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<td><strong>Author(s)</strong></td>
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<tr>
<td><strong>Citation</strong></td>
<td>IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society, Montreal, QC, 25-28 October 2012, p. 6200 - 6205</td>
</tr>
<tr>
<td><strong>Issued Date</strong></td>
<td>2012</td>
</tr>
<tr>
<td><strong>URL</strong></td>
<td><a href="http://hdl.handle.net/10722/225405">http://hdl.handle.net/10722/225405</a></td>
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Optimal Design and Implementation of a Permanent Magnet Linear Vernier Machine for Direct-drive Wave Energy Extraction

Wenlong Li, K.T. Chau, and Christopher H.T. Lee
Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong
wlli@eee.hku.hk, ktchau@eee.hku.hk, htlee@eee.hku.hk

Abstract—This paper presents a permanent magnet linear vernier (PMLV) machine which is dedicated for low-speed direct-drive applications. Firstly, the machine operation principle is discussed, and the preliminary design approach is formulated. Then, by applying the analytical calculation method for solving the magnetostatic field problem, the machine structure, especially the stator toothed-pole structure which acts on the field modulation function is optimized. Finally, a PMLV machine is prototyped and implemented for direct-drive wave power generation. Both analytical calculation and experimental verification are given to verify its performances.

1. INTRODUCTION

Linear machines are a class of electromagnetic devices which operates at a linear oscillatory or progressive motion. As their rotational counterparts, they can also be used for mechanical energy and electrical energy inter-conversion. In recent years, development and application of linear machines are in an accelerated pace in various fields, such as industrial automation, robotics, power generation, and transportation [1]-[5].

Low-speed drives, such as wind power generation and electric vehicle motor drives, attract more and more attention in recent years [6]-[12]. For the conventional machine topologies, at the same power rating, low-speed direct drives usually mean a large physical volume and relatively low power density compared to the high-speed drives. In order to solve these problems, mechanical gearboxes for speed reduction and torque transmission are widely applied, thus the power density of the whole driving system is improved accordingly. However, the mechanical transmission units inevitably incur the system complexity, operation cost and further deteriorate its control performance and reliability. For solving the problems raised by mechanical gears but still attaining their merits, the coaxial magnetic gears were proposed and applied for direct-drive applications [13]-[18]. The coaxial magnetic gear consists of two rotors and one stationary ring. One rotor operates at a low speed and other at a high speed. Thus, the high-speed drives can be worked as the low-speed drives via the coaxial magnetic gear. Fig. 1 shows its linear tubular morphology of the coaxial magnetic gear [19]. The two movers have surface-mounted PMs, and the stationary parts consist of iron rings and airspaces interpolating each other. By integration of the magnetic gear with the conventional high-speed machine, the integrated machine can enable low-speed operation but with high power density. Thus, the magnetic-geared machine becomes a hot research topic in recent several years [20]-[21]. By extending its operation principle, the permanent magnet vernier machine is developed which uses a toothed-pole structure for magnetic field modulation and exhibits a low-speed and high-torque nature [22]-[24].

![Fig. 1. Linear tubular magnetic gear.](image1)

Air-gaps Low-speed mover Field modulation rings
High-speed mover PMs

![Fig. 2. Linear tubular vernier machine.](image2)

Air-gap mover Toothed-pole structure Coil Stator PMs

The purpose of this paper is to present a permanent magnet linear vernier (PMLV) machine. By studying its operation principle, the design procedure will be introduced. Then, by
analytically modeling the field modulation effect of the toothed-pole structure, the dimensions of the toothed-pole structure are optimized and the performance is improved accordingly. Finally, a PMLV machine will be designed and prototyped for direct-drive wave energy conversion. By using analytical and experimental evaluations, the proposed machine performances will be verified.

II. PRELIMINARY DESIGN

The concept of vernier machine lies in its toothed-pole stator configuration which resembles to the field modulation rings in the magnetic gear, as shown in Fig. 2. Due to the distinct difference in the permeability of ferromagnetic material and air space, the toothed-pole structure can enable different pole-pair numbers of magnetomotive forces (MMFs) of stator and mover to develop a steady force. In order to realize this function, the pole-pair numbers of MMFs of PMs and armature winding and the stator tooth number should satisfy the following relationship:

\[ Z_3 = \left[ Z_1 \pm P \right] \tag{1} \]

where \( Z_1 \) is the number of teeth of the stator, \( Z_2 \) and \( P \) are the pole-pair numbers of PMs of the mover and armature winding respectively.

When AC current with an angular frequency \( \omega \) is fed into the armature winding, the resultant magnetic field of armature excitation rotates at a speed of \( \omega Z_2 \), and the mover speed is \( \omega Z_2 \). By borrowing the concept of gear ratio in magnetic gears, the gear ratio of this machine is \( Z_2/P \).

A. Mover Design

The mover configuration with PMs can be classified into three categories depending on the location of PMs, namely surface-mounted PM (SPM), surface-inset PM (STPM), and interior PM (IPM). In this design, only the SPM topology is considered and mathematically modeled.

Under the current frequency \( f \) in the armature winding, the mover speed \( v \) can be expressed as:

\[ v = v_p = 2 f r_m \frac{Z_2}{P} \tag{2} \]

where \( v_p \) is the synchronous speed, \( r_m \) is the pole-pitch of PMs on the mover. For a given operation speed and current frequency, the PM pole-pitch can be easily determined.

By using the Ampere’s circuital law and assuming that the magnetic drop occurs only in the air-gap, the fundamental component of air-gap flux density \( B_{ag1} \) can be expressed as:

\[ B_{ag1} = \frac{4}{\pi} B_0 h_m \sin \left( \frac{w_m \pi}{2 r_m} \right) \tag{3} \]

where \( B_0 \) is the PM remanence, \( h_m \) is the PM thickness, \( \mu_r \) is the PM relative recoil permeability, \( g_0 \) is the air-gap length, \( w_m \) is the PM length, and \( r_m \) is the PM pole-pitch.

B. Stator Design

According to (1), the stator tooth pitch \( \tau_t \) can be written in terms of the PM pole-pitch:

\[ \tau_t = \frac{2 Z_1}{Z_1^2 - \tau_m} \tau_m \tag{4} \]

The developed thrust force of the linear machine is determined by the electric loading which is limited by the cooling method. Most of the heat is caused by the copper loss expressed as:

\[ P_c = 2 m N I^2 \rho \frac{I_0^2}{A_c} \tag{5} \]

where \( m \) is the phase number, \( N \) is the turns of coil of each phase, \( I \) is the phase current, \( \rho \) is the copper resistivity, \( I_0 \) is the conductor effective length, and \( A_c \) is the cross-sectional area of the conductor.

The heat produced by armature winding dissipates to the air space via the stator yoke. The relationship of the total copper loss with the coil temperature rise \( T \) and the heat transfer coefficient \( h \) can be expressed as [25]:

\[ P_c = 2 P \rho h T I_0 \tag{6} \]

According to (5) and (6), the machine electric loading can be determined, depending on the cooling method and its temperature limit.

C. Thrust Estimation

Based on the air-gap flux density and the electric loading in the windings, the pull-out thrust force can be estimated from the magnetic field interaction:

\[ F_m = \frac{4 m}{\pi} \int R_1 B_0 h_m \sin^2 \left( \frac{\pi}{\tau_m} z \right) dz \tag{7} \]

where \( R_1 \) is the amplitude of the fundamental component of magnetic reluctance of the toothed-pole structure.

Therefore, given the basic design requirements, the initial specifications of the linear machine can be determined according to the above equations.

III. OPTIMAL SIZING

As shown in Fig. 3, the toothed-pole structure of the vernier machine takes the field modulation function which inherently governs the machine operation. Therefore, it is necessary to study the underlaid relationship between its structure dimensions and the field modulation effect. In order to take a physical insight of contributions of the toothed-pole structure for the field modulation, an analytical calculation is carried out. By taking the toothed-pole structure as an anisotropic material, the calculation can be easily handled. In the calculation, the tubular linear machine configuration is considered. The permeabilities of the toothed-pole structure along \( z \) and \( r \) direction are described as:

\[ \mu_{\phi,z} = \frac{\mu_0 \mu_t \mu_r \tau_t}{\mu_0 \tau_t + b_i (\mu_0 - \mu_r)} \tag{8} \]

\[ \mu_{\phi,r} = \mu_0 + b_i (\mu_r - \mu_0) \tag{9} \]

where \( \mu_0 \) is the air space permeability, \( \mu_r \) is the stator iron core permeability, \( \tau_t \) is the tooth pitch and \( b_i \) is the tooth pitch.
By solving Maxwell’s equations in each region, the magnetic field can be analytically obtained [26]-[28]. In order
to ease the computation, some assumptions have been taken
into consideration: the permeability of the back iron of the
mover and the yoke of the stator is considered to be infinite;
the relative recoil permeability of PMs is considered to be
unity; the saturation effect of the ferromagnetic material is
ignored.

As shown in Fig. 4, the three regions are the region I (PM),
region II (air-gap) and region III (toothed-pole structure). The
flux density \( B \) and field intensity \( H \) in
the above three regions can be expressed as:

In the region I:

\[
B_I = \mu_0 (H_I + M_{res}) \tag{10}
\]

In the other two regions where no PMs involved:

\[
B_{II,III} = \mu_{r,II,III} H_{II,III} \tag{11}
\]

where \( M_{res} \) is the PM residual magnetization vector. It should
be noted that the permeability in region III is not the same in
the \( r \) and \( z \) directions.

For the optimization of toothed-pole structure dimensions,
the magnetic field excited by PMs is evaluated. Thus, there is
no current involved, and the magnetic scalar potential is
applied for solving the Maxwell’s equations. The governing
Maxwell’s equations in the aforementioned three regions are
expressed as:

In the region I:

\[
\nabla^2 \varphi_I(r,z) = \nabla \cdot (M_{res}) \tag{12}
\]

In the region II:

\[
\nabla^2 \varphi_{II}(r,z) = 0 \tag{13}
\]

In the region III:

\[
\frac{\partial^2 \varphi_{III}(r,z)}{\partial r^2} + \frac{\mu_{r,III}}{r} \frac{\partial \varphi_{III}(r,z)}{\partial r} + \frac{\partial^2 \varphi_{III}(r,z)}{\partial z^2} = 0 \tag{14}
\]

where \( M_r \) is the PM residual magnetization vector in the \( r \)
direction which can be expressed as:

\[
M_r(r,z) = \left( \frac{P_0}{r} + r \sum_{n=1}^{\infty} M_n \sin(\omega_n z) \right) \tag{15}
\]

where

\[
P_0 = \frac{1}{2n_0 + h_w}
\]

\[
P_1 = \frac{1}{r_0 + r_0 h_w}
\]

\[
\omega_n = \frac{(2n-1)\pi}{\tau_w}
\]

\[
M_n = \frac{4B_0}{\mu_0 \pi (2n-1)} \tag{16}
\]

The unique solutions of the above partial differential
equations are governed by the boundary conditions. There are
four boundaries formed by three regions and its conditions
are given by:

\[
B_I(r_0,z) = 0 \tag{17}
\]

\[
B_{II}(r_1,z) = B_{III}(r_1,z) \tag{18}
\]

\[
H_{II}(r_1,z) = H_{III}(r_1,z) \tag{19}
\]

\[
B_{II}(r_2,z) = B_{III}(r_2,z) \tag{20}
\]

\[
H_{II}(r_2,z) = H_{III}(r_2,z) \tag{21}
\]

\[
B_{II}(r_3,z) = 0 \tag{22}
\]

Using separation of variables, the general solution of the
Laplace’s equations can be easily obtained. By ignoring the
right term of (12), the Poisson’s equation can be reduced to
the Laplace’s equation. Thus, the general solution can be
simplified as:

\[
\psi_I^p(r,z) = \sum_{n=1}^{\infty} \left[ a_{n0}^I I_0(k_n r) + b_{n0}^I K_0(k_n r) \right] \times \sin(k_n z) \tag{23}
\]

where \( I_0(\cdot) \) and \( K_0(\cdot) \) are the modified Bessel functions of
the first kind and the second kind of order zero respectively. The
special solution of this Poisson’s equation is:

\[
\psi_I^p(r,z) = \sum_{n=1}^{\infty} - \frac{2P_0 M_n}{\omega_n^2} \sin(\omega_n z) \tag{24}
\]

Therefore, the solution of (12) is the summation of (23) and
(24).

The general solution of the region II has the same form as
that of the region I:

\[
\psi_{II}^p(r,z) = \sum_{n=1}^{\infty} \left[ a_{n0}^{II} I_0(k_n r) + b_{n0}^{II} K_0(k_n r) \right] \times \sin(k_n z) \tag{25}
\]

By using the same approach, the general solution of region
III is described as:

\[
\psi_{III}^p(r,z) = \sum_{n=1}^{\infty} \left[ a_{n0}^{III} I_0(m_n r) + b_{n0}^{III} K_0(m_n r) \right] \times \sin(k_n z) \tag{26}
\]

where
After calculating the magnetic field in each region, the machine performance can be evaluated analytically. For assessing the field modulation of the toothed-pole structure, the \( P \)th order harmonic component of flux density in the stator back-iron is investigated. The flux density of \( r \) and \( z \) components of region III can be expressed as:

\[
B_{IIIr} = -\mu_{0,r} \frac{\partial \phi_m(r,z)}{\partial r}
\]

\[
B_{IIIz} = -\mu_{0,z} \frac{\partial \phi_m(r,z)}{\partial z}
\]

The unknown coefficients can be determined from the boundary conditions \( (17)-(22) \) as given by:

\[
a_{III} = \frac{K_0(m_{n,r})}{l_0(m_{n,r})} \tag{30}
\]

\[
\frac{1}{b_{III}^n} = \frac{XC_0 l_1(m_{n,r})K_0(m_{n,r}) - l_0(m_{n,r})K_0(m_{n,r})}{[l_0(\omega_0r_2) - \omega_0 l_0(\omega_0r_0)] l_0(\omega_0r_2) + l_0(\omega_0r_0) X C_0 K_1(m_{n,r})l_1(m_{n,r}) + l_0(m_{n,r})K_0(m_{n,r})} \tag{31}
\]

where

\[
C_0 = \sqrt{\mu_{0,r}\mu_{0,z}}/\mu_0
\]

\[
S = M_{a}[2P_0 l_0(\omega_0r_0) + \omega_0 l_0(\omega_0r_0)]/\omega_0 l_0(\omega_0r_0)
\]

\[
T = 2P_0 M_{a}[l_1(\omega_0r_0) - \omega_0 l_0(\omega_0r_0)]/\omega_0^2 l_0(\omega_0r_0)
\]

\[
U = l_0(\omega_0r_0)K_0(\omega_0r_0) + l_0(\omega_0r_0)K_1(\omega_0r_0)/l_0(\omega_0r_0)K_0(\omega_0r_0) - l_0(\omega_0r_0)K_1(\omega_0r_0)/l_0(\omega_0r_0)K_0(\omega_0r_0)
\]

\[
V = l_1(\omega_0r_0) + UK_0(\omega_0r_0)/l_1(\omega_0r_0) - UL_0(\omega_0r_0)
\]

\[
W = \frac{S - TU}{l_0(\omega_0r_0) - UL_0(\omega_0r_0)}
\]

\[
X = \frac{VI_1(\omega_0r_0) + K_0(\omega_0r_0)}{VI_1(\omega_0r_0) - K_1(\omega_0r_0)}
\]

where \( l_1(\cdot) \) and \( K_1(\cdot) \) are the modified Bessel functions of the first kind and the second kind of order one respectively.

Fig. 5 shows the \( P \)th harmonic component flux density versus the PM remanence according to the variation of the tooth width \( b_t \) and slot depth \( h_t \) over the tooth pitch \( i_t \). It can be observed that when the tooth width equals to one half of the tooth pitch and the slot depth equals about 0.55 of the tooth pitch, the field modulation efficiency of the toothed-pole structure is optimum.

IV. IMPLEMENTATION FOR DIRECT-DRIVE WAVE POWER GENERATION

Because of its clean, sustainable, and tremendous features, the oceanic wave energy is accepted as one of the promising renewable energy sources in the future. Its low-frequency and reciprocating motion nature makes it hard to harvest directly. Till now, people usually capture this kind of energy by intermediate devices or facilities and then use conventional electric generators for electricity production [29]. This approach has low conversion efficiency and high initial cost. Therefore, the direct-drive approach attracts more and more attention [30]. For direct-drive energy harvesting, a generator of high thrust density is highly expected. The PMLV machine which possesses the magnetic gear effect is a suitable candidate for this application [31].

Based on the above design approach, a PMLV machine is dimensioned and prototyped. For easing the fabrication, the single-sided flat type is adopted. The toothed-pole structure dimensions are set, namely the tooth pitch \( i_t = 12 \text{ mm}, \) tooth width \( b_t = 6 \text{ mm}, \) and slot depth \( h_t = 6 \text{ mm}. \) The number of teeth of the stator \( Z_1 = 27, \) pole-pair number of PMs \( Z_2 = 24 \) and pole-pair number of armature MMFs \( P = 3. \)

In order to assess the performance of the proposed machine, the finite element analysis (FEA) is applied for the field computation and a set-up is built for testing the prototype machine. Fig. 6 shows the test-bed of the proposed machine which consists of a DC reducer motor, the prototype machine and some mechanical components for rotational-
linear motion inter-conversion. The DC reducer motor serves as the prime mover. By using the cam and connecting rod, the rotational motion can be converted into linear motion which imitates the incident wave force. When the DC reducer motor operates at a constant speed, the prototype machine operates in an oscillatory mode via the cam and the linkage.

Fig. 7 shows the magnetic field distributions at no-load condition. It can be seen that there are multi-pole-pair MMFs at the mover side, but at the stator side, only several pole-pairs are found which confirms the field modulation function of the toothed-pole structure. As indicated by the arrow in Fig. 7, even with a tiny movement of the mover, the flux linkage in the armature winding is changed dramatically which confirms the high force production.

When the prototype machine operates in the oscillatory mode, the no-load induced voltage waveform is measured as shown in Fig. 8. It can be observed that the amplitude of the voltage depends on the mover instantaneous velocity. Also, the measured no-load induced voltage waveform at 0.5 m/s is compared with the simulated no-load induced voltage waveform under the same mover velocity. As shown in Fig. 9, the agreement is very good.

Fig. 10 depicts the static force waveforms when the 3-phase winding is fed with the rated current and without current. It shows that the cogging force as a parasitic force component is low compared with the rated thrust. Fig. 11 shows the static force characteristic at different currents.
V. CONCLUSION

This paper has presented the design, optimization and application of a PMLV machine. Firstly, the basic specifications of the machine are determined by the traditional design approach. Then, by using the analytical calculation method, the relationship of the machine performance and the toothed-pole structure dimension is unveiled. According to the analytical result, a PMLV machine is designed and prototyped. Finally, the prototype machine is implemented for the direct-drive wave power generation. Both the numerical analysis and experimental results have been used to verify the machine performances.

ACKNOWLEDGMENT

This work was supported and funded by a research grant (Project No. HKU 710711E) of the Research Grants Council in Hong Kong Special Administrative Region, China.

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