2\pi\lambda_{b1}\int_{R_b}^\infty\frac{v}{1+(r^\beta\bar{T})^{-1}v^\beta}\mathrm{d}v\Big]
$$
$$
\stackrel{(b)}{=} \exp\Big[-\pi\lambda_{b1}r^2\bar{T}^{\frac{2}{\beta}}\int_{[(\frac{r}{R_b})^\beta\bar{T}]^{-\frac{2}{\beta}}}^\infty(1+u^{\frac{\beta}{2}})^{-1}\mathrm{d}u\Big]
$$
$$
= \exp\Big\{-\pi\lambda_{b1}r^2\mathcal{Z}_1\Big[\Big(\frac{r}{R_b}\Big)^\beta\bar{T}\Big]\Big\}. \tag{19}
$$

By noting the Rayleigh channel fading we have Step (a). Using change of variables with $u = [r^\beta\bar{T}]^{-\frac{2}{\beta}}v^2$, we have Step (b). Denote $\mathcal{Z}_1(x) \triangleq \frac{2\bar{T}}{\beta-2}{}_2F_1[1, 1-\frac{2}{\beta}; 2-\frac{2}{\beta}; -x]$, where ${}_2F_1[\cdot]$ is the Gauss hypergeometric function. Furthermore, interference $I_{u,s}$ comes from the BSs which are spatially distributed as PPP Φ_{b2} and outside the circle centered at the origin with radius R. With the same approach of (19), the simplified derivation for the Laplace transform $\mathcal{L}_{I_{u,s}}[r^\beta P^{-1}\bar{T}]$ is

$$
\mathcal{L}_{I_{u,s}}\left[r^\beta P^{-1}\bar{T}\right] = \mathbb{E}_{I_{u,s}}\left[\exp(-r^\beta P^{-1}\bar{T}I_u)\right]
$$
$$
= \mathbb{E}_{\Phi_{b2},\{|h_{k,0}|^2\}}\Big[\exp\Big(-r^\beta P^{-1}\bar{T}\sum_{k\in\Phi_{b2}\odot R} P|h_{k,0}|^2 d_{k,0}^{-\beta}\Big)\Big]
$$
$$
= \exp[-\pi\lambda_{b2}r^2\mathcal{Z}_1(\bar{T})]. \tag{20}
$$

Substituting (18), (19) and (20) into (17), we have $\mathcal{P}_{s,i}$ in (6).

Similarly, $\mathcal{P}_{b,i}$ can be obtained with the same approach of deriving $\mathcal{P}_{s,i}$, and the proof is omitted here for the limited spaces. We then get Theorem 1 and the proof is finished. ∎

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