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<th>Measuring dark energy with the Eiso–Ep correlation of gamma-ray bursts using model-independent methods</th>
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<td>Wang, JS; Wang, F; Cheng, KS; Dai, ZG</td>
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Measuring dark energy with the $E_{\text{iso}} - E_p$ correlation of gamma-ray bursts using model-independent methods

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ABSTRACT

We use two model-independent methods to standardize long gamma-ray bursts (GRBs) using the $E_{\text{iso}} - E_p$ correlation ($\log E_{\text{iso}} = a + b \log E_p$), where $E_{\text{iso}}$ is the isotropic-equivalent gamma-ray energy and $E_p$ is the spectral peak energy. We update 42 long GRBs and attempt to constrain the cosmological parameters. The full sample contains 151 long GRBs with redshifts from 0.0331 to 8.2. The first method is the simultaneous fitting method. We take the extrinsic scatter $\sigma_{\text{ext}}$ into account and assign it to the parameter $E_{\text{iso}}$. The best-fitting values are $a = 49.15 \pm 0.26$, $b = 1.42 \pm 0.11$, $\sigma_{\text{ext}} = 0.34 \pm 0.03$ and $\Omega_m = 0.79$ in the flat $\Lambda$CDM model. The constraint on $\Omega_m$ is $0.55 < \Omega_m < 1$ at the 1σ confidence level. If reduced $\chi^2$ method is used, the best-fit results are $a = 48.96 \pm 0.18$, $b = 1.52 \pm 0.08$, and $\Omega_m = 0.50 \pm 0.12$. The second method uses type Ia supernovae (SNe Ia) to calibrate the $E_{\text{iso}} - E_p$ correlation. We calibrate 90 high-redshift GRBs in the redshift range from 1.44 to 8.1. The cosmological constraints from these 90 GRBs are $\Omega_m = 0.23^{+0.06}_{-0.04}$ for flat $\Lambda$CDM and $\Omega_m = 0.18 \pm 0.11$ and $\Omega_m = 0.46 \pm 0.51$ for non-flat $\Lambda$CDM. For the combination of GRB and SNe Ia sample, we obtain $\Omega_m = 0.271 \pm 0.019$ and $h = 0.701 \pm 0.002$ for the flat $\Lambda$CDM and the non-flat $\Lambda$CDM, and the results are $\Omega_m = 0.225 \pm 0.044$, $\Omega_m = 0.640 \pm 0.082$, and $h = 0.698 \pm 0.004$. These results from calibrated GRBs are consistent with that of SNe Ia. Meanwhile, the combined data can improve cosmological constraints significantly, compared to SNe Ia alone. Our results show that the $E_{\text{iso}} - E_p$ correlation is promising to probe the high-redshift Universe.

Key words. cosmological parameters – dark energy – gamma rays: general

1. Introduction

Gamma-ray bursts (GRBs) are the most violent explosions in the Universe, with the highest isotropic energy up to $10^{54}$ erg (for reviews, see Mészáros 2006; Zhang 2007; Gehrels et al. 2009). Thus, they can be detected to the edge of the visible Universe (Ciardi & Loeb 2000; Lamb & Reichart 2000; Wang et al. 2012). For instance, the spectroscopically confirmed redshift of GRB090423 is about 8.2 (Tanvir et al. 2009; Salvaterra et al. 2009). Therefore, they are promising probes for the high-redshift Universe (for a recent review, see Wang et al. 2015). Many studies have been carried out to use GRBs for cosmological purposes, such as the star formation rate (Totani 1997; Wijers et al. 1998; Porciani & Madau 2001; Wang & Dai 2009, 2011a), the intergalactic medium metal enrichment (Barkana & Loeb 2004; Wang et al. 2012), dark energy (Dai et al. 2004; Friedman & Bloom 2005; Schaefer 2007; Basilakos & Perivolaropoulos 2008; Wang et al. 2011b), reionization (Totani et al. 2006; Gallarini et al. 2008; Wang 2013), possible anisotropic acceleration (Wang & Wang 2014a), and the two-point correlation (Li & Lin 2015).

To constrain the cosmological parameters, standard rulers or candles such as baryon acoustic oscillations (BAO; Cole et al. 2005; Eisenstein et al. 2005; Anderson et al. 2014), cosmic microwave background (CMB; Komatsu et al. 2011; Planck Collaboration XVI 2013; Planck Collaboration XIII 2015) and SNe Ia (Riess et al. 1998; Perlmutter et al. 1999; Suzuki et al. 2012) are required. The redshifts of BAO and SNe Ia are low, however, and the CMB is only a snapshot of cosmic expansion. Some parameters, such as the density and EOS parameter of dark energy (Wang 2012; Wang & Dai 2014; Wang & Wang 2014b), might evolve with redshift. GRBs can probe the evolution of these parameters at high redshifts and serve as complementary tools for SNe Ia. The study of these evolutions can differentiate dark energy models. Some luminosity correlations have been proposed to standardize GRBs (Amati et al. 2002; Ghirlanda et al. 2004a; Liang & Zhang 2005). Ghirlanda et al. (2004a) found a tight correlation between collimated energy $E_t$ and the peak energy $E_p$ of $\nu F_\nu$ spectrum. Dai et al. (2004) used this correlation to constrain cosmological parameters with 12 GRBs. Liang & Zhang (2005) found the $E_{\text{iso}} - E_p - b_0$ correlation and used this correlation to constrain cosmological parameters. Recently, Wang et al. (2011b) constrained cosmological parameters with 109 GRBs using six GRB empirical correlations, and found $\Omega_m = 0.31^{+0.10}_{-0.09}$ in the flat $\Lambda$ cold dark matter (CDM) model. Other attempts have also been made to standardize GRBs (Ghirlanda et al. 2004b; Friedman & Bloom 2005; Schaefer 2007; Wang et al. 2007; Liang et al. 2008; Kodama et al. 2008; Qiu et al. 2009; Cardone et al. 2010; Wang & Dai 2011c). These methods of standardizing the long GRBs are mainly based on some empirical correlations, such as the $E_{\text{iso}} - E_p$ (Amati et al. 2002), $E_p - L_p$ (Schaefer 2003; Wei & Gao 2003), and $E_p - E_{\gamma}$ correlation.
GRB sample can help to constrain cosmological models better. In this paper, we focus on the usage of the $E_{\text{iso}} - E_p$ correlation. Amati et al. (2002) discovered this correlation with a small sample of BeppoSAX GRBs. Since many more GRBs are detected, attempts have been made to use this correlation for the purpose of cosmology. Amati et al. (2008) used a simultaneous fitting method to constrain the $E_{\text{iso}} - E_p$ correlation coefficients and cosmological parameters together with 70 long GRBs. The extrinsic scatter $\sigma_{\text{ext}}$ was taken into consideration in this method (D’Agostini 2005). Amati et al. (2008) assigned $\sigma_{\text{ext}}$ to $E_p$ and found 0.04 $< \Omega_m < 0.40$ and $\sigma_{\text{ext}} = 0.17 \pm 0.02$ at 1$\sigma$ confidence level in the flat $\Lambda$CDM Universe. For non-flat $\Lambda$CDM model, the results are $\Omega_m \in [0.04, 0.40]$ and $\Omega_\Lambda < 1.05$ (Amati et al. 2008). However, Ghirlanda (2009) doubted this result. He claimed that the extrinsic scatter term should be assigned to $E_{\text{iso}}$. This is consistent with D’Agostini (2005), who described that the extrinsic scatter $\sigma_{\text{ext}}$ should be assigned to the parameter that also depends on hidden variables (cosmological parameters in our study). We discuss this point in detail in Sect. 3.1. However, this would lead to no constraint on cosmological parameters with the same 70 GRBs from Amati et al. (2008). We test it again with a larger sample in this paper.

The calibration method is also helpful to standardize GRBs. Imitating the example of standardizing the center of SNe Ia with Cepheid variables, GRBs can also be calibrated with SNe Ia (Liang et al. 2008; Kodama et al. 2008; Wei 2010; Lin et al. 2016). This method is also cosmological model independent. Liang et al. (2008) calibrated 42 high redshift GRBs with SNe Ia. Five interpolation methods were used and the results were consistent with each other. Wei (2010) standardized 59 high-redshift GRBs with SNe Ia, using the $E_{\text{iso}} - E_p$ correlation, and found that GRBs can improve the constraint on cosmological parameters. Wang & Dai (2011c) calibrated 116 GRBs with Union 2 SNe Ia with cosmographic parameters.

We use 151 GRBs, 109 of which are taken from Amati et al. (2008) and Amati et al. (2009). The remaining 42 GRBs are the updated long GRBs, which were detected by Fermi GBM, Konus-Wind, Swift-BAT, and Suzaku-WAM. The energy band, fluence, low ($\alpha$), high ($\beta$) energy photon indices, spectral peak energy, and redshift are taken from the refined analysis of the corresponding GRB team. We test whether this larger sample in this paper.

In Sect. 3.1. The $E_{\text{iso}} - E_p$ correlation and constraints on cosmological parameters

3.1. The $E_{\text{iso}} - E_p$ correlation

To constrain cosmological models more precisely, we combine our updated 42 GRBs with 109 GRBs from Amati et al. (2008) and Amati et al. (2009). The full sample contains 151 GRBs and

$\Phi(E) = \begin{cases} AE_{\text{obs}}E_{\text{obs}}^{-\alpha} & \text{if } E \leq \frac{E_\text{iso}}{2\sigma_{\text{ext}}} \\ BE_\text{iso}^{-\beta} & \text{otherwise} \end{cases}$

where $E_{\text{obs}}$ is the observed peak energy, $\alpha$ and $\beta$ are the low and high energy photon indices, respectively. We take the typical spectral index values for those GRB whose indices are not given out in the references, i.e., $\alpha = -1.0$ and $\beta = -2.2$ (Salvaterra et al. 2009).

With these spectra parameters, we can obtain the peak energy in the cosmological rest frame by $E_p = E_{\text{obs}} \times (1 + z)$ and the bolometric fluence in the band of $1 - 10^4$ keV by (Bloom et al. 2001)

$S_{\text{bolo}} = S \times \frac{\int_{E_{\text{min}}}^{E_{\text{max}}} E \Phi(E) dE}{\int_{E_{\text{min}}}^{E_{\text{max}}} E \Phi(E) dE}$

where $S$ is the observed fluence, $E_{\text{min}}$ and $E_{\text{max}}$ are the detection limits of the instrument, and $z$ is the redshift.

In the $E_{\text{iso}} - E_p$ plane, $E_p$ is an observed value, which is not dependent on the cosmological model. However, $E_{\text{iso}}$ depends on the cosmological model from

$E_{\text{iso}} = 4\pi d_L^2 S_{\text{bolo}}(1 + z)^{-1}$

where $d_L$ is the luminosity distance. Assuming a flat $\Lambda$CDM model, the $d_L$ can be expressed with Hubble expansion rate

$d_L(\Omega_m, z) = (1 + z) \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1 + z')^3 + 1 - \Omega_m}}$

where $\Omega_m$ is the matter density at present, and $H_0$ is the Hubble constant. Since the Hubble constant is precisely measured, we take $H_0 = 67.8$ km s$^{-1}$ Mpc$^{-1}$ (Planck Collaboration XVI 2013; Planck Collaboration XIII 2015), except when we use the combination data of SNe and GRB to constrain cosmological models.

We list 42 updated GRBs in Table 1. The isotropic energy $E_{\text{iso}}$ is calculated with benchmark parameters with $\Omega_m = 0.308$ for the flat $\Lambda$CDM Universe (Planck Collaboration XVI 2013; Planck Collaboration XIII 2015). During the calculation, we only take the errors propagating from the spectrum parameters, namely observed fluence $S$ and peak energy $E_{\text{obs}}$. The uncertainties from other parameters are attributed into the extrinsic scatter $\sigma_{\text{ext}}$. A&A 585, A68 (2016)
covers the redshift range from 0.0331 to 8.2. We parameterize the $E_{\text{iso}} - E_{p}$ correlation as follows:

$$\log E_{\text{iso}} \text{ erg} = a + b \log E_{p} \text{ keV},$$

where $a$ and $b$ are the intercept and slope. Here $E_{p}$ has been corrected into the cosmological rest frame.

Before constraining cosmological models, we test the possible redshift evolution of the $E_{\text{iso}} - E_{p}$ correlation using the maximum likelihood method. The full data is divided into four redshift bins: [0.0331, 0.958], [0.966, 1.613], [1.619, 2.671], and [2.69, 8.2]. Each bin almost includes the same number of GRBs. The results are shown in Table 2. We give out the best-fit values and $1\sigma$ uncertainties in the coefficients $a$, $b$, and the extrinsic scatter $\sigma_{\text{ext}}$. The $\sigma_{\text{ext}}$ is almost constant in different bins.
Since the correlation. This result is consistent with those of Basilakos & Perivolaropoulos (2008) and Wang et al. (2011b). The full data result are also shown in Figure 1. This result illustrates that the $E_{\text{iso}} - E_p$ correlation fits the data well.

Table 2. $E_{\text{iso}} - E_p$ correlation fitting results of full data and four redshift bins.

<table>
<thead>
<tr>
<th>Redshift range</th>
<th>$a$</th>
<th>$b$</th>
<th>$\sigma_{\text{ext}}$</th>
<th>GRB number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full data</td>
<td>0.24</td>
<td>1.48 ± 0.09</td>
<td>0.34 ± 0.04</td>
<td>151</td>
</tr>
<tr>
<td>[0.0331, 0.958]</td>
<td>0.36</td>
<td>1.58 ± 0.15</td>
<td>0.34 ± 0.07</td>
<td>37</td>
</tr>
<tr>
<td>[0.966, 1.613]</td>
<td>0.61</td>
<td>1.37 ± 0.23</td>
<td>0.40 ± 0.07</td>
<td>38</td>
</tr>
<tr>
<td>[1.619, 2.671]</td>
<td>0.64</td>
<td>1.33 ± 0.24</td>
<td>0.33 ± 0.07</td>
<td>38</td>
</tr>
<tr>
<td>[2.69, 8.1]</td>
<td>0.60</td>
<td>1.34 ± 0.22</td>
<td>0.25 ± 0.07</td>
<td>38</td>
</tr>
</tbody>
</table>

Notes. The best-fit value, 1σ uncertainties, and extrinsic scatter $\sigma_{\text{ext}}$ are given.

Its value is about 0.34, which implies that the extrinsic scatter dominates the error size. The results show no statistically significant evidence for the redshift evolution of the $E_{\text{iso}} - E_p$ correlation. This result is consistent with those of Basilakos & Perivolaropoulos (2008) and Wang et al. (2011b). The full data result are also shown in Figure 1. This result illustrates that the $E_{\text{iso}} - E_p$ correlation fits the data well.

As discussed by D'Agostini (2005), we use the following likelihood to fit the linear relation $y = a + bx$.

$$L(\Omega_m, a, b, \sigma_{\text{ext}}) \propto \prod_i \frac{1}{\sqrt{\sigma_{\text{ext}}^2 + \sigma_x^2 + b^2 \sigma_y^2}} \times \exp \left[ -\frac{(y_i - a - bx_i)^2}{2(\sigma_{\text{ext}}^2 + \sigma_x^2 + b^2 \sigma_y^2)} \right].$$

(6)

Following the description of D’Agostini (2005), the parameter $y$ should not only depend on $x$, but also depend on some hidden variables ($\Omega_m$ here). Thus, the expression of the $E_{\text{iso}} - E_p$ plane should be written as $y = \log \frac{E_{\text{iso}}}{\text{erg}}$ and $x = \log \frac{E_p}{\text{keV}}$. However, Amati et al. (2008) set $y = \log \frac{E_{\text{iso}}}{\text{erg}}$, thus the extrinsic scatter $\sigma_{\text{ext}}$ does not contain the error from the cosmological models.

3.2. Simultaneous fitting

Since the $E_{\text{iso}} - E_p$ correlation does not evolve with redshift, it can be used to constrain parameters directly. We emphasize that there is no circularity problem in the simultaneous fitting method because we do not assume any cosmological model.

In this section, we focus on the constraint on the flat $\Lambda$CDM model. The luminosity distance is expressed as Eq. (4).

Using the likelihood expressed in equation (6), we can constrain the current matter density $\Omega_m$, the extrinsic scatter parameter $\sigma_{\text{ext}}$, and the coefficients of the $E_{\text{iso}} - E_p$ correlation simultaneously. In our calculations, the best-fit values are $a = 49.15 \pm 0.26$, $b = 1.42 \pm 0.12$, $\sigma_{\text{ext}} = 0.34 \pm 0.03$, and $\Omega_m = 0.76$. We show the constraint on $\Omega_m$ in Fig. 2 with a solid line. The 1σ uncertainty is $\Omega_m \in [0.55, 1]$. We also use the reduced $\chi^2$ method to constrain the matter density. This method also includes the effect of extrinsic scatter

$$\chi^2 = \sum_i \frac{(y_i - a - bx_i)^2}{(\sigma_y^2 + b^2 \sigma_x^2 + \sigma_{\text{ext}}^2)}.$$  

(7)

The hidden variables (cosmological parameters) are included in $E_{\text{iso}}$. The extrinsic scatter is used to set the reduced $\chi^2$ to unity, which is also used in SNe Ia cosmology (Suzuki et al. 2012). The value of $\sigma_{\text{ext}}$ is 0.34 when the reduced $\chi^2$ is unity. The best-fit results are $a = 48.96 \pm 0.18$, $b = 1.52 \pm 0.08$, and $\Omega_m = 0.50 \pm 0.12$. The constraint from reduced $\chi^2$ method is roughly consistent with the likelihood method. The $\chi^2 / \chi^2_{\text{min}}$ evolution with $\Omega_m$ are shown in Fig. 2 with a dashed line. If the extrinsic scatter is not considered, the results are $a = 48.50 \pm 0.05$, $b = 1.81 \pm 0.02$, and $\Omega_m = 0.19 \pm 0.05$.

There is a mild tension between the results from the likelihood method and the reduced $\chi^2$ method. The extrinsic scatter is large, which loosely constrains the cosmological parameters.

When we calculate the parameter $E_{\text{iso}}$, a cosmological model and a spectrum model are used, while the uncertainties from the...
are not well established, thus we take these uncertainties into a scatter parameter \( \sigma_{ext} \). This scatter can be calibrated on the parameter \( E_{iso} \). In the future, this scatter can be reduced, since precise observation and data analysis will be performed by the team of Sino-French space-based multiband astronomical variable objects monitor (SVOM; Basa et al. 2008; Götz et al. 2009; Paul et al. 2011).

We also compare our results to the current precise measurements, such as the results from Planck+WMAP (Planck Collaboration XVI 2013), BAO (Beutler et al. 2011; Anderson et al. 2014; Kazin et al. 2014; Ross et al. 2015), and SNe Ia (Conley et al. 2011; Suzuki et al. 2012). We show them in Table 3. The best-fit \( \Omega_m \) by GRBs, using \( \chi^2 \) method, conflicts with the observation of CMB and BAO. For the results from SNe Ia, however, if both statistical and systematic errors are included, the constraints on cosmological parameters are loose (Kowalski et al. 2008; Amanullah et al. 2010; Suzuki et al. 2012). In this case, the best-fit \( \Omega_m \) with GRBs, using \( \chi^2 \) method, is consistent with those from SNe Ia at 1\( \sigma \) confidence level; see Fig. 12 of Kowalski et al. (2008), Fig. 10 of Amanullah et al. (2010) and Fig. 5 of Suzuki et al. (2012).

### 4. Calibration of the \( E_{iso} – E_p \) correlation

#### 4.1. Standardizing GRBs with SNe Ia

Just as using Cepheid variables to standardize SNe Ia, the GRBs can be calibrated with SNe Ia. We can use the calibration method to standardize the GRBs with the \( E_{iso} – E_p \) correlation. With this approach, the parameters \( a \) and \( b \) are obtained and only cosmological parameters remain free. We use the latest Union 2.1 data from Suzuki et al. (2012). This method is also cosmological model independent (Liang et al. 2008; Kodama et al. 2008; Wei 2010). The extrinsic scatter is also taken into account when calculating the error propagation of \( E_{iso} \). The full GRB data is separated into two groups. The dividing line is the highest redshift in SNe Ia Union 2.1 data, namely, \( z = 1.414 \). The low-redshift group \((z < 1.414)\) includes 61 GRBs and the high-redshift group \((z > 1.414)\) contains 90 GRBs.

Firstly, the linear interpolation method is used to calculate the distance moduli \( \mu \) of 61 low-redshift GRBs. Liang et al. (2008) have shown that there are no differences on the final result between the linear interpolation and the cubic interpolation. The 1\( \sigma \) error of the distance moduli \( \sigma_{\mu,i} \) can be obtained as follows:

\[
\sigma_{\mu,i} = \frac{z_{i+1} - z_i}{z_{i+1} - z_i} \sigma_{\mu,i+1} + \left( \frac{z - z_i}{z_{i+1} - z_i} \right) \sigma_{\mu,i+1}, \tag{8}
\]

where \( z_{i+1} \) and \( z_i \) are the redshift of the two nearest SNe Ia and \( \sigma_{\mu,i+1} \) and \( \sigma_{\mu,i} \) are the errors of these two SNe Ia. The redshift of interpolated GRB lies between \( z_i \) and \( z_{i+1} \).

After the distance moduli of 61 low-redshift GRBs are obtained, the luminosity distance can be derived from

\[
\mu = 5 \log \frac{d_L}{\text{Mpc}} + 25. \tag{9}
\]

Then the isotropic-equivalent energy \( E_{iso} \) can be calculated from Eq. (3). Following Schaefer (2007) and Liang et al. (2008), we use the bisector of the two ordinary least squares method (Isobe et al. 1990) to fit the \( E_{iso} – E_p \) correlation. The best-fit values are \( a = 48.46 \pm 0.033 \) and \( b = 1.766 \pm 0.007 \). The result is shown in Fig. 3. The errors of distance moduli are not taken into consideration because the extrinsic scatter \( \sigma_{ext} \) dominates the error size in the regression analysis (Schaefer 2007). Thus, we take \( \sigma_{ext} \) directly into account during the calculations of the uncertainties of high-redshift GRBs \( (\sigma_{log E_{iso}}) \). From the previous section, the value of \( \sigma_{ext} \) is nearly constant, so we typically set \( \sigma_{ext} = 0.34 \).

We have shown that the \( E_{iso} – E_p \) correlation does not evolve with redshift in the previous section. Thus, the calibrated \( E_{iso} – E_p \) correlation can be extrapolated to the high-redshift sample, namely, \( z > 1.414 \) group. Using Eq. (5), we can derive \( E_{iso} \) of...
### Table 4. 90 calibrated GRBs with redshift, bolometric fluence, peak energy in cosmological rest frame, and distance moduli.

<table>
<thead>
<tr>
<th>GRB</th>
<th>Redshift $z$</th>
<th>$S_{bolo}$ (10^{-5} \text{ erg cm}^{-2})$</th>
<th>$E_p$ (keV)</th>
<th>$\mu_{cal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>005318</td>
<td>0.42 ± 0.03</td>
<td>115 ± 25</td>
<td>44.48 ± 1.28</td>
<td></td>
</tr>
<tr>
<td>100814</td>
<td>1.39 ± 0.23</td>
<td>312.32 ± 48.80</td>
<td>45.09 ± 1.11</td>
<td></td>
</tr>
<tr>
<td>110213</td>
<td>1.55 ± 0.23</td>
<td>223.86 ± 70.11</td>
<td>44.34 ± 1.63</td>
<td></td>
</tr>
<tr>
<td>010222</td>
<td>1.46 ± 1.5</td>
<td>766 ± 30</td>
<td>44.28 ± 0.88</td>
<td></td>
</tr>
<tr>
<td>120724</td>
<td>0.15 ± 0.02</td>
<td>68.45 ± 18.60</td>
<td>46.62 ± 1.48</td>
<td></td>
</tr>
<tr>
<td>060418</td>
<td>0.3 ± 0.5</td>
<td>572 ± 143</td>
<td>45.73 ± 1.41</td>
<td></td>
</tr>
<tr>
<td>030328</td>
<td>0.52</td>
<td>328 ± 55</td>
<td>45.56 ± 1.13</td>
<td></td>
</tr>
<tr>
<td>070125</td>
<td>1.33 ± 1.3</td>
<td>934 ± 148</td>
<td>44.79 ± 1.11</td>
<td></td>
</tr>
<tr>
<td>090102</td>
<td>1.54 ± 0.83</td>
<td>1149 ± 166</td>
<td>46.64 ± 1.08</td>
<td></td>
</tr>
<tr>
<td>040912</td>
<td>0.21 ± 0.06</td>
<td>44 ± 33</td>
<td>43.44 ± 3.43</td>
<td></td>
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<tr>
<td>990123</td>
<td>1.6</td>
<td>35.8 ± 5.8</td>
<td>44.91 ± 1.8</td>
<td></td>
</tr>
<tr>
<td>010703</td>
<td>1.6</td>
<td>35.2 ± 0.59</td>
<td>47.34 ± 1.05</td>
<td></td>
</tr>
<tr>
<td>090418</td>
<td>2.35 ± 0.59</td>
<td>1567 ± 384</td>
<td>46.79 ± 1.40</td>
<td></td>
</tr>
<tr>
<td>110503</td>
<td>2.76 ± 0.21</td>
<td>5725.25 ± 50.95</td>
<td>45.58 ± 0.94</td>
<td></td>
</tr>
<tr>
<td>990510</td>
<td>1.61</td>
<td>2.6 ± 0.04</td>
<td>423 ± 42</td>
<td></td>
</tr>
<tr>
<td>080605</td>
<td>1.6398</td>
<td>0.34 ± 0.28</td>
<td>460 ± 35</td>
<td></td>
</tr>
<tr>
<td>08110A</td>
<td>1.86</td>
<td>4.75 ± 0.16</td>
<td>46.49 ± 1.09</td>
<td></td>
</tr>
<tr>
<td>090120</td>
<td>0.11 ± 0.034</td>
<td>13.99 ± 19.41</td>
<td>47.75 ± 1.31</td>
<td></td>
</tr>
<tr>
<td>100906</td>
<td>1.727</td>
<td>3.56 ± 0.55</td>
<td>437.23 ± 244.40</td>
<td>4.6 ± 2.91</td>
</tr>
<tr>
<td>120119</td>
<td>1.728</td>
<td>4.62 ± 0.59</td>
<td>451.37 ± 45.64</td>
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### Notes
- The $1\sigma$ uncertainties are also given.

Compared with Wei (2010), the error bars of distance moduli of our results are smaller. The main reason is that we use a larger sample, which leads to a smaller $\sigma_{\text{int}}$. The extrinsic scatter parameter has been taken into consideration during the calculation of the error size of $E_{\text{iso}}$.

#### 4.2. Constraining cosmological models

These GRBs carry the information of high-redshift Universe, and can be taken as good complements to the Union 2.1 data set. We test if these high-redshift GRBs alone can constrain the $\Lambda$CDM model. Using the distance modulus in Eq. (4) and Eq. (9), the $\chi^2$ is

$$
\chi^2_{GRB}(\Omega_m) = \sum_{i=1}^{80} \left[ \mu_{\text{cal}}(z_i) - \mu(z_i) \right]^2 / \sigma_{\text{int}}^2.
$$

**References**
where \( \mu_{\text{cal}} \) is the calibrated GRB distance modulus listed in Table 4. The best-fit result is \( \Omega_m = 0.23_{-0.04}^{+0.06} \) with 1\( \sigma \) uncertainty. The \( \chi^2 \) evolution with \( \Omega_m \) is shown in Fig. 2. This result is consistent with the constraints from SNe Ia (Conley et al. 2011; Suzuki et al. 2012), CMB (Planck Collaboration XVI 2013; Planck Collaboration XIII 2015), and BAO (Beutler et al. 2011; Anderson et al. 2014; Kazin et al. 2014; Ross et al. 2015) at 1\( \sigma \) confidence level, as shown in Table 3.

Since this GRB sample can constrain cosmological parameters successfully, we also combine the calibrated GRB data with SNe Ia from Union 2.1 sample to constrain cosmological models. For the flat \( \Lambda \)CDM, we obtain \( \Omega_m = 0.271 \pm 0.019 \) and \( h = 0.701 \pm 0.002 \), where \( h \) is the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). This is very consistent with the Union 2.1 SNe Ia data. For the non-flat \( \Lambda \)CDM, the luminosity distance is different and can be expressed as follows:

\[
\begin{align*}
    d_L &= \begin{cases} 
    cH_0^{-1}(1+z)(-\Omega_k)^{-1/2} \sin[(-\Omega_k)^{1/2}z], & \Omega_k < 0, \\
    cH_0^{-1}(1+z)I, & \Omega_k = 0, \\
    cH_0^{-1}(1+z)\Omega_k^{-1/2} \sin[\Omega_k^{1/2}z], & \Omega_k > 0,
    \end{cases}
\end{align*}
\]

where

\[ \Omega_k = 1 - \Omega_m - \Omega_\Lambda, \]

and

\[
I = \int_0^\infty \frac{dz}{\sqrt{(1+z)^2 \Omega_m + \Omega_\Lambda + (1+z)^2 \Omega_k}}
\]

The \( \chi^2 \) of SNe Ia is constructed as follows:

\[
\chi^2_{\text{SNe}}(h, \Omega_m, \Omega_\Lambda) = \sum_{i=1}^{880} \frac{[\mu_{\text{obs}}(z_i) - \mu(z_i)]^2}{\sigma^2(z_i)}.
\]

Then the total \( \chi^2 \) is

\[
\chi^2_{\text{total}}(h, \Omega_m, \Omega_\Lambda) = \chi^2_{\text{SNe}}(h, \Omega_m, \Omega_\Lambda) + \chi^2_{\text{GRB}}(h, \Omega_m, \Omega_\Lambda).
\]

The best-fit values with 1\( \sigma \) uncertainties are \( \Omega_m = 0.225 \pm 0.044 \), \( \Omega_\Lambda = 0.640 \pm 0.082 \), and \( h = 0.698 \pm 0.004 \) for the combined sample (SNe+GRB). For the GRB sample, we obtain \( \Omega_m = 0.18 \pm 0.11 \) and \( \Omega_\Lambda = 0.46 \pm 0.51 \), which is consistent with the SNe Ia results at 1\( \sigma \) confidence level. The combined sample can help to constrain cosmological parameters much tighter because not only is the sample enlarged, but also the redshift covers a much wider. The flatness of the Universe depends on the curvature parameter, that is to say, \( \Omega_k = 1 - \Omega_m - \Omega_\Lambda \).

In Fig. 4, we use three samples, GRB, SNe, and combination of GRB+SNe to constrain the cosmological model. Both results prefer a flat Universe at the 1\( \sigma \) confidence level. The constraint from the GRB is almost perpendicular to that from SNe Ia in the \( \Omega_m - \Omega_\Lambda \) plane. Thus GRBs can significantly help to constrain \( \Omega_m \) because, in this redshift domain, the dark matter dominates the evolution of the Universe. We also show constraints on \( \Omega_m - h \) in Fig. 5, and \( \Omega_\Lambda - h \) in Fig. 6.

5. Discussions and summary

In this paper, we update 42 long GRBs for the \( E_{\text{iso}} - E_p \) correlation and combine them with 109 long GRBs from Amati et al. (2008) and Amati et al. (2009). This sample contains GRBs detected by different detectors with different sensitivities. Thus, the sample might be biased, but this bias should only have a weak effect on our results. We also use the complete sample to perform our analysis. We use the same criteria as Salvaterra et al. (2012) and Pescalli et al. (2015) to collect GRBs. The results are \( a = 49.45 \pm 0.61, b = 1.24 \pm 0.22 \) and \( \sigma_{\text{ext}} = 0.38 \pm 0.06 \), while no constraint on \( \Omega_m \) is found. These results are in tension with that of our updated full sample with a larger extrinsic scatter. No statistical evidence for the redshift evolution of the \( E_{\text{iso}} - E_p \) is found in the full sample.

For cosmological purposes, we fit the \( E_{\text{iso}} - E_p \) plane and the cosmological parameters simultaneously. Using a likelihood
function we obtain $a = 49.15 \pm 0.26$, $b = 1.42 \pm 0.11$, $\sigma_{\text{ext}} = 0.34 \pm 0.03$, and $\Omega_m \in [0.55, 1]$. Using the reduced $\chi^2$, we obtain $a = 48.96 \pm 0.18$, $b = 1.52 \pm 0.08$, and $\Omega_m = 0.50 \pm 0.12$. The results from these two fitting methods are in mild tension. The main reason is that the extrinsic scatter of this correlation is too large. Thus, Ghirlanda (2009) finds no constraint with a smaller sample using the likelihood method. We also use a calibrating method. Based on the SNe Ia data, we obtain 90 calibrated GRBs. From these calibrated GRBs, we acquire $\Omega_m = 0.23^{+0.06}_{-0.04}$ for flat $\Lambda$CDM and for the non-flat $\Lambda$CDM, we obtain $\Omega_m = 0.18 \pm 0.11$ and $\Omega_m = 0.46 \pm 0.51$. We also combine the GRB sample with SNe Ia Union 2.1 data and obtain $\Omega_m = 0.271 \pm 0.019$ and $h = 0.701 \pm 0.002$ for the flat $\Lambda$CDM. For the non-flat $\Lambda$CDM, the results are $\Omega_m = 0.225 \pm 0.044$, $\Omega_m = 0.640 \pm 0.082$, and $h = 0.698 \pm 0.004$. We list our results in Table 3, and compare them with the results from other current measurements. The results from GRBs are consistent with results from SNe Ia in 1\sigma confidence level (Conley et al. 2011; Suzuki et al. 2012), while they conflict with CMB (Planck Collaboration XVI 2013; Planck Collaboration XIII 2015) and BAO (Beutler et al. 2011; Anderson et al. 2014; Kazin et al. 2014; Ross et al. 2015). We also found that the GRBs can help to constrain dark matter better. The constraint from GRB are almost perpendicular to that from SNe Ia in the $\Omega_m$–$\Lambda$ plane. The main reason might be that at high redshift, the dark matter dominates the Universe.

The extrinsic scatter is taken into account in both the simultaneous fitting method and the calibrating method. Our results shows that tighter constraints on cosmological model can be obtained with the calibrating method. For the simultaneous fitting method, the reduced $\chi^2$ method gives a more stringent constraint on cosmological parameters than the likelihood method, but the constraint is still loose because of the large extrinsic scatter. This scatter is introduced by both cosmological models and the GRB spectrum parameters, such as $E_p$, fluence, and photon index. The spectrum parameters can be precisely measured by SVOM (Basa et al. 2008; Götz et al. 2009; Paul et al. 2011), which can reduce the extrinsic scatter. The GRBs from SVOM would help shed light on the properties of early Universe.

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