A SMOOTH AMBIGUITY MODEL OF THE COMPETITIVE FIRM

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ABSTRACT

This paper examines the optimal production decision of the competitive firm under price uncertainty when the firm's preferences exhibit smooth ambiguity aversion. Ambiguity is modeled by a second-order probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. Ambiguity preferences are modeled by the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk. Within this framework, we derive necessary and sufficient conditions under which the ambiguity-averse firm optimally produces less in response either to the introduction of ambiguity or to greater ambiguity aversion when ambiguity prevails. In the case that the price risk is binary, we show that ambiguity and greater ambiguity aversion always adversely affect the firm's production decision.

Keywords: ambiguity; ambiguity aversion; production

JEL classification numbers: D21, D24, D81

I. INTRODUCTION

Since the seminal work of Sandmo (1971), the behavior of the competitive firm has been the subject of considerable research in decision making under uncertainty (Batra and Ullah, 1974; Broll, 1992; Broll and Zilcha, 1992; Chavas, 1985; Viaene and Zilcha, 1998; Wong, 1996; to name just a few). Most of the extant models in the literature assume that the firm's preferences admit the standard von Neumann-Morgenstern expected utility representation.[1]

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[1]Notable exceptions are Paroush and Venezia (1979) and Wong (2012, 2014) who incorporate regret theory into Sandmo's (1971) model of the competitive firm under price uncertainty.

Such a modeling approach rules out the possibility that the firm is unable to unambiguously assign a probability distribution that uniquely describes the price risk, which gives rise to ambiguity, or uncertainty in the sense of Knight (1921).

The purpose of this paper is to incorporate ambiguity into the model of the competitive firm under price uncertainty. We define uncertainty to be made up of two components, risk and ambiguity. Risk aversion is the aversion to a set of outcomes with a known probability distribution. Ambiguity aversion is the additional aversion to being unsure about the probabilities of outcomes. Dated back to the Ellsberg's (1961) paradox, ambiguity has been alluded to the violation of the independence axiom, which is responsible for the decision criterion being linear in the outcome probabilities.[2] The distinction between the known-unknown and the unknown-unknown is relevant since individuals appear to prefer gambles with known rather than unknown probabilities. Indeed, ample experiments (Chow and Sarin, 2001; Einhorn and Hogarth, 1986; Sarin and Weber, 1993) and surveys (Chesson and Viscusi, 2003; Viscusi and Chesson, 1999) have documented convincing evidence that ambiguity aversion prevails.

Klibanoff et al. (2005) have recently developed a powerful decision criterion known as "smooth ambiguity aversion" that is compatible with ambiguity averse preferences under uncertainty (hereafter referred to as the KMM model). The KMM model features the recursive structure that is far more tractable in comparison to other models of ambiguity such as the pioneering maxmin expected utility (or multiple-prior) model of Gilboa and Schmeidler (1989).[3] Specifically, the KMM model represents ambiguity by a second-order

[2]A vivid description of the Ellsberg's (1961) paradox is from Keynes (1921). Consider the following experiment with two urns, K and U. Urn K contains 50 red balls and 50 blue balls. Urn U contains 100 balls (all balls are either red or blue), but the exact numbers of red and blue balls are not disclosed. Subjects are asked to select from which urn they would like to draw a ball. They are rewarded if the color of their choice is drawn. In this experiment, subjects typically select urn K, revealing aversion to ambiguity. To see this, suppose that subjects believe that the probability of drawing a blue ball from urn U is p. Subjects should prefer to draw a red ball or a blue ball from urn U than from urn K, depending on whether p is smaller or greater than $1/2$, respectively. If $p = 1/2$, subjects should be indifferent between the two urns. Since subjects choose to draw from urn K, such paradoxical behavior can only be rationalized by allowing multiple priors to be held by subjects on urn U. See also Dillenberger and Segal (2015) and Machina (2014).

[3]Skiadas (2013) shows that smooth ambiguity preferences can be approximated by preferences admitting an expected utility representation in continuous-time or high-frequency models under Brownian or Poisson

probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. The KMM model then measures the firm's expected utility under ambiguity by taking the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk. This recursive structure creates a crisp separation between ambiguity and ambiguity aversion, i.e., between beliefs and tastes, which allows us to study these two attributes independently.[4] Another nice feature of the KMM model is that we can apply the conventional techniques in the decision making under uncertainty in the context of ambiguity (Alary et al., 2013; Gollier, 2011, 2014; Iwaki and Osaki, 2014; Osaki et al., 2015; Snow, 2010, 2011; Taboga, 2005; Treich, 2010; Wong, 2015).

Within the KMM model, we derive necessary and sufficient conditions under which the ambiguity-averse firm optimally produces less in response either to the introduction of ambiguity or to greater ambiguity aversion when ambiguity prevails. These conditions ensure that the ambiguity premium is negative (more negative as the firm becomes more ambiguity averse), which reduces the firm's certainty equivalent marginal revenue. The firm as such is induced to cut down its optimal output level so as to restore the optimality condition that the certainty equivalent marginal revenue is equated to the marginal cost of production.

Since the conditions that ensure the ambiguity premium to be negative (more negative as the firm becomes more ambiguity averse) are rather complicated, we derive more tangible sufficient conditions under which ambiguity and greater ambiguity aversion adversely affect the firm's production decision. These conditions include the firm's coefficient of relative risk aversion not to exceed unity and its subjective beliefs to be ranked in the sense of first-order stochastic dominance. Barham et al. (2014) document that the average coefficient of relative risk aversion is 0.8 in their sample of Midwestern grain farmers in the U.S., which

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uncertainty.

[4]The model of Choquet expected utility developed by Schmeidler (1989) and the maxmin expected utility model of Gilboa and Schmeidler (1989) do not separate ambiguity preferences from ambiguous beliefs, and therefore cannot provide a sensible notion of greater ambiguity aversion.

is in line with the magnitudes of relative risk aversion found in many developing countries (Cardenas and Carpenter, 2008). In the context of a portfolio choice problem, Gollier (2011) derives similar sufficient conditions under which a more ambiguity-averse individual optimally invests less in a risky asset whose return is ambiguous. In the case that the price risk is binary (Alary et al., 2013; Snow, 2011), we show that ambiguity and greater ambiguity aversion always adversely affect the firm's production decision. These results are consistent with the results of Alary et al. (2013) and Snow (2011) on the propensities for self-insurance and self-protection.

The rest of this paper is organized as follows. Section II delineates the KMM model of the competitive firm under price uncertainty. Sections III and IV examine how ambiguity and ambiguity aversion affect the firm's optimal production decision, respectively. Section V concludes.

II. THE MODEL

Consider the competitive firm of Sandmo (1971) within the context of the KMM model. There is one period with two dates, 0 and 1. To begin, the firm produces a single commodity according to a deterministic cost function, $C(Q)$, where $Q \geq 0$ is the output level, and $C(Q)$ is compounded to date 1 with the properties that $C(0) = C'(0) = 0$, and $C'(Q) > 0$ and $C''(Q) > 0$ for all $Q > 0$.[5] At date 1, the firm sells its entire output, Q, at the then prevailing per-unit price, $\tilde{P}$, which is not known ex ante.[6] The firm's profit at date 1 is therefore uncertain and given by $\tilde{\Pi} = \tilde{P}Q - C(Q)$. The firm possesses a von Neumann-Morgenstern utility function, $u(\Pi)$, defined over its profit at date 1, Π, with $u'(\Pi) > 0$ and $u''(\Pi) < 0$, indicating the presence of risk aversion.

The price risk, $\tilde{P}$, is distributed according to an objective cumulative distribution func-

[5]The strict convexity of the cost function reflects the fact that the firm's production technology exhibits decreasing returns to scale.

[6]Throughout the paper, random variables have a tilde ($\tilde{\ }$) while their realizations do not.

tion (CDF), $F^\circ(P)$, over support $[\underline{P}, \overline{P}]$, where $0 < \underline{P} < \overline{P}$. The firm, however, is uncertain about $F^\circ(P)$ and thus faces ambiguity. Let $F(P|\theta)$ be the firm's subjective CDF of $\tilde{P}$ over support $[\underline{P}, \overline{P}]$, where θ is the realization of an unknown parameter, $\tilde{\theta}$. The KMM model represents ambiguity by a second-order subjective CDF of $\tilde{\theta}$, $G(\theta)$, over support $[\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} < \overline{\theta}$, which captures the firm's uncertainty about which of the subjective CDF, $F(P|\theta)$, governs the price risk, $\tilde{P}$. Following Snow (2010, 2011) and Wong (2015), we assume that the firm's ambiguous beliefs are unbiased in the sense that the expected price risk is equal to the objective price risk:

$$\int_{\underline{\theta}}^{\overline{\theta}} F(P|\theta)\mathrm{d}G(\theta) = F^\circ(P), \tag{1}$$

for all $P \in [\underline{P}, \overline{P}]$.[7]

The recursive structure of the KMM model implies that we can compute the firm's expected utility under ambiguity in three steps. First, we calculate the firm's expected utility for each subjective CDF of $\tilde{P}$:

$$U(Q, \theta) = \int_{\underline{P}}^{\overline{P}} u[PQ - C(Q)]\mathrm{d}F(P|\theta). \tag{2}$$

Second, we transform each (first-order) expected utility obtained in Eq. (2) via an ambiguity function, $\varphi(U)$, where U is the firm's utility level and $\varphi'(U) > 0$. Finally, we take the (second-order) expectation of the transformed expected utility obtained in the second step with respect to the second-order subjective CDF of $\tilde{\theta}$. The firm's ex-ante decision problem as such is given by

$$\max_{Q \geq 0} \int_{\underline{\theta}}^{\overline{\theta}} \varphi[U(Q, \theta)]\mathrm{d}G(\theta). \tag{3}$$

Inspection of the objective function of program (3) reveals that we can separate the effect of ambiguity, represented by the CDF, $G(\theta)$, from that of ambiguity preferences, represented by the shape of the ambiguity function, $\varphi(U)$.

[7] The assumption that the expected price risk is equal to the objective price risk is motivated by the premise that the behavior of an ambiguity-neutral decision maker should be unaffected by the introduction of, or changes in, ambiguity.

We say that the firm is ambiguity averse if, for any given output level, Q, the objective function of program (3) decreases when the firm's ambiguous beliefs, specified by $G(\theta)$, change in a way that induces a mean-preserving spread in the distribution of the firm's expected utility. According to this definition, Klibanoff et al. (2005) show that ambiguity aversion implies concavity for $\varphi(U)$ (see also Neilson, 2010). They show further that the maxmin expected utility model of Gilboa and Schmeidler (1989) is the limiting case when $\varphi(U)$ is extremely concave.[8] Throughout the paper, we assume the firm to be ambiguity averse so that $\varphi''(U) < 0$.

The first-order condition for program (3) is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta) = 0, \tag{4}$$

where Q^* is the firm's optimal output level, $U(Q,\theta)$ is given in Eq. (2), and

$$U_Q(Q,\theta) = \int_{\underline{P}}^{\overline{P}} u'[PQ - C(Q)][P - C'(Q)]\mathrm{d}F(P|\theta). \tag{5}$$

Differentiating the objective function of program (3) twice yields

$$\frac{\partial^2}{\partial Q^2}\int_{\underline{\theta}}^{\overline{\theta}} \varphi[U(Q,\theta)]\mathrm{d}G(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \varphi''[U(Q,\theta)]U_Q(Q,\theta)^2\mathrm{d}G(\theta)$$

$$+\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{P}}^{\overline{P}} \varphi'[U(Q,\theta)]u''[PQ - C(Q)][P - C'(Q)]^2\mathrm{d}F(P|\theta)\mathrm{d}G(\theta)$$

$$-\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{P}}^{\overline{P}} \varphi'[U(Q,\theta)]u'[PQ - C(Q)]C''(Q)\mathrm{d}F(P|\theta)\mathrm{d}G(\theta) < 0, \tag{6}$$

for all $Q \geq 0$, where the inequality follows from the assumed properties of $\varphi(U)$, $u(\Pi)$, and $C(Q)$. It then follows from Eq. (6) that Eq. (4) is both necessary and sufficient for Q^* to the unique maximum solution to program (3).

[8] When $\varphi(U) = [1-\exp(-\alpha U)]/\alpha$, Klibanoff et al. (2005) show that the maxmin expected utility model of Gilboa and Schmeidler (1989) is the limiting case as the constant absolute ambiguity aversion, α, approaches infinity under some conditions.

III. AMBIGUITY AND PRODUCTION

In this section, we examine the effect of ambiguity on the firm's optimal production decision. To this end, we consider the benchmark case wherein there is no ambiguity in that the firm knows the objective CDF of $\tilde{P}$, i.e., $F(P|\theta) = F^\circ(P)$ for all $P \in [\underline{P}, \overline{P}]$ and $\theta \in [\underline{\theta}, \overline{\theta}]$. Eq. (4) as such reduces to

$$\int_{\underline{P}}^{\overline{P}} u'[PQ^\circ - C(Q^\circ)][P - C'(Q^\circ)]\mathrm{d}F^\circ(P) = 0, \tag{7}$$

where Q° is the firm's optimal output level in the absence of ambiguity. Comparing Eq. (4) with Eq. (7) yields our first proposition.

Proposition 1. Introducing ambiguity to the ambiguity-averse competitive firm reduces the optimal output level, i.e., $Q^* < Q^\circ$*, if, and only if, the covariance between* $\varphi'[U(Q^\circ, \tilde{\theta})]$ *and* $U_Q(Q^\circ, \tilde{\theta})$ *is negative.*

Proof. Differentiating the objective function of program (3) with respect to Q, and evaluating the resulting derivative at $Q = Q^\circ$ yields

$$\frac{\partial}{\partial Q} \int_{\underline{\theta}}^{\overline{\theta}} \varphi[U(Q, \theta)]\mathrm{d}G(\theta)\bigg|_{Q=Q^\circ} = \int_{\underline{\theta}}^{\overline{\theta}} \varphi'[U(Q^\circ, \theta)]U_Q(Q^\circ, \theta)\mathrm{d}G(\theta). \tag{8}$$

Taking expectations on both side of Eq. (5) with $Q = Q^\circ$ with respect to the CDF, $G(\theta)$, yields

$$\int_{\underline{\theta}}^{\overline{\theta}} U_Q(Q^\circ, \theta)\mathrm{d}G(\theta) = \int_{\underline{P}}^{\overline{P}} u'[PQ^\circ - C(Q^\circ)][P - C'(Q^\circ)]\mathrm{d}F^\circ(P) = 0, \tag{9}$$

where the first equality follows from Eq. (1), and the second equality follows from Eq. (7). Using Eq. (9), we can write Eq. (8) as[9]

$$\frac{\partial}{\partial Q} \int_{\underline{\theta}}^{\overline{\theta}} \varphi[U(Q, \theta)]\mathrm{d}G(\theta)\bigg|_{Q=Q^\circ} = \mathrm{Cov}_G\{\varphi'[U(Q^\circ, \tilde{\theta})], U_Q(Q^\circ, \tilde{\theta})\}, \tag{10}$$

[9]For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}(\tilde{X}, \tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

where $\mathrm{Cov}_G(\cdot,\cdot)$ is the covariance operator with respect to the CDF, $G(\theta)$. It then follows from Eqs. (4) and (6) that $Q^* < Q^\circ$ if, and only if, the covariance term on the right-hand side of Eq. (10) is negative. $\square$

Proposition 1 provides a necessary and sufficient condition under which the presence of ambiguity adversely affects the firm's production decision. This condition states that the marginal ambiguity, $\varphi'[U(Q^\circ,\tilde{\theta})]$ is negatively correlated with the marginal expected utility, $U_Q(Q^\circ,\tilde{\theta})$, at the optimal output level, Q°, when ambiguity is absent. To see the intuition for Proposition 1, we write Eq. (7) as

$$\mathrm{E}_{F^\circ}(\tilde{P}) + \frac{\mathrm{Cov}_{F^\circ}\{u'[\tilde{P}Q^\circ - C(Q^\circ)],\tilde{P}\}}{\mathrm{E}_{F^\circ}\{u'[\tilde{P}Q^\circ - C(Q^\circ)]\}} = C'(Q^\circ), \tag{11}$$

where $\mathrm{E}_{F^\circ}(\cdot)$ and $\mathrm{Cov}_{F^\circ}(\cdot,\cdot)$ are the expectation and covariance operators with respect to the CDF, $F^\circ(P)$. Eq. (11) states that the risk-averse firm optimally produces at the output level, Q°, that equates the certainty equivalent marginal revenue to the marginal cost of production. We can interpret the second term on the left-hand side of Eq. (11) as the risk premium demanded by the firm to compensate for its exposure to the price risk. Given risk aversion, i.e., $u''(\Pi) < 0$, this term is negative so that the certainty equivalent marginal revenue is below the objective expected price, $\mathrm{E}_{F^\circ}(\tilde{P})$. The risk-averse firm as such produces less than the optimal output level under certainty. This is a well-known result of Sandmo (1971).

Using Eq. (1), we can write the right-hand side of Eq. (8) as

$$\mathrm{E}_{F^\circ}(\tilde{P}) + \frac{\mathrm{Cov}_{F^\circ}\{u'[\tilde{P}Q^\circ - C(Q^\circ)],\tilde{P}\}}{\mathrm{E}_{F^\circ}\{u'[\tilde{P}Q^\circ - C(Q^\circ)]\}}$$

$$+\frac{\mathrm{Cov}_G\{\varphi'[U(Q^\circ,\tilde{\theta})],U_Q(Q^\circ,\tilde{\theta})\}}{\mathrm{E}_{F^\circ}\{u'[\tilde{P}Q^\circ - C(Q^\circ)]\}\mathrm{E}_G\{\varphi'[U(Q^\circ,\tilde{\theta})]\}} - C'(Q^\circ), \tag{12}$$

where $\mathrm{E}_G(\cdot)$ is the expectation operator with respect to the CDF, $G(\theta)$. We can interpret the third term of expression (12) as the ambiguity premium demanded by the firm to compensate for its exposure to ambiguity at $Q = Q^\circ$. Given that the covariance between $\varphi'[U(Q^\circ, \tilde{\theta})]$ and $U_Q(Q^\circ, \tilde{\theta})$ is negative, the ambiguity premium is negative so that the certainty equivalent marginal revenue is further reduced as compared to that in the absence of ambiguity. To restore the optimality condition that the certainty equivalent marginal revenue is equated to the marginal cost of production, the ambiguity-averse firm is induced to produce less than Q° in response to the introduction of ambiguity.

While the result of Proposition 1 is intuitive, the necessary and sufficient condition that a negative ambiguity premium induces the ambiguity-averse firm to produce less is by no means informative. In the following proposition, we derive more tangible sufficient conditions under which $\mathrm{Cov}_G\{\varphi'[U(Q^\circ, \tilde{\theta})], U_Q(Q^\circ, \tilde{\theta})\} < 0$.

Proposition 2. Introducing ambiguity to the ambiguity-averse competitive firm reduces the optimal output level, i.e., $Q^* < Q^\circ$, if the firm's coefficient of relative risk aversion, $R(\Pi) = -\Pi u''(\Pi)/u'(\Pi)$, does not exceed unity, and if the parameter, θ, ranks the subjective cumulative distribution function, $F(P|\theta)$, in the sense of first-order stochastic dominance.

Proof. Differentiating $\varphi'[U(Q, \theta)]$ with respect to θ yields

$$\frac{\partial}{\partial \theta}\varphi'[U(Q, \theta)] = -\int_{\underline{P}}^{\overline{P}} \varphi''[U(Q, \theta)]u'[PQ - C(Q)]QF_\theta(P|\theta)\mathrm{d}P, \tag{13}$$

where $F_\theta(P|\theta) = \partial F(P|\theta)/\partial\theta$, and we have used integration by parts. Likewise, differentiating $U_Q(Q, \theta)$ with respect to θ yields

$$\frac{\partial}{\partial \theta}U_Q(Q, \theta) = \int_{\underline{P}}^{\overline{P}} \Big\{u'[PQ - C(Q)]\{R[PQ - C(Q)] - 1\}$$
$$+[C'(Q)Q - C(Q)]u''[PQ - C(Q)]\Big\}F_\theta(P|\theta)\mathrm{d}P, \tag{14}$$

where $R(\Pi) = -\Pi u''(\Pi)/u'(\Pi)$ is the firm's coefficient of relative risk aversion, and we have used integration by parts. Since $C(0) = 0$, the strict convexity of $C(Q)$ implies that $C'(Q) > C(Q)/Q$ for all $Q > 0$. If $R(\Pi) \leq 1$ and θ ranks $F(P|\theta)$ in the sense of first-order stochastic dominance, it follows from Eqs. (13) and (14) with $Q = Q^\circ$ that $\mathrm{Cov}_G\{\varphi'[U(Q^\circ, \tilde{\theta})], U_Q(Q^\circ, \tilde{\theta})\} < 0$. From Proposition 1, we have $Q^* < Q^\circ$. □

To see the intuition for Proposition 2, we follow Gollier (2011) and Taboga (2005) to define:

$$H(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\varphi'[U(Q^\circ, X)]}{\mathrm{E}_G\{\varphi'[U(Q^\circ, \tilde{\theta})]\}} \mathrm{d}G(X) = \frac{\mathrm{E}_G\{\varphi'[U(Q^\circ, \tilde{X})]|\tilde{X} \leq \theta\}}{\mathrm{E}_G\{\varphi'[U(Q^\circ, \tilde{\theta})]\}} G(\theta), \tag{15}$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]$. It is evident from Eq. (15) that $H(\underline{\theta}) = 0$, $H(\overline{\theta}) = 1$, and $H'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ so that we can interpret $H(\theta)$ as the distorted second-order CDF of $\tilde{\theta}$, where the distortion factor, $\varphi'[U(Q^\circ, \theta)]/\mathrm{E}_G\{\varphi'[U(Q^\circ, \tilde{\theta})]\}$ is a Radon-Nikodym derivative. From Eqs. (4), (6), (8), (9), and (15), we have $Q^* < Q^\circ$ if, and only if,

$$\int_{\underline{\theta}}^{\overline{\theta}} U_Q(Q^\circ, \theta) \mathrm{d}G(\theta) = 0 \Rightarrow \int_{\underline{\theta}}^{\overline{\theta}} U_Q(Q^\circ, \theta) \mathrm{d}H(\theta) < 0. \tag{16}$$

We can interpret the left-hand side of condition (16) as the first-order condition for program (3) when the firm is ambiguity neutral. The right-hand side of condition (16) is then the condition that ensures the ambiguity-neutral firm to reduce its optimal output level when the second-order CDF of $\tilde{\theta}$ is shifted from $G(\theta)$ to $H(\theta)$. Hence, the effect of introducing ambiguity on the ambiguity-averse firm's production decision is identical to the comparative static result of a shift in ambiguity from $G(\theta)$ to $H(\theta)$ on the ambiguity-neutral firm's optimal output level.

Using Leibniz's rule, we differentiate $\mathrm{E}_G\{\varphi'[U(Q^\circ, \tilde{X})]|\tilde{X} \leq \theta\}$ with respect to θ to yield

$$\frac{\partial}{\partial\theta} \mathrm{E}_G\{\varphi'[U(Q^\circ, \tilde{X})]|\tilde{X} \leq \theta\} = \frac{G'(\theta)}{G(\theta)} \Big\{\varphi'[U(Q^\circ, \theta)] - \mathrm{E}_G\{\varphi'[U(Q^\circ, \tilde{X})]|\tilde{X} \leq \theta\}\Big\}, \tag{17}$$

which, from Eq. (13), is positive (negative) if an increase in θ deteriorates (improves) $F(P|\theta)$ in the sense of first-order stochastic dominance, i.e., $F_\theta(P|\theta) > (<)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.[10]

[10] Proposition 2 holds even in the case that the sign of $F_\theta(P|\theta)$ varies as θ changes. See the proof of Proposition 2.

It then follows from Eqs. (15) and (17) that $H(\theta) < (>)\ G(\theta)$ for all $\theta \in (\underline{\theta}, \overline{\theta})$ so that $H(\theta)$ dominates (is dominated by) $G(\theta)$ in the sense of first-order stochastic dominance if $F_\theta(P|\theta) > (<)\ 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.[11] In this case, condition (16) holds if, and only if, $U_Q(Q^\circ, \theta)$ is decreasing (increasing) in θ, which, from Eq. (14), is true if the firm's coefficient of relative risk aversion does not exceed unity, i.e., $R(\Pi) \leq 1$.[12]

Following Alary et al. (2013) and Snow (2011), we consider the case that the underlying risk is binary: $\tilde{P} = \underline{P}$ with the objective probability p, and subjective probability, $\rho(\theta)$, and $\tilde{P} = \overline{P}$ with the complementary objective probability, $1-p$, and subjective probability, $1-\rho(\theta)$.[13] It follows from Eq. (1) that $\mathrm{E}_G[\rho(\tilde{\theta})] = p$. In this case, Eq. (7) becomes

$$pu'[\underline{P}Q^\circ - C(Q^\circ)][\underline{P} - C'(Q^\circ)] + (1-p)u'[\overline{P}Q^\circ - C(Q^\circ)][\overline{P} - C'(Q^\circ)] = 0. \tag{18}$$

We show in the following proposition that $\mathrm{Cov}_G\{\varphi'[U(Q^\circ, \tilde{\theta})], U_Q(Q^\circ, \tilde{\theta})\} < 0$.

Proposition 3. Given that the price risk, $\tilde{P}$, is binary, introducing ambiguity to the ambiguity-averse competitive firm always reduces the optimal output level, i.e., $Q^* < Q^\circ$.

Proof. Differentiating $\varphi'[U(Q^\circ, \theta)]$ with respect to θ yields

$$\frac{\partial}{\partial\theta}\varphi'[U(Q^\circ, \theta)] = -\varphi''[U(Q^\circ, \theta)]\rho'(\theta)\{u[\overline{P}Q^\circ - C(Q^\circ)] - u[\underline{P}Q^\circ - C(Q^\circ)]\}, \tag{19}$$

which has the same sign as that of $\rho'(\theta)$. Differentiating $U_Q(Q^\circ, \theta)$ with respect to θ yields

$$\begin{aligned}\frac{\partial}{\partial\theta}U_Q(Q^\circ, \theta) = -\rho'(\theta)\{&u'[\underline{P}Q^\circ - C(Q^\circ)][C'(Q^\circ) - \underline{P}] \\ &+ u'[\overline{P}Q^\circ - C(Q^\circ)][\overline{P} - C'(Q^\circ)]\},\end{aligned} \tag{20}$$

[11] If θ ranks $F(P|\theta)$ in the sense of second-order stochastic dominance, one can analogously show that $G(\theta)$ and $H(\theta)$ are also ranked in the sense of second-order stochastic dominance. In this case, $Q^* < Q^\circ$ if the firm's coefficient of relative prudence, $P(\Pi) = -\Pi u'''(\Pi)/u''(\Pi)$ is positive and less than two, i.e., $0 < P(\Pi) < 2$. See also Gollier (2011).

[12] Gollier (2011) derives similar sufficient conditions under which an ambiguity-averse investor demands a smaller amount of a risky asset whose return becomes ambiguous.

[13] Alary et al. (2013) and Snow (2011) examine the effect of ambiguity aversion on self-insurance and self-protection, where a binary structure (loss versus no-loss states) is natural.

which has the opposite sign to that of $\rho'(\theta)$ because $\underline{P} < C'(Q^\circ) < \overline{P}$. It follows immediately from Eqs. (19) and (20) that $\mathrm{Cov}_G\{\varphi'[U(Q^\circ, \tilde{\theta})], U_Q(Q^\circ, \tilde{\theta})\} < 0$ irrespective of the sign of $\rho'(\theta)$. From Proposition 1, we have $Q^* < Q^\circ$. $\square$

To see the intuition for Proposition 3, we write the marginal expected utility, $U_Q(Q^\circ, \theta)$, as

$$U_Q(Q^\circ, \theta) = u'[\overline{P}Q^\circ - C(Q^\circ)][\overline{P} - C'(Q^\circ)]$$

$$-\rho(\theta)\{u'[\overline{P}Q^\circ - C(Q^\circ)][\overline{P} - C'(Q^\circ)] + u'[\underline{P}Q^\circ - C(Q^\circ)][C'(Q^\circ) - \underline{P}]\}. \tag{21}$$

Since $\underline{P} < C'(Q^\circ) < \overline{P}$, it follows from Eq. (21) that $U_Q(Q^\circ, \theta)$ decreases with an increase in $\rho(\theta)$. Given Eq. (18), $U_Q(Q^\circ, \theta)$ changes sign from positive to negative at $\rho(\theta) = p$ as $\rho(\theta)$ increases. Hence, a marginal reduction in output from Q° reduces $U(Q^\circ, \theta)$ when the expected utility is high, i.e., when $\rho(\theta) < p$, and increases $U(Q^\circ, \theta)$ when the expected utility is low, i.e., when $\rho(\theta) > p$. Since $\mathrm{E}_G[\rho(\tilde{\theta})] = p$, Eq. (18) implies that the expected value of $U(Q^\circ, \theta)$ remains unchanged for a marginal change in output from Q°. Hence, a marginal reduction in output from Q° gives rise to a mean-preserving contraction in the distribution of $U(Q^\circ, \tilde{\theta})$. The firm, being ambiguity averse, has a concave ambiguity function, $\varphi(U)$, thereby choosing to cut down its production, i.e., $Q^* < Q^\circ$, in response to the introduction of ambiguity.

IV. AMBIGUITY AVERSION AND PRODUCTION

In this section, we examine the effect of greater ambiguity aversion on the firm's optimal production decision when ambiguity prevails. To this end, we follow the comparative notion of "more ambiguity aversion" defined by Klibanoff et al. (2005). Specifically, an ambiguity function, $\hat{\varphi}(U)$, is said to represent more ambiguity aversion than the original ambiguity function, $\varphi(U)$, if the firm with $\varphi(U)$ prefers an uncertain act over a pure risky act whenever the firm with $\hat{\varphi}(U)$ does so. As shown by Klibanoff et al. (2005), this is true if, and only if, $\hat{\varphi}(U)$ is more concave than $\varphi(U)$ in the Arrow-Pratt sense, i.e., $-\hat{\varphi}''(U)/\hat{\varphi}'(U) > -\varphi''(U)/\varphi'(U)$ for all U. Hence, the firm becomes more ambiguity averse when there is a concave transformation of $\varphi(U)$, i.e., $\hat{\varphi}(U) = K[\varphi(U)]$, where $K(\cdot)$ satisfies that $K'(\cdot) > 0$ and $K''(\cdot) < 0$.

The more ambiguity-averse firm's ex-ante decision problem is given by

$$\max_{Q\geq 0} \int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta). \tag{22}$$

The first-order condition for program (22) is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} K'\{\varphi[U(Q^{\dagger},\theta)]\}\varphi'[U(Q^{\dagger},\theta)]U_Q(Q^{\dagger},\theta)\mathrm{d}G(\theta) = 0, \tag{23}$$

where $Q^{\dagger}$ is the firm's optimal output level. It is evident that Eq. (6) remains valid when we replace $\varphi(U)$ by $K[\varphi(U)]$. Comparing Eq. (4) with Eq. (23) yields the following proposition.

Proposition 4. Making the ambiguity-averse competitive firm more ambiguity averse by replacing $\varphi(U)$ by $K[\varphi(U)]$ reduces the optimal output level, i.e., $Q^{\dagger} < Q^*$, if, and only if, the covariance between $K'\{\varphi[U(Q^*,\theta)]\}$ and $\varphi'[U(Q^*,\tilde{\theta})]U_Q(Q^*,\tilde{\theta})$ is negative.

Proof. Differentiating the objective function of program (22) with respect to Q, and evaluating the resulting derivative at $Q = Q^*$ yields

$$\frac{\partial}{\partial Q}\int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta)\Big|_{Q=Q^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} K'\{\varphi[U(Q^*,\theta)]\}\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta). \tag{24}$$

Using Eq. (4), we can write Eq. (24) as

$$\frac{\partial}{\partial Q}\int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta)\Big|_{Q=Q^*}$$

$$= \mathrm{Cov}_G\Big\{K'\{\varphi[U(Q^*,\tilde{\theta})]\}, \varphi'[U(Q^*,\tilde{\theta})]U_Q(Q^*,\tilde{\theta})\Big\}. \tag{25}$$

It then follows from Eqs. (6) and (23) that $Q^\dagger < Q^*$ if, and only if, the covariance term on the right-hand side of Eq. (25) is negative. □

Proposition 4 provides a necessary and sufficient condition under which greater ambiguity aversion adversely affects the firm's production decision. To see the intuition for Proposition 4, we use Eq. (12) to compare the ambiguity premium under $\varphi(U)$ and that under $K[\varphi(U)]$, both of which are evaluated at $Q = Q^*$:

$$\frac{\mathrm{Cov}_G\{\varphi'[U(Q^*,\tilde{\theta})], U_Q(Q^*,\tilde{\theta})\}}{\mathrm{E}_{F^\circ}\{u'[\tilde{P}Q^* - C(Q^*)]\}\mathrm{E}_G\{\varphi'[U(Q^*,\tilde{\theta})]\}}$$

$$-\frac{\mathrm{Cov}_G\{K'\{\varphi[U(Q^*,\tilde{\theta})]\}\varphi'[U(Q^*,\tilde{\theta})], U_Q(Q^*,\tilde{\theta})\}}{\mathrm{E}_{F^\circ}\{u'[\tilde{P}Q^* - C(Q^*)]\}\mathrm{E}_G\{K'\{\varphi[U(Q^*,\tilde{\theta})]\}\varphi'[U(Q^*,\tilde{\theta})]\}}$$

$$= -\frac{\mathrm{Cov}_G\{K'\{\varphi[U(Q^*,\tilde{\theta})]\}, \varphi'[U(Q^*,\tilde{\theta})]U_Q(Q^*,\tilde{\theta})\}}{\mathrm{E}_{F^\circ}\{u'[\tilde{P}Q^* - C(Q^*)]\}\mathrm{E}_G\{K'\{\varphi[U(Q^*,\tilde{\theta})]\}\varphi'[U(Q^*,\tilde{\theta})]\}}, \tag{26}$$

where we have used Eq. (4). Given that the covariance between $K'\{\varphi[U(Q^*,\theta)]\}$ and $\varphi'[U(Q^*,\tilde{\theta})]U_Q(Q^*,\tilde{\theta})$ is negative, it follows from Eq. (26) that the ambiguity premium is more negative under $K[\varphi(U)]$ than under $\varphi(U)$. Greater ambiguity aversion as such reduces the certainty equivalent marginal revenue. To restore the optimality condition that the certainty equivalent marginal revenue is equated to the marginal cost of production, the more ambiguity-averse firm finds it optimal to produce less than Q^*.

The necessary and sufficient condition as stated in Proposition 4, which ensures the ambiguity premium to be more negative as the firm becomes more ambiguity averse, is rather complicated. As such , we derive in the following proposition more tangible sufficient conditions under which greater ambiguity aversion adversely affects the firm's production decision.

Proposition 5. Making the ambiguity-averse competitive firm more ambiguity averse reduces the optimal output level, i.e., $Q^\dagger < Q^*$, if the firm's coefficient of relative risk aversion, $R(\Pi) = -\Pi u''(\Pi)/u'(\Pi)$, does not exceed unity, and if an increase in the parameter, θ, always deteriorates (improves) the subjective cumulative distribution function, $F(P|\theta)$, in the sense of first-order stochastic dominance.

Proof. Differentiating $K'\{\varphi'[U(Q,\theta)\}$ with respect to θ yields

$$\frac{\partial}{\partial\theta}K'\{\varphi[U(Q,\theta)]\}$$

$$= -\int_{\underline{P}}^{\overline{P}} K''\{\varphi[U(Q,\theta)]\}\varphi'[U(Q,\theta)]u'[PQ - C(Q)]QF_\theta(P|\theta)\mathrm{d}P, \tag{27}$$

where $F_\theta(P|\theta) = \partial F(P|\theta)/\partial\theta$, and we have used integration by parts. If $R(\Pi) \leq 1$ and an increase in θ always deteriorates (improves) $F(P|\theta)$ in the sense of first-order stochastic dominance, Eq. (14) implies that $U_Q(Q^*,\theta)$ is decreasing (increasing) in θ, and Eq. (27) implies that $K'\{\varphi[U(Q^*,\theta)]\}$ is increasing (decreasing) in θ. From Proposition 2, we have $Q^* < Q^\circ$ so that Eqs. (6) and (7) imply that

$$\int_{\underline{\theta}}^{\overline{\theta}} U_Q(Q^*,\theta)\mathrm{d}G(\theta) > 0, \tag{28}$$

Consider first the case that an increase in θ always deteriorates $F(P|\theta)$ in the sense of first-order stochastic dominance. Since $\partial U_Q(Q^*,\theta)/\partial\theta < 0$, there are two possible cases that are consistent with Eq. (28): (i) $U_Q(Q^*,\theta) > 0$ for all $\theta \in [\underline{\theta},\overline{\theta})$, and (ii) there exists a unique point, $\theta_1 \in (\underline{\theta},\overline{\theta})$ such that $U_Q(Q^*,\theta) > (<)\ 0$ for all $\theta < (>)\ \theta_1$. In case (i), the right-hand side of Eq. (24) must be positive. Using Eq. (4), we can write Eq. (24) as

$$\frac{\partial}{\partial Q}\int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta)\Big|_{Q=Q^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\varphi[U(Q^*,\theta)]\} - K'\{\varphi[U(Q^*,\overline{\theta})]\}\Big\}\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta). \tag{29}$$

Since $\partial K'\{\varphi[U(Q^*,\theta)]\}/\partial\theta > 0$, we have $K'\{\varphi[U(Q^*,\theta)]\} < K'\{\varphi[U(Q^*,\overline{\theta})]\}$ for all $\theta \in [\underline{\theta},\overline{\theta})$. It then follows from $U_Q(Q^*,\theta) > 0$ for all $\theta \in [\underline{\theta},\overline{\theta})$ that the right-hand side of Eq. (29) is negative, a contradiction. Hence, case (i) cannot hold. In case (ii), we can use Eq. (4) to write Eq. (24) as

$$\frac{\partial}{\partial Q}\int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta)\bigg|_{Q=Q^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\varphi[U(Q^*,\theta)]\} - K'\{\varphi[U(Q^*,\theta_1)]\}\Big\}\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta). \tag{30}$$

Since $K'\{\varphi[U(Q^*,\theta)]\} < (>)\ K'\{\varphi[U(Q^*,\theta_1)]\}$ and $U_Q(Q^*,\theta) > (<)\ 0$ for all $\theta < (>)\ \theta_1$, the right-hand side of Eq. (30) is negative. It then follows from Eqs. (6) and (23) that $Q^\dagger < Q^*$.

Consider now the case that an increase in θ always improves $F(P|\theta)$ in the sense of first-order stochastic dominance. Since $\partial U_Q(Q^*,\theta)/\partial\theta > 0$, there are two possible cases that are consistent with Eq. (28): (i) $U_Q(Q^*,\theta) > 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$, and (ii) there exists a unique point, $\theta_2 \in (\underline{\theta},\overline{\theta})$ such that $U_Q(Q^*,\theta) < (>)\ 0$ for all $\theta < (>)\ \theta_2$. In case (i), the right-hand side of Eq. (24) must be positive. Using Eq. (4), we can write Eq. (24) as

$$\frac{\partial}{\partial Q}\int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta)\bigg|_{Q=Q^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\varphi[U(Q^*,\theta)]\} - K'\{\varphi[U(Q^*,\underline{\theta})]\}\Big\}\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta). \tag{31}$$

Since $\partial K'\{\varphi[U(Q^*,\theta)]\}/\partial\theta < 0$, we have $K'\{\varphi[U(Q^*,\theta)]\} < K'\{\varphi[U(Q^*,\underline{\theta})]\}$ for all $\theta \in (\underline{\theta},\overline{\theta}]$. It then follows from $U_Q(Q^*,\theta) > 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$ that the right-hand side of Eq. (31) is negative, a contradiction. Hence, case (i) cannot hold. In case (ii), we can use Eq. (4) to write Eq. (24) as

$$\frac{\partial}{\partial Q}\int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta)\bigg|_{Q=Q^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\varphi[U(Q^*,\theta)]\} - K'\{\varphi[U(Q^*,\theta_2)]\}\Big\}\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta). \tag{32}$$

Since $K'\{\varphi[U(Q^*,\theta)]\} > (<)\ K'\{\varphi[U(Q^*,\theta_2)]\}$ and $U_Q(Q^*,\theta) < (>)\ 0$ for all $\theta < (>)\ \theta_2$, the right-hand side of Eq. (32) is negative. It then follows from Eqs. (6) and (23) that $Q^\dagger < Q^*$. $\square$

To see the intuition for Proposition 5, we consider the distorted second-order CDF, $H(\theta)$, given by Eq. (15) with Q° replaced by Q^*. Define the following function:

$$\hat{H}(\theta) = \int_{\underline{\theta}}^{\theta} \frac{K'\{\varphi[U(Q^*,X)]\}}{\mathrm{E}_H\{K'\{\varphi[U(Q^*,\tilde{\theta})]\}\}}\mathrm{d}H(X) = \frac{\mathrm{E}_H\{K'\{\varphi[U(Q^*,\tilde{X})]\}|\tilde{X} \leq \theta\}}{\mathrm{E}_H\{K'\{\varphi[U(Q^*,\tilde{\theta})]\}\}}H(\theta), \tag{33}$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]$, where $\mathrm{E}_H(\cdot)$ is the expectation operator with respect to the CDF, $H(\theta)$. It is evident from Eq. (33) that $\hat{H}(\underline{\theta}) = 0$, $\hat{H}(\overline{\theta}) = 1$, and $\hat{H}'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ so that we can interpret $\hat{H}(\theta)$ as another distorted second-order CDF of $\tilde{\theta}$. From Eqs. (4), (6), (24), and (33), we have $Q^\dagger < Q^*$ if, and only if,

$$\int_{\underline{\theta}}^{\overline{\theta}} U_Q(Q^*,\theta)\mathrm{d}H(\theta) = 0 \Rightarrow \int_{\underline{\theta}}^{\overline{\theta}} U_Q(Q^*,\theta)\mathrm{d}\hat{H}(\theta) < 0. \tag{34}$$

We can interpret the left-hand side of condition (34) as the optimality condition when the firm is ambiguity neutral and has the second-order CDF, $H(\theta)$. The right-hand side of condition (16) is then the condition that ensures the ambiguity-neutral firm to reduce its optimal output level when the second-order CDF of $\tilde{\theta}$ is shifted from $H(\theta)$ to $\hat{H}(\theta)$. Hence, the effect of greater ambiguity aversion on the ambiguity-averse firm's production decision is identical to the comparative static result of a shift in ambiguity from $H(\theta)$ to $\hat{H}(\theta)$ on the ambiguity-neutral firm's optimal output level.

Using Leibniz's rule, we differentiate $\mathrm{E}_H\{K'\{\varphi[U(Q^*,\tilde{X})]\}|\tilde{X} \leq \theta\}$ with respect to θ to yield

$$\frac{\partial}{\partial\theta}\mathrm{E}_H\Big\{K'\{\varphi[U(Q^*,\tilde{X})]\}\Big|\tilde{X} \leq \theta\Big\}$$

$$= \frac{H'(\theta)}{H(\theta)}\left\{K'\{\varphi[U(Q^*,\theta)]\} - \mathrm{E}_H\left\{K'\{\varphi[U(Q^*,\tilde{X})]\}\middle|\tilde{X} \le \theta\right\}\right\}. \tag{35}$$

Differentiating $K'\{\varphi[U(Q,\theta)]\}$ with respect to θ yields

$$\frac{\partial}{\partial\theta}K'\{\varphi[U(Q,\theta)]\}$$

$$= -\int_{\underline{P}}^{\overline{P}} K''\{\varphi[U(Q,\theta)]\}\varphi'[U(Q,\theta)]u'[PQ - C(Q)]QF_\theta(P|\theta)\mathrm{d}P. \tag{36}$$

It follows from Eq. (36) that the right-hand side of Eq. (35) is positive (negative) if an increase in θ deteriorates (improves) $F(P|\theta)$ in the sense of first-order stochastic dominance, i.e., $F_\theta(P|\theta) > (<)\ 0$ for all $\theta \in [\underline{\theta},\overline{\theta}]$. It then follows from Eqs. (33) and (35) that $\hat{H}(\theta) < (>)\ H(\theta)$ for all $\theta \in (\underline{\theta},\overline{\theta})$ so that $\hat{H}(\theta)$ dominates (is dominated by) $H(\theta)$ in the sense of first-order stochastic dominance if $F_\theta(P|\theta) > (<)\ 0$ for all $\theta \in [\underline{\theta},\overline{\theta}]$. In this case, condition (34) holds if, and only if, $U_Q(Q^*,\theta)$ is decreasing (increasing) in θ, which, from Eq. (14), is true if the firm's coefficient of relative risk aversion does not exceed unity, i.e., $R(\Pi) \le 1$.

Following Alary et al. (2013) and Snow (2011), we consider the case that the price risk is binary: $\tilde{P} = \underline{P}$ with the subjective probability, $\rho(\theta)$, and $\tilde{P} = \overline{P}$ with the complementary subjective probability, $1 - \rho(\theta)$. We show in the following proposition that the greater ambiguity aversion always induces the firm to reduce its optimal output.

Proposition 6. Given that the price risk, $\tilde{P}$, is binary, making the ambiguity-averse competitive firm more ambiguity averse always reduces the optimal output level, i.e., $Q^\dagger < Q^*$.

Proof. We write $K'\{\varphi[U(Q^*,\theta)]\}$ as

$$K'\{\varphi[U(Q^*,\theta)]\} = K'\Big\{\varphi\{\rho(\theta)u[\underline{P}Q^* - C(Q^*)] + [1-\rho(\theta)]u[\overline{P}Q^* - C(Q^*)]\}\Big\}. \tag{37}$$

Since $K''(\cdot) < 0$, it follows from Eq. (37) that $K'\{\varphi[U(Q^*,\theta)]\}$ increases with an increase in $\rho(\theta)$. Likewise, we write $\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)$ as

$$\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta) = \varphi'[U(Q^*,\theta)]\Big\{u'[\overline{P}Q^* - C(Q^*)][\overline{P} - C'(Q^*)]$$

$$-\rho(\theta)\{u'[\overline{P}Q^* - C(Q^*)][\overline{P} - C'(Q^*)] + u'[\underline{P}Q^\circ - C(Q^*)][C'(Q^*) - \underline{P}]\}\Big\}. \tag{38}$$

Since $\underline{P} < C'(Q^*) < \overline{P}$, it follows from Eq. (38) that $\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)$ decreases with an increase in $\rho(\theta)$. Given Eq. (4), $\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)$ changes sign from positive to negative at $\rho(\theta) = \rho(\theta^*)$ as $\rho(\theta)$ increases for a unique value of $\theta^* \in (\underline{\theta},\overline{\theta})$. Using Eq. (4), we can write Eq. (24) as

$$\frac{\partial}{\partial Q}\int_{\underline{\theta}}^{\overline{\theta}} K\{\varphi[U(Q,\theta)]\}\mathrm{d}G(\theta)\bigg|_{Q=Q^*}$$

$$= \int_{\{\theta:\rho(\theta)\le\rho(\theta^*)\}}\Big\{K'\{\varphi[U(Q^*,\theta)]\} - K'\{\varphi[U(Q^*,\theta^*)]\}\Big\}\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta)$$

$$+ \int_{\{\theta:\rho(\theta)>\rho(\theta^*)\}}\Big\{K'\{\varphi[U(Q^*,\theta)]\} - K'\{\varphi[U(Q^*,\theta^*)]\}\Big\}\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta)\mathrm{d}G(\theta),$$

which is negative. It then follows from Eqs. (6) and (23) that $Q^\dagger < Q^*$. $\square$

The intuition for Proposition 6 is as follows. From Eqs. (4) and (38), it is true that $\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta) > 0$ when $\varphi[U(Q^*,\theta)]$ is high, i.e., when $\rho(\theta) < \rho(\theta^*)$, and that $\varphi'[U(Q^*,\theta)]U_Q(Q^*,\theta) < 0$ when $\varphi[U(Q^*,\theta)]$ is low, i.e., when $\rho(\theta) > \rho(\theta^*)$. Given Eq. (4), the expected value of $\varphi[U(Q^*,\theta)]$ remains unchanged for a marginal change in output from Q^*. Hence, a marginal reduction in output from Q^* gives rise to a mean-preserving contraction in the distribution of $\varphi[U(Q^*,\tilde{\theta})]$. The firm, being more ambiguity averse, has a concave transformation of $\varphi(U)$, thereby choosing to cut down its production, i.e., $Q^* < Q^\circ$, in response to greater ambiguity aversion.

V. CONCLUSION

In this paper, we examine the production decision of the competitive firm under price uncertainty *à la* Sandmo (1971) when the firm's preferences exhibit smooth ambiguity aversion developed by Klibanoff et al. (2005). The KMM model represents ambiguity by a second-order probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. The KMM model then measures the firm's expected utility under ambiguity by taking the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk.

Within the KMM model, we derive necessary and sufficient conditions under which the ambiguity-averse firm optimally produces less in response either to the introduction of ambiguity or to greater ambiguity aversion when ambiguity prevails. These conditions ensure that the ambiguity premium is negative (more negative as the firm becomes more ambiguity averse), thereby inducing the firm to cut down its optimal output level so as to equate the certainty equivalent marginal revenue to the marginal cost of production. We further derive more tangible sufficient conditions under which ambiguity and greater ambiguity aversion adversely affect the firm's production decision. These conditions include the firm's coefficient of relative risk aversion not to exceed unity and its subjective beliefs to be ranked in the sense of first-order stochastic dominance. Finally, we show that the adverse effect of ambiguity and ambiguity aversion on output always prevails when the price risk is binary.

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