| Title | Limits on Active to Sterile Neutrino Oscillations from Disappearance Searches in the MINOS, Daya Bay, and Bugey-3 Experiments |
Limits on Active to Sterile Neutrino Oscillations from Disappearance Searches in the MINOS, Daya Bay, and Bugey-3 Experiments


(Daya Bay Collaboration)

(MINOS Collaboration)
The discovery of neutrino flavor oscillations \cite{1,2} marked a crucial milestone in the history of particle physics. It indicates neutrinos undergo mixing between flavor and mass eigenstates and hence carry nonzero mass. It also represents the first evidence of physics beyond the standard model of particle physics. Since then, neutrino oscillations have been confirmed and precisely measured – with data from natural (atmospheric and solar) and man-made (reactor and accelerator) neutrino sources.

The majority of neutrino oscillation data available can be well described by a three-flavor neutrino model \cite{3–5} in agreement with precision electroweak measurements from collider experiments \cite{6,7}. A few experimental results, however, including those from the Liquid Scintillator Neutrino Detector (LSND) \cite{8} and MiniBooNE \cite{9} experiments, cannot be explained by three-neutrino mixing. Both experiments observed an antineutrino excess in a reactor experiment to constrain oscillations into light sterile neutrinos. The three experiments are sensitive to complementary regions of parameter space, enabling the combined analysis to probe regions allowed by the LSND and MiniBooNE experiments in a minimally extended four-neutrino flavor framework. Stringent limits on \( \sin^2 2\theta_{13} \) are set over 6 orders of magnitude in the sterile mass-squared splitting \( \Delta m_{41}^2 \). The sterile-neutrino mixing phase space allowed by the LSND and MiniBooNE experiments is excluded for \( \Delta m_{41}^2 < 0.8 \text{ eV}^2 \) at 95% CL.

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In this Letter, we report results from a joint analysis developed in parallel to the independent sterile neutrino searches from the Daya Bay \cite{14} and the MINOS experiments \cite{15}. In this analysis, the measurement of muon (anti)neutrino disappearance by the MINOS experiment is combined with electron antineutrino disappearance measurements from the Daya Bay and Bugey-3 experiments \cite{16} using the signal confidence level (CL_{s}) method \cite{17,18}. The combined results are analyzed in light of the muon (anti)neutrino to electron (anti)neutrino appearance indications from the LSND \cite{8} and MiniBooNE \cite{9} experiments. The independent MINOS, Daya Bay, and Bugey-3 results are all obtained from disappearance measurements and therefore are insensitive to CP-violating effects due to mixing between the three active flavors. Under the assumption of CPT invariance, the combined results shown constrain both neutrino and antineutrino appearance.

The results reported here required several novel improvements developed independently from the Daya Bay–only \cite{14} and MINOS-only \cite{15} analyses, specifically, a full reanalysis of the MINOS data to search for sterile neutrino mixing, based on the CL_{s} method, a CL_{s}-based analysis of the Bugey-3 results taking into account new reactor flux calculations and the Daya Bay experiment’s reactor flux measurement, the combination of the Daya Bay results

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with the Bugey-3 results taking into account correlated systematics between the experiments, and, finally, the combination of the Daya Bay + Bugey-3 and MINOS results to place stringent constraints on electron neutrino and antineutrino appearance driven by sterile neutrino oscillations.

We adopt a minimal extension of the three-flavor neutrino model by including one sterile flavor and one additional mass eigenstate. This $3 + 1$ sterile neutrino scenario is referred to as the four-flavor model in the text. In this model, the muon to electron neutrino appearance probability $P_{\nu_\mu \rightarrow \nu_e}(L/E)$ as a function of the propagation length $L$, divided by the neutrino energy $E$, can be expressed using a $4 \times 4$ unitary mixing matrix $U$ by

$$P_{\nu_\mu \rightarrow \nu_e}(L/E) = \left| \sum_i U_{\nu_\mu i}^* e^{-i(m_i^2/2E)L} \right|^2.$$  

(1)

In the region where $\Delta m_{41}^2 \gg |\Delta m_{32}^2|$ and for short baselines ($|\Delta m_{32}^2 L/4E| \sim 0$), Eq. (1) can be simplified to

$$P_{\nu_\mu \rightarrow \nu_e}(L/E) \approx 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right) \approx P_{\nu_e \rightarrow \nu_\mu}.$$  

(2)

A nonzero amplitude for the appearance probability, $4|U_{e4}|^2|U_{\mu 4}|^2$, is a possible explanation for the MiniBooNE and LSND results. The matrix element $|U_{e4}|^2$ can be constrained with measurements of electron antineutrino disappearance, as in the Daya Bay [14] and Bugey-3 [16] experiments. Likewise, $|U_{\mu 4}|^2$ can be constrained with measurements of muon neutrino and antineutrino disappearance, as in the MINOS [15] experiment. For these experiments, the general four-neutrino survival probabilities $P_{\nu_\mu \rightarrow \nu_\mu}(L/E)$ and $P_{\nu_e \rightarrow \nu_e}(L/E)$ are

$$P_{\nu_\mu \rightarrow \nu_\mu}(L/E) = 1 - 4 \sum_{k>j} |U_{e k}|^2 |U_{\mu j}|^2 \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right),$$  

(3)

$$P_{\nu_e \rightarrow \nu_e}(L/E) = 1 - 4 \sum_{k>j} |U_{\mu k}|^2 |U_{\mu j}|^2 \sin^2 \left( \frac{\Delta m_{32}^2 L}{4E} \right).$$  

(4)

The mixing matrix augmented with one sterile state can be parametrized by $U = R_{34} R_{24} R_{14} R_{23} R_{13} R_{12}$ [19], where $R_{ij}$ is the rotational matrix for the mixing angle $\theta_{ij}$, yielding

$$|U_{e4}|^2 = \sin^2 \theta_{14},$$

$$|U_{\mu 4}|^2 = \sin^2 \theta_{24} \cos^2 \theta_{14},$$

$$4|U_{e4}|^2|U_{\mu 4}|^2 = \sin^2 2\theta_{14} \sin^2 \theta_{24} \equiv \sin^2 2\theta_{\mu e}.$$  

(5)

Searches for sterile neutrinos are carried out by using the reconstructed energy spectra to look for evidence of oscillations driven by the sterile mass-squared difference $\Delta m_{41}^2$. For small values of $\Delta m_{41}^2$, corresponding to slow oscillations, the energy-dependent shape of the oscillation probability could be measured in the reconstructed energy spectra. For large values corresponding to rapid oscillations, an overall reduction in neutrino flux would be seen.

The $\text{CL}_s$ method [17,18] is a two-hypothesis test that compares the three-flavor (null) hypothesis (labeled 3$\nu$) to an alternate four-flavor hypothesis (labeled 4$\nu$). To determine if the four-flavor hypothesis can be excluded, we construct the test statistic $\Delta \chi^2 = \chi^2_{3\nu} - \chi^2_{4\nu}$, where $\chi^2_{3\nu}$ is the $\chi^2$ value resulting from a fit to a four-flavor hypothesis, and $\chi^2_{4\nu}$ is the $\chi^2$ value from a fit to the three-flavor hypothesis. The $\Delta \chi^2$ value observed with data, $\Delta \chi^2_{\text{obs}}$, is compared to the $\Delta \chi^2$ distributions expected if the three-flavor hypothesis is true, or the four-flavor hypothesis is true. To quantify this, we construct

$$\text{CL}_{3\nu} = P(\Delta \chi^2 \geq \Delta \chi^2_{\text{obs}}, 3\nu),$$

$$\text{CL}_{3\nu+b} = P(\Delta \chi^2 \geq \Delta \chi^2_{\text{obs}}, 4\nu),$$

$$\text{CL}_{s} = \frac{\text{CL}_{3\nu+b}}{\text{CL}_{b}}.$$  

(6)

over a grid of $(\sin^2 \theta_{14}, \Delta m_{41}^2)$ points for the Daya Bay + Bugey-3 experiments and a grid of $(\sin^2 \theta_{24}, \Delta m_{41}^2)$ for the MINOS experiment. $\text{CL}_{3\nu}$ measures consistency with the three-flavor hypothesis, and $\text{CL}_{3\nu+b}$ measures the agreement with the four-flavor hypothesis. The alternate hypothesis is excluded at the $\alpha$ confidence level if $\text{CL}_{s} \leq 1 - \alpha$. The construction of $\text{CL}_{s}$ ensures that even if $\text{CL}_{3\nu+b}$ is small, indicating disagreement with the four-flavor hypothesis, this hypothesis can only be excluded when $\text{CL}_{b}$ is large, indicating consistency with the three-flavor hypothesis. Thus, the $\text{CL}_{s}$ construction ensures the four-flavor hypothesis can only be excluded if the experiment is sensitive to it. Calculating $\text{CL}_{3\nu}$ and $\text{CL}_{3\nu+b}$ can be done in two ways. The first method is the Gaussian $\text{CL}_{s}$ method [20], which uses two Gaussian $\Delta \chi^2$ distributions. The first distribution is obtained by fitting toy Monte Carlo (MC) data assuming the three-flavor hypothesis is true, and the second distribution is obtained by assuming the four-flavor hypothesis is true ($\Delta \chi^2_{4\nu}$). The mean of each distribution is obtained from a fit to the Asimov data set, an infinite statistics sample where the relevant parameters are set to best-fit values for each hypothesis [21]. The Gaussian width for the Asimov data set is derived analytically. In the second method, the distributions of $\Delta \chi^2$ are approximated by MC simulations of pseudoexperiments.
The Gaussian method is used to obtain the Daya Bay and Bugey-3 combined results, while the second method is used to obtain the MINOS results.

The MINOS experiment [22] operates two functionally equivalent detectors separated by 734 km. The detectors sample the NuMI neutrino beam [23], which yields events with an energy spectrum that peaks at about 3 GeV. Both detectors are magnetized steel and scintillator calorimeters, with the 1 kton Near Detector (ND) situated 1 km downstream of the NuMI production target, and the 5.4 kton Far Detector (FD) located at the Soudan Underground Laboratory [22]. The analysis reported here uses data from an exposure of $10.56 \times 10^{20}$ protons on target, for which the neutrino beam composition is $91.8\% \bar{\nu}_\mu$, $6.9\% \bar{\nu}_\mu$, and $1.3\% (\nu_\mu + \bar{\nu}_\mu)$.

To look for sterile neutrino mixing, the MINOS experiment uses the reconstructed energy spectra in the ND and FD of both charged-current (CC) and neutral-current (NC) neutrino interactions. The sterile mixing signature differs depending on the range of $\Delta m^2_{41}$ values considered. For $\Delta m^2_{41} \in (0.005, 0.05) \text{ eV}^2$, the muon neutrino CC spectrum in the FD would display deviations from three-flavor oscillations. For rapid oscillations driven by $\Delta m^2_{41} \in (0.05, 0.5) \text{ eV}^2$, the combination of finite detector energy resolution and rapid oscillations at the FD location would result in an apparent event rate depletion between the ND and FD. For larger sterile neutrino masses, corresponding to $\Delta m^2_{41} > 0.5 \text{ eV}^2$, oscillations into sterile neutrinos would distort the ND CC energy spectrum. Additional sensitivity is obtained by analyzing the reconstructed energy spectrum for NC candidates. The NC cross sections and interaction topologies are identical for all three active neutrino flavors, rendering the NC spectrum insensitive to standard oscillations, but mixing with a sterile neutrino state would deplete the NC energy spectrum at the FD, as the sterile neutrino would not interact in the detector. For large sterile neutrino masses, such a depletion would also be measurable at the ND.

The simulated FD-to-ND ratios of the reconstructed energy spectra for $\nu_\mu$ CC and NC selected events, including four-flavor oscillations for both the ND and FD, are fit to the equivalent FD-to-ND ratios obtained from data [15]. Current and previous results of the MINOS sterile neutrino searches, along with further analysis details, are described in Refs. [15,24–26]. The MINOS experiment employs the Feldman-Cousins ordering principle [27] in obtaining exclusion limits in the four-flavor parameter space. However, this approach requires a computationally impractical joint fit to be consistent, since it requires minimizing $\chi^2$ over $\Delta m^2_{41}$, a shared parameter between the MINOS and Daya Bay + Bugey-3 experiments. Thus, the CL$_s$ method described above is used.

While the MINOS experiment does not have any sensitivity to $\sin^2 \theta_{14}$, there is a small sensitivity to $\sin^2 \theta_{34}$ due to the inclusion of the NC channel. During the fit, $\sin^2 \theta_{34}$ is allowed to vary freely in addition to $\Delta m^2_{32}$ and $\sin^2 \theta_{23}$, while $\sin^2 \theta_{24}$ and $\Delta m^2_{41}$ are held fixed to define the particular four-flavor hypothesis that is being tested. Since the constraint on $\sin^2 \theta_{34}$ is relatively weak, the distribution of $\Delta \chi^2$ deviates from the normal distribution and the Gaussian CL$_s$ method cannot be used. The $\Delta \chi^2_{3\nu}$ and $\Delta \chi^2_{4\nu}$ distributions are constructed by fitting pseudoexperiments.

In the three-flavor case, pseudoexperiments are simulated using the same parameters listed in Ref. [15], i.e., $\sin^2 \theta_{12} = 0.307$, $\Delta m^2_{21} = 7.54 \times 10^{-5} \text{ eV}^2$ based on a global fit to neutrino data [28], and $\sin^2 \theta_{13} = 0.022$, based on a weighted average of results from reactor experiments [29–31]. For the atmospheric oscillation parameters, equal numbers of pseudoexperiments are simulated in the upper and lower octant ($\sin^2 \theta_{23} = 0.61$ and $\sin^2 \theta_{23} = 0.41$) with $|\Delta m^2_{32}| = 2.37 \times 10^{-3} \text{ eV}^2$, based on the most recent MINOS results [32]. The uncertainties on solar oscillation parameters have negligible effect on the analysis, so fixed values are used. In the four-flavor case, $|\Delta m^2_{32}|$, $\sin^2 \theta_{23}$, and $\sin^2 \theta_{34}$ are taken from fits to data at each $(\sin^2 \theta_{24}, \Delta m^2_{41})$ grid point. In both the three- and four-flavor cases, half of the pseudoexperiments are generated in each mass hierarchy. A comparison of MINOS exclusion contours obtained using the Feldman-Cousins procedure [15] with those obtained using the CL$_s$ method is shown in Fig. 1. Note that if $\Delta m^2_{41} = 2 \Delta m^2_{31}$ or $\Delta m^2_{41} \ll \Delta m^2_{31}$ and $\sin^2 \theta_{23} = \sin^2 \theta_{34} = 1$, $\theta_{24}$ can take on the role normally played by $\theta_{23}$. In these cases, the four-flavor model is degenerate with the three-flavor model, leading to regions of parameter space that cannot be excluded.

The Daya Bay experiment measures electron antineutrinos via inverse $\beta$ decay (IBD): $\bar{\nu}_e + p \rightarrow e^+ + n$. 

[FIG. 1. Comparison of the MINOS 90% CL contour using the Feldman-Cousins method [15] and the CL$_s$ method. The region to the right of the curve is excluded at the 90% CL (CL$_s$).]
The antineutrinos are produced by six reactor cores and detected in eight identical Gd-doped liquid-scintillator antineutrino detector (ADs) [33] in three underground experimental halls (EHs). The flux-averaged baselines for EH1, EH2, and EH3 are 520, 570, and 1590 m, respectively. The target mass in each of the two near EHs is 40 tons, and that in the far EH is 80 tons. Details of the IBD event selection, background estimates, and assessment of systematic uncertainties can be found in Refs. [29,34]. By searching for distortions in the \( \bar{\nu}_e \) energy spectra, the experiment is sensitive to \( \sin^2 2\theta_{14} \) for a mass-squared splitting \( \Delta m^2_{41} \in (0.0003, 0.2) \text{ eV}^2 \). For \( \Delta m^2_{41} > 0.2 \text{ eV}^2 \), spectral distortions cannot be resolved by the detector. Instead, the measured antineutrino flux can be compared with the predicted flux to constrain the sterile neutrino parameter space. Recently, the Daya Bay Collaboration published its measurement of the overall antineutrino flux [35]. The result is consistent with previous measurements at short baselines, which prefer 5% lower values than the latest calculations [36,37], a deficit commonly referred to as the reactor antineutrino anomaly [38]. However, the reactor spectrum measurement from the Daya Bay Collaboration [35] (and from the RENO Collaboration [30] and the Double Chooz Collaboration [31]) shows clear discrepancies with the latest calculations, which indicates an underestimation of their uncertainties. The uncertainties on the antineutrino flux models for this analysis are increased to 5% from the original 2% as suggested by Refs. [39,40]. The Daya Bay Collaboration has recently updated the sterile neutrino search result in Ref. [14] with limits on \( \sin^2 2\theta_{14} \) improved by about a factor of 2 with respect to previous results [41]. This data set is used in producing the combined results presented here.

Two independent sterile neutrino search analyses are conducted by the Daya Bay Collaboration. The first analysis uses the predicted \( \bar{\nu}_e \) spectrum to generate the predicted prompt spectrum for each antineutrino detector simultaneously, taking into account detector effects such as energy resolution, nonlinearity, detector efficiency, and oscillation parameters described in Ref. [29]. A log-likelihood function is constructed with nuisance parameters to include the detector-related uncertainties and a covariance matrix to incorporate the uncertainties on reactor neutrino flux prediction. The Gaussian CL\( _s \) method is used to calculate the excluded region. The second analysis uses the observed spectra at the near sites to predict the far site spectra to further reduce the dependency on reactor antineutrino flux models. Both analyses yield consistent results [14].

The Bugey-3 experiment was performed in the early 1990s and its main goal was to search for neutrino oscillations using reactor antineutrinos. In this experiment, two \( ^6\text{Li} \)-doped liquid scintillator detectors measured \( \bar{\nu}_e \) generated from two reactors at three different baselines (15, 40, and 95 m) [16]. The Bugey-3 experiment detected IBD interactions with the recoil neutron capturing on \( ^6\text{Li}\ (n + ^6\text{Li} \rightarrow ^4\text{He} + ^3\text{H} + 4.8 \text{ MeV}) \). Probing shorter baselines than the Daya Bay experiment, the Bugey-3 experiment is sensitive to regions of parameter space with larger \( \Delta m^2_{41} \) values.

The original Bugey-3 results obtained using the raster scan technique are first reproduced employing a \( \chi^2 \) definition used in the original Bugey-3 analysis [16]:

\[
\chi^2 = \sum_{i} \sum_{j} \frac{\left( [Aa_i + b(E_j - 1.0)]R^\text{pre}_{i,j} - R^\text{obs}_{i,j} \right)^2}{\sigma^2_{i,j}} + \sum_{i} \frac{(a_i - 1)^2}{\sigma^2_{a_i}} + \frac{(A - 1)^2}{\sigma^2_A} + \frac{b^2}{\sigma^2_b},
\]

where \( A \) is the overall normalization, \( a_i \) is the relative detection efficiency, \( b \) is an empirical factor to include the uncertainties of the energy scale, \( i \) represents the data from three baselines, and \( j \) sums over the \( N_i \) bins at each baseline. The values of \( \sigma_{a_i} \) and \( \sigma_A \) are set at 0.014 MeV\(^{-1}\) and 0.020 MeV\(^{-1}\), respectively, according to the reported values in Ref. [16]. The \( \sigma_{i,j} \) are the statistical uncertainties. The uncertainty on the overall normalization \( \sigma_a \) is set to 5% to be consistent with the constraint employed in the Daya Bay analysis [14]. The ratio of the observed spectrum to the predicted unoscillated spectrum is denoted by \( R^\text{obs}_{i,j} \), while \( R^\text{pre}_{i,j} \) is the predicted ratio of the spectrum including oscillations to the one without oscillations. To predict the energy spectra, the average fission fractions are used [42], and the energy resolution is set to 5% at 4.2 MeV [16] with a functional form similar to the Daya Bay result [14].

![Excluded regions for the original Bugey-3 raster scan (RS) result [16], for the reproduced Bugey-3 result with adjusted fluxes, for the Daya Bay result [14], and for the combined Daya Bay and reproduced Bugey-3 results. The region to the right of the curve is excluded at the 90% CL.](image-url)
experiment’s. The predicted energy spectra are validated against the published Bugey-3 spectra [16].

In the Bugey-3 experiment, the change in the oscillation probability over the baselines of the detectors and the reactors is studied with MC simulations assuming that antineutrinos are uniformly generated in the reactor cores and uniformly measured in the detectors, and approximated by treating the baselines as normal distributions. To achieve the combination with the Daya Bay experiment, two changes are made in the reproduced Bugey-3 analysis: the change in the cross section of the IBD process due to the updated neutron decay time [6] is applied, and the antineutrino flux is adjusted from the ILL + Vogel model [43,44] to that of Huber [36] and Mueller [37], for consistency with the prediction used by the Daya Bay experiment. These adjustments change the reproduced contour with respect to the original Bugey-3 one, in particular by reducing the sensitivity to regions with \( \Delta m^2_{41} > 3 \, \text{eV}^2 \), with less noticeable effects for smaller \( \Delta m^2_{41} \) values. The reproduced Bugey-3 limit on the sterile neutrino mixing, and the limit obtained by combining the Bugey-3 with the Daya Bay results through a \( \chi^2 \) fit, with common overall normalization and oscillation parameters, are shown in Fig. 2.

Individually, the MINOS and Bugey-3 experiments are both sensitive to regions of parameter space allowed by the LSND measurement through constraints on \( \theta_{24} \) and \( \theta_{14} \), shown in Figs. 1 and 2, respectively. We illustrate this sensitivity in Fig. 3, which displays a comparison of the energy spectra for the Bugey-3 and MINOS data to four-flavor \((4\nu)\) predictions produced at the LSND best-fit point [8] as an example. For the Bugey-3 experiment, a \( \chi^2 \) value of 48.2 is found between the data and the four-flavor prediction. Taking equal priors between these two models, the posterior likelihood for \( 3\nu \) vs \( 4\nu \) is 1 vs \( 3.4 \times 10^{-11} \) in the Bayesian framework. For the MINOS experiment, a \( \chi^2 \) value of 38.0 is obtained between data and prediction. The posterior likelihood for \( 3\nu \) vs \( 4\nu \) is 1 vs \( 5.6 \times 10^{-9} \).

In our combined analysis, we obtain \( \Delta \chi^2_{4\nu} \) as well as \( \Delta \chi^2_{3\nu} \) and \( \Delta \chi^2_{4\nu} \) distributions for each \((\sin^2 2\theta_{14}, \Delta m^2_{41})\) grid point of the Daya Bay and Bugey-3 combination, and for each \((\sin^2 2\theta_{24}, \Delta m^2_{41})\) grid point from the MINOS experiment. We then combine pairs of grid points from the MINOS and the Daya Bay and Bugey-3 results at fixed values of \( \Delta m^2_{41} \) to obtain constraints on electron neutrino or antineutrino appearance due to oscillations into sterile neutrinos. Since the systematic uncertainties of accelerator and reactor experiments are largely uncorrelated, for each \((\sin^2 2\theta_{14}, \sin^2 2\theta_{24}, \Delta m^2_{41})\) grid point, a combined \( \Delta \chi^2_{\text{obs}} \) is constructed from the sum of the corresponding MINOS and Daya Bay + Bugey-3 \( \Delta \chi^2_{\text{obs}} \) values. Similarly, the combined \( \Delta \chi^2_{3\nu} \) and \( \Delta \chi^2_{4\nu} \) distributions are constructed by adding random samples drawn from the corresponding MINOS and Daya Bay + Bugey-3 distributions.
the CL$_s$ value at every ($\sin^2 2\theta_{14}, \sin^2 2\theta_{24}$) point is calculated using Eq. (6), while the $\Delta m^2_{31}$ value is fixed. While CL$_s$ is single valued at every ($\sin^2 2\theta_{14}, \sin^2 2\theta_{24}$) point for a given value of $\Delta m^2_{31}$, it is multivalued as a function of $\sin^2 2\theta_{\mu e}$ [cf. Eq. (5)]. To obtain a single-valued function, we make the conservative choice of selecting the largest CL$_s$ value for any given $\sin^2 2\theta_{\mu e}$. The 90% CL$_s$ exclusion contour resulting from this procedure is shown in Fig. 4. Under the assumption of CPT conservation, the combined constraints are equally valid in constraining electron neutrino or antineutrino appearance. The combined results of the Daya Bay + Bugey-3 and MINOS experiments constrain $\sin^2 2\theta_{\mu e} < [3.0 \times 10^{-4} \ (90\% \ \text{CL}_s), \ 4.5 \times 10^{-4} \ (95\% \ \text{CL}_s)]$ for $\Delta m^2_{31} = 1.2$ eV$^2$.

In conclusion, we have combined constraints on $\sin^2 2\theta_{14}$ derived from a search for electron antineutrino disappearance at the Daya Bay and Bugey-3 reactor experiments with constraints on $\sin^2 2\theta_{24}$ derived from a search for muon (anti)neutrino disappearance in the NuMI beam at the MINOS experiment. Assuming a four-flavor model of active-sterile oscillations, we constrain $\sin^2 2\theta_{\mu e}$, the parameter controlling electron (anti)neutrino appearance at short-baseline experiments, over 6 orders of magnitude in $\Delta m^2_{31}$. We set the strongest constraint to date and exclude the sterile neutrino mixing phase space allowed by the LSND and MiniBooNE experiments for $\Delta m^2_{31} < 0.8$ eV$^2$ at a 95% CL$_s$. Our results are in good agreement with results from global fits (see Refs. [13,47] and references therein) at specific parameter choices; however, they differ in detail over the range of parameter space. The results explicitly show the strong tension between null results from disappearance searches and appearance-based indications for the existence of light sterile neutrinos.

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Note added.—Recently, a paper appeared by the IceCube Collaboration that sets limits using sterile-driven disappearance of muon neutrinos [48]. The results place strong constraints on $\sin^2 2\theta_{24}$ for $\Delta m^2_{31} \in (0.1, 10) \text{ eV}^2$. Further, a paper that reanalyses the same IceCube data in a model including nonstandard neutrino interactions also appeared [49].

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