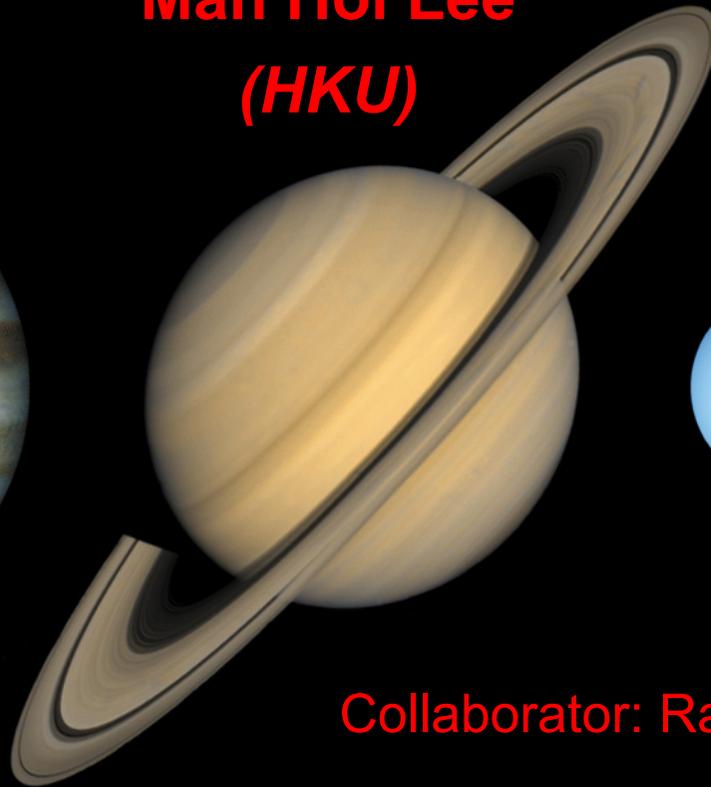
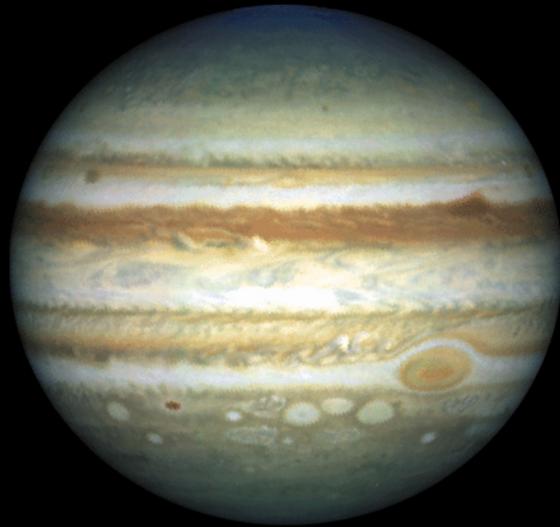


# Did our Solar System Lose a Giant Planet?

Man Hoi Lee  
(HKU)



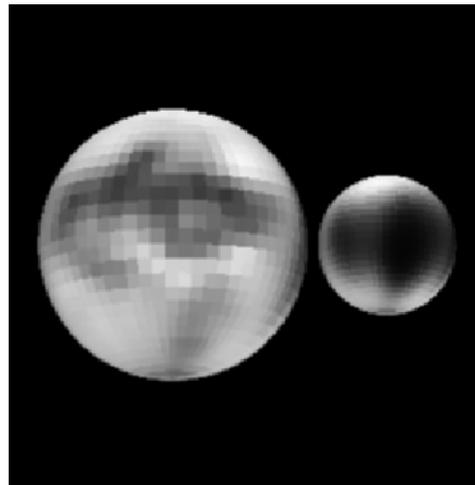
Collaborator: Ramon Brasser (ASIAA)

# Outline

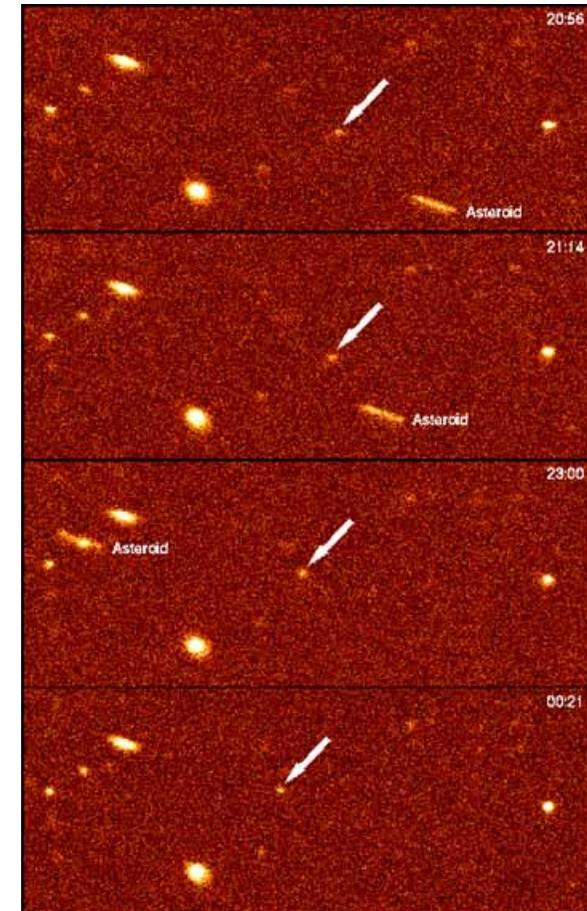
- Evidence for migration of giant planets in our Solar System.
- Migration models.
- Orbital and obliquity (spin axis tilt) constraints: what's the connection between planet migration and obliquities?
- Simulation results.

# Kuiper Belt Objects

- Large population of small bodies orbiting the Sun beyond Neptune's orbit.
- Pluto discovered in 1930.
- Many more discovered since 1992.
- Debris ([planetesimals](#)) left over from formation of Solar System.

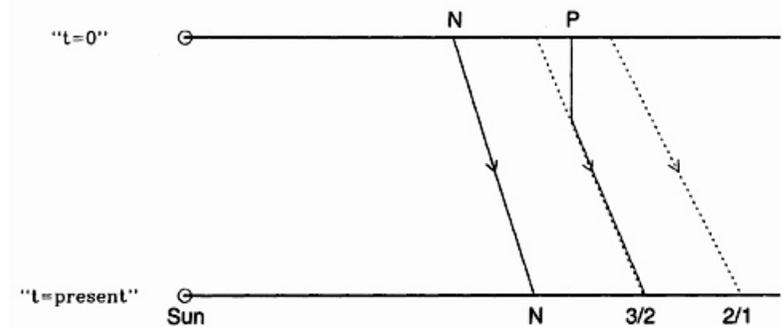
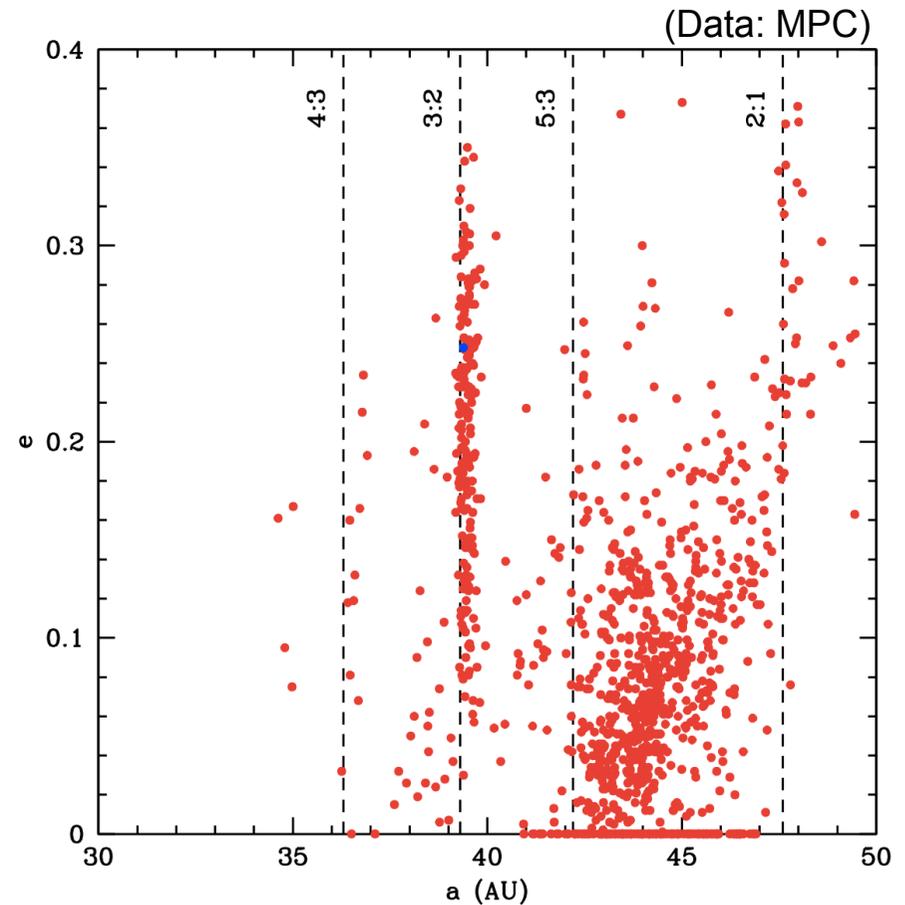


Pluto and its satellite Charon



KBO 1992 QB1

- Many Kuiper belt objects in orbital **mean-motion resonance** with Neptune: Orbital period is a simple fraction of Neptune's.
- E.g., Pluto in 3:2.
- Also, 4:3, 3:2, 5:3, 2:1, etc.
- Evidence for **outward** migration of Neptune driven by planetesimals.
- To pump up Plutino orbital eccentricities to  $e \sim 0.3$ , Neptune's orbit must have migrated by  $\Delta a \sim 7$  AU (Malhotra 1995; Malhotra & Hahn 1999).



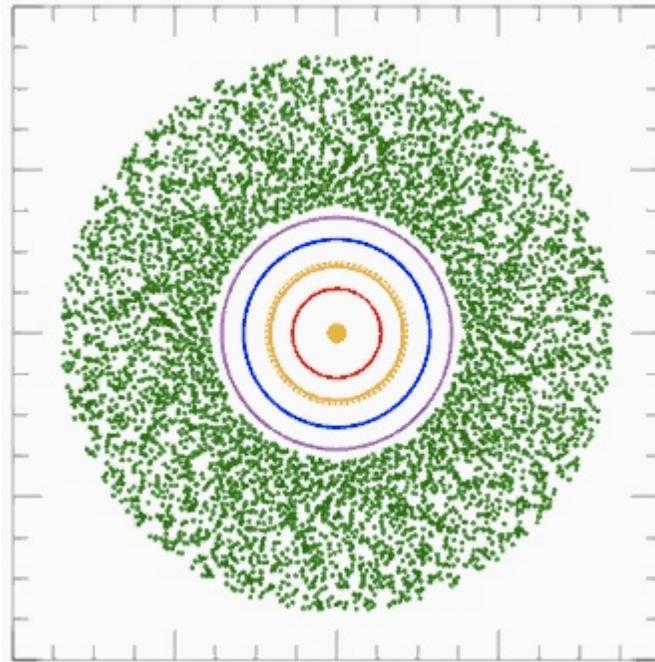
(Malhotra 1995)

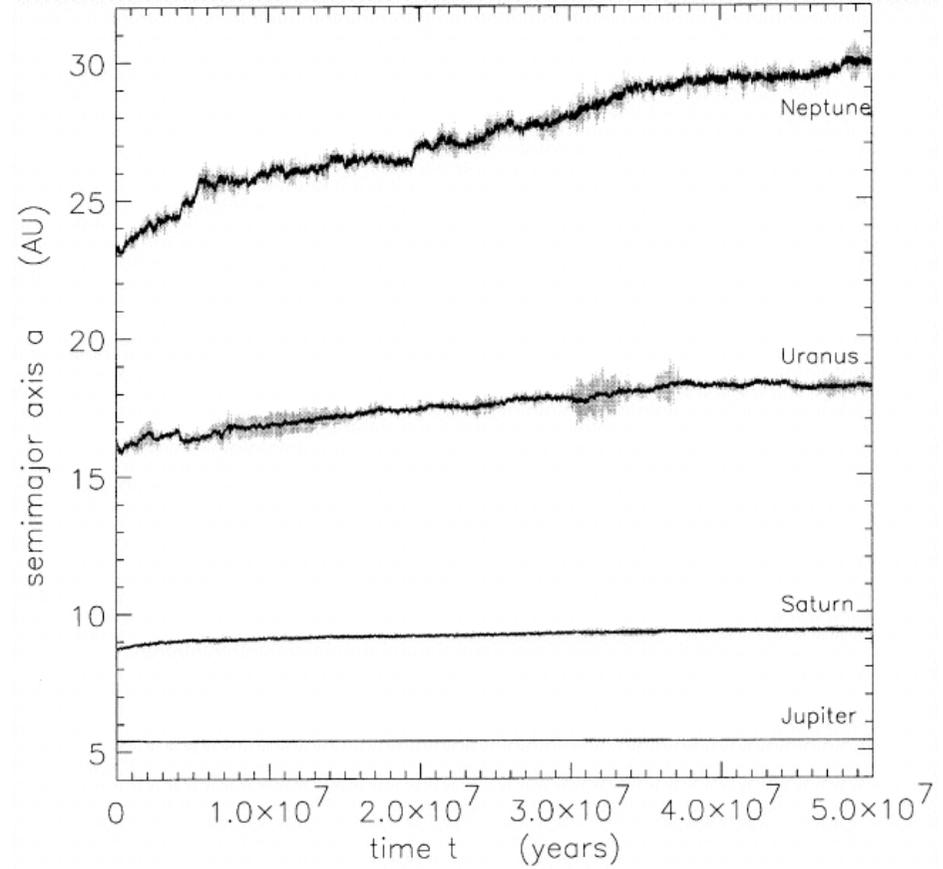
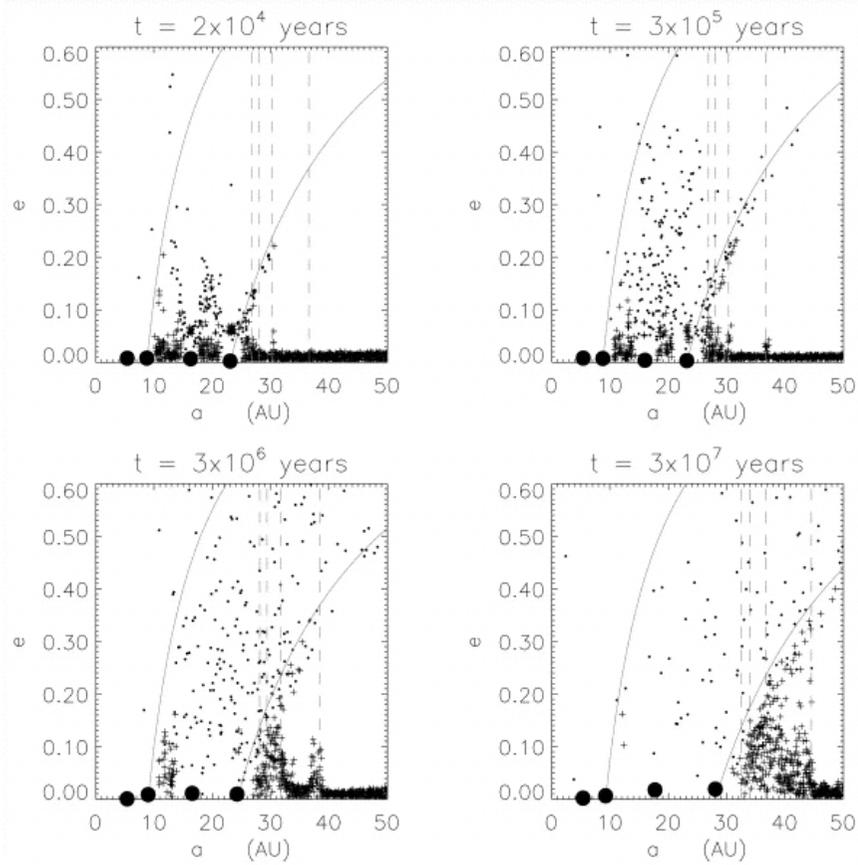
# Giant Planet Migration Models

- Smooth Migration Model
- Classic Nice Model
- Resonant Nice Model
- Resonant 5-Planet Model

# Model I: Smooth Migration

- Even before Kuiper Belt was discovered, Fernandez & Ip (1984) asked
  - what happened to the orbits of the giant planets when they interacted gravitationally with a disk of leftover planetesimals?





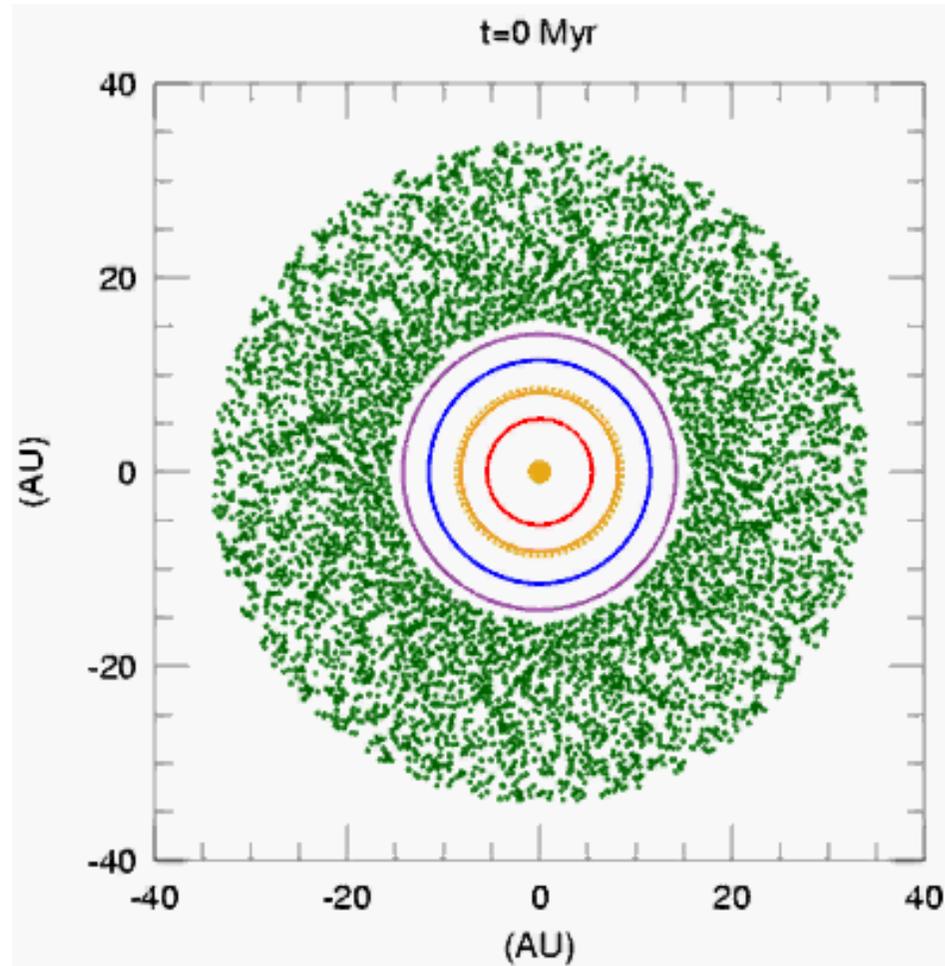
(Hahn & Malhotra 1999)

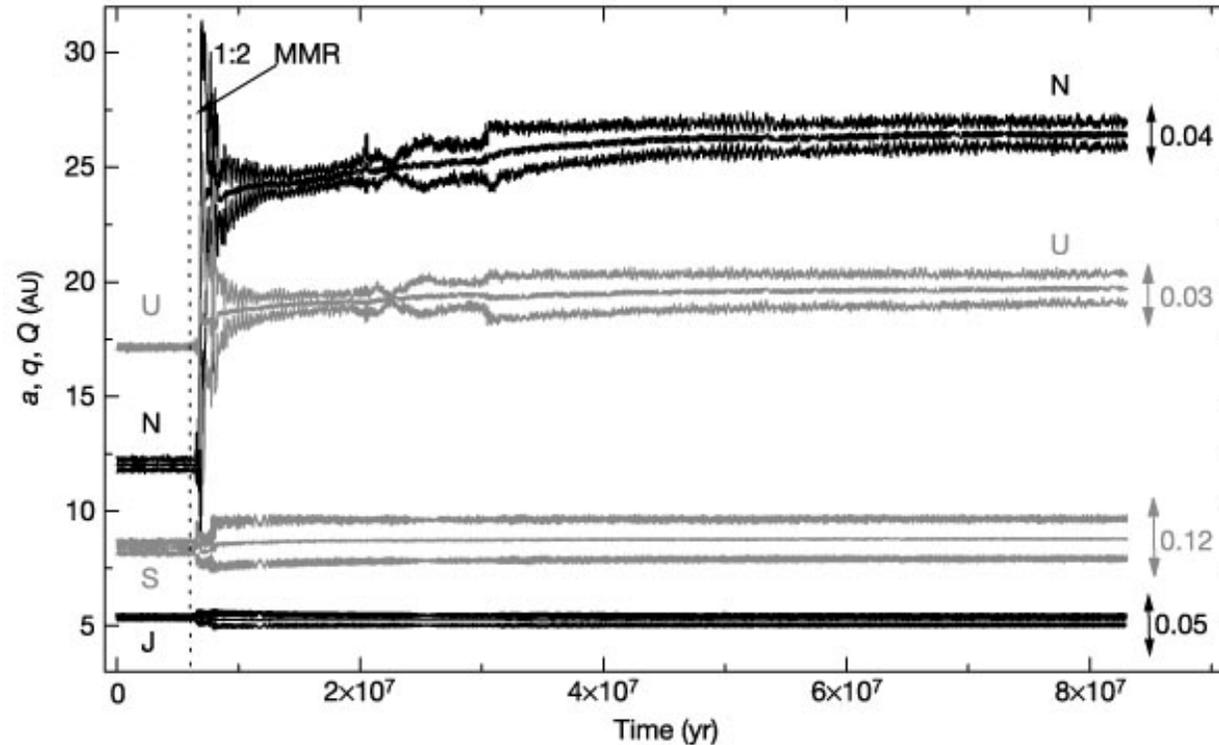
- Saturn, Uranus, and Neptune evolve radially outward as they scatter planetesimals, while Jupiter's orbit shrinks as it ejects planetesimals.
- Outer Solar System initially more compact (but Saturn outside 2:1 mean-motion resonance with Jupiter).
- Disk mass  $\sim 50 M_{\oplus}$  is required to expand Neptune's orbit by  $\Delta a \sim 7 \text{ AU}$  (Malhotra 1995; Malhotra & Hahn 1999).

## Model II: Classic Nice

Tsiganis et al. (2005)

- Saturn closer than 2:1 mean-motion resonance with Jupiter.
- Ice giants within  $\sim 18$  AU.

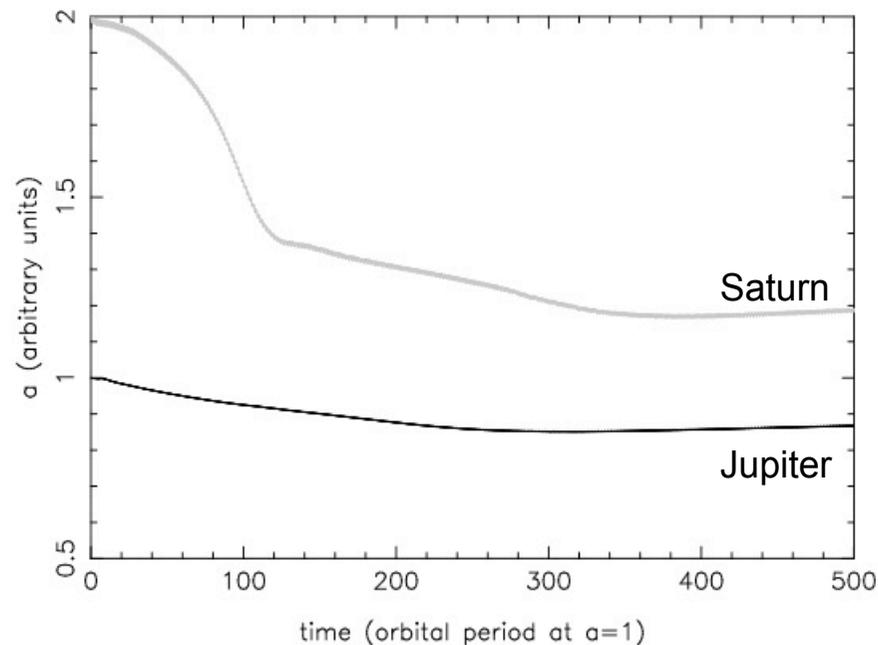




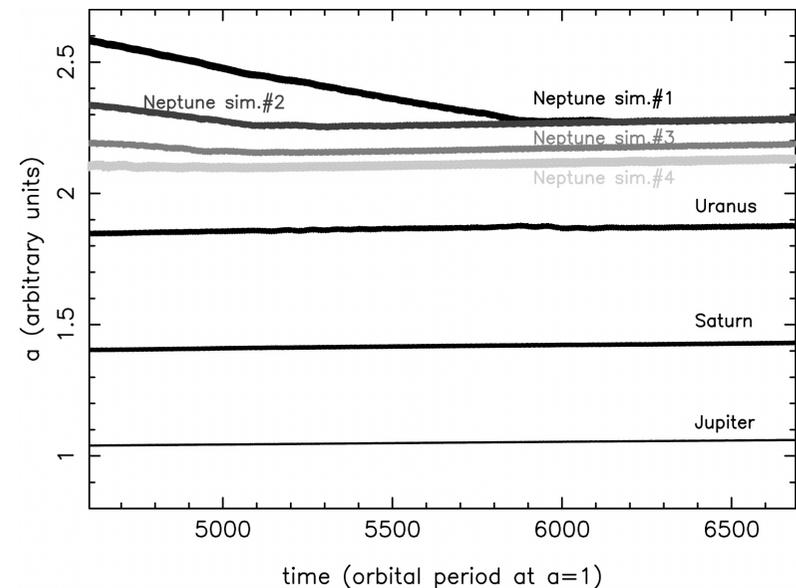
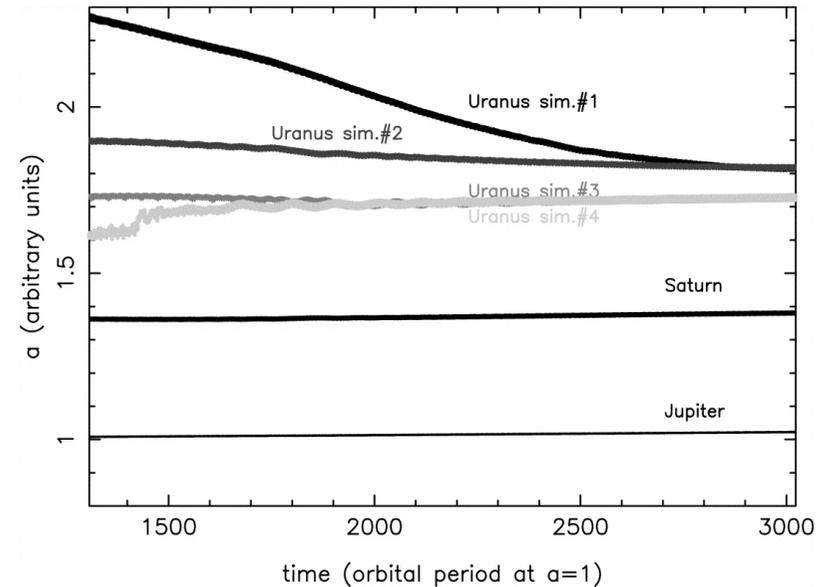
- Jupiter and Saturn crossed 2:1 orbital resonance by divergent migration.
- Close encounters among giant planets scattered Uranus and Neptune out.
- Eccentricities and inclinations damped to present values by interactions with planetesimals.

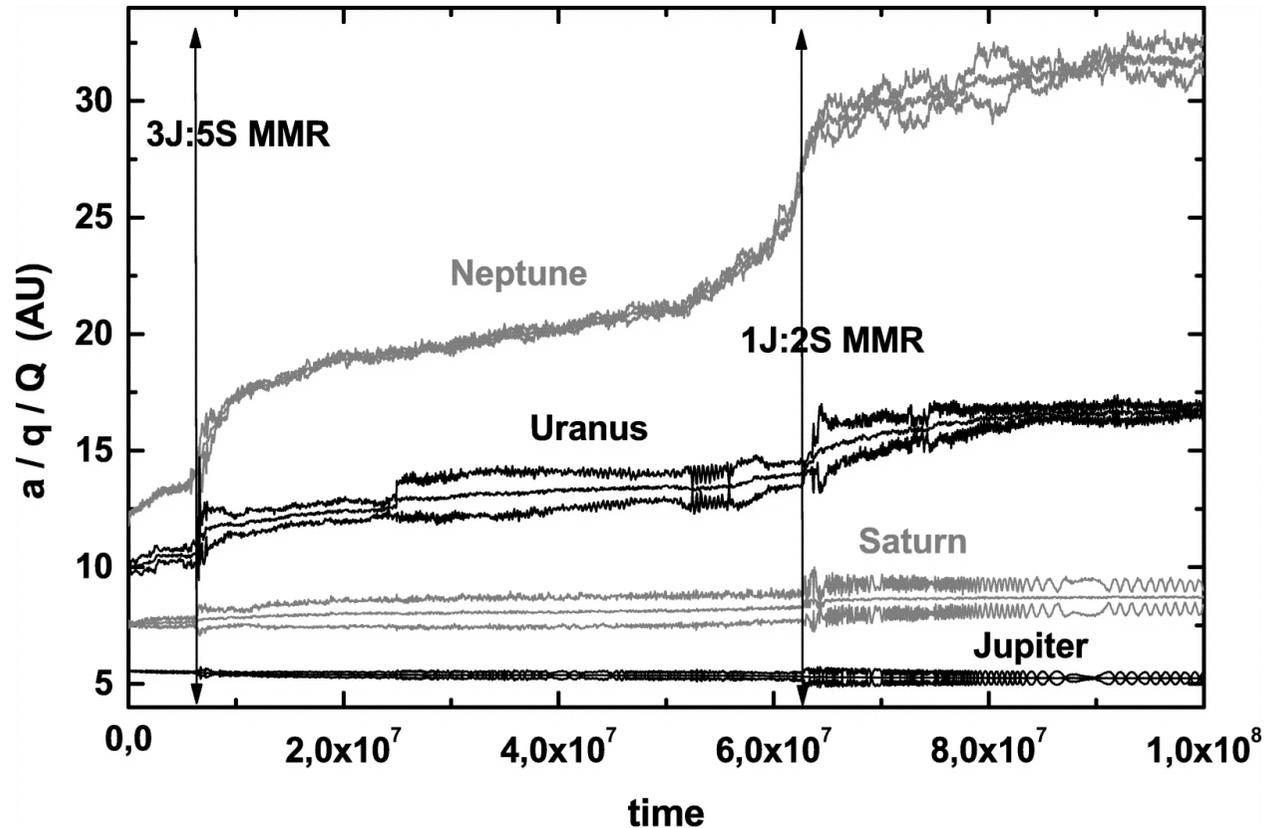
## Model III: Resonant Nice

- Before the protoplanetary gas disk dispersed
  - Interactions with the gas disk can cause inward migration of Jupiter and Saturn.
  - Capture of Jupiter and Saturn into 3:2 mean-motion resonance can stop migration (Masset & Snellgrove 2001; Morbidelli & Crida 2007).



- Morbidelli et al. (2007) proposed that earlier migration in gas disk put all four giant planets in mean-motion resonances:
  - Jupiter and Saturn in 3:2.
  - Saturn and inner ice giant in 3:2.
  - Ice giants in 4:3.

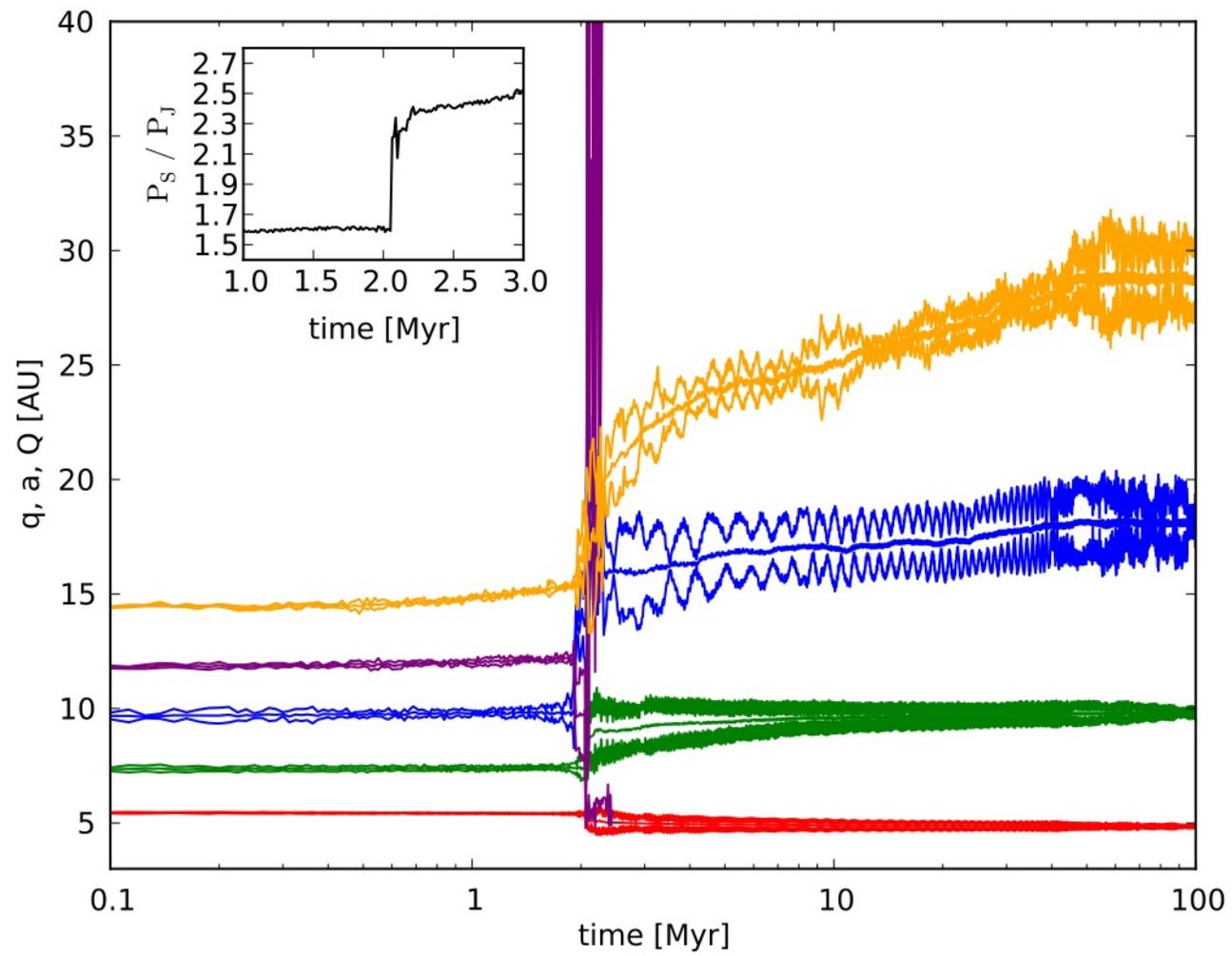




- Subsequent divergent migration driven by planetesimals.
- *N*-body simulations with planetesimals show that
  - Probability that all four giant planets survive is small.
  - It is very difficult to match constraints on orbital history.

## Model IV: Resonant 5-Planet

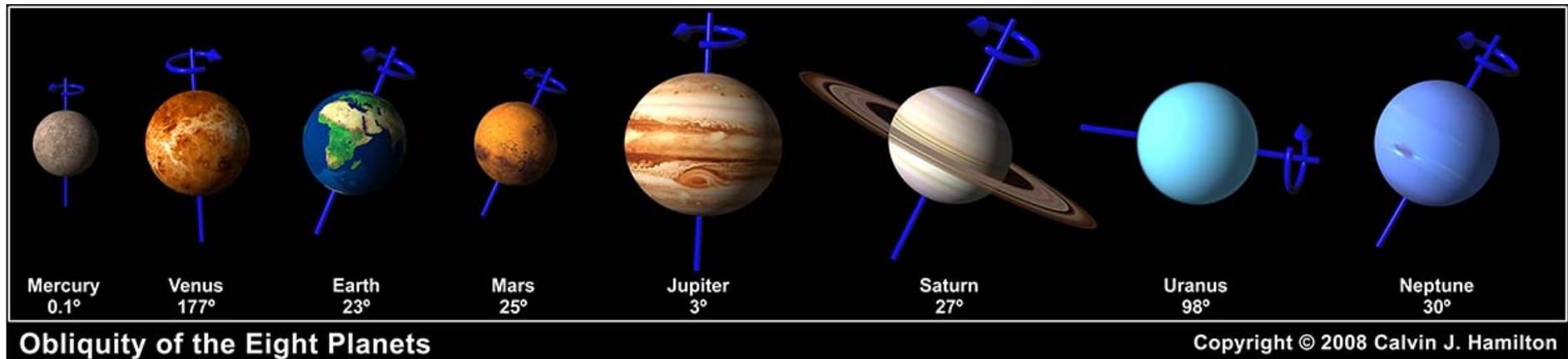
- Since a giant planet is often ejected in the resonant Nice model, what if the Solar System started with an extra ice giant, which was ejected during the encounter phase? (Nesvorný 2011; Batygin et al. 2012)
- We consider two initial resonance configurations:
  - **Compact**: J and S in 3:2, S and  $I_1$  in 3:2,  $I_1$  and  $I_2$  in 4:3,  $I_2$  and  $I_3$  in 4:3.
  - **Loose**: 3:2, 3:2, 2:1, 3:2.



# Orbital Constraints

- Four giant planets (Jupiter, Saturn and two ice giants) left at the end of the evolution.
- Planets are within 10% of their current semimajor axes, with  $P_S/P_J > 2.35$ .
- $P_S/P_J$  must jump from  $\sim 2.1$  to  $\sim 2.3$  in a short time to avoid making terrestrial planet orbits too eccentric and secular resonance sweeping through asteroid belt.
- $\Delta\varpi_{JS} = \varpi_J - \varpi_S$  circulates.

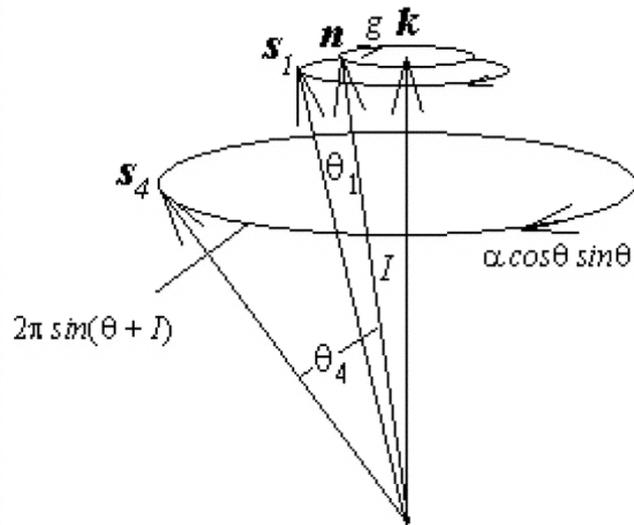
# Obliquity Constraints



- Obliquity  $\epsilon$  = Angle between spin axis and orbit normal

- Similarity between Saturn's spin-axis precession rate ( $\alpha \cos \varepsilon$ ) and nodal regression rate of Neptune's orbit (eigenfrequency  $-s_8$ ) and between  $27^\circ$  obliquity and Cassini state 2 of **secular spin-orbit resonance** (Ward & Hamilton 2004; Hamilton & Ward 2004):
  - Secular resonant interaction responsible for tilting Saturn.
  - Capture into Cassini state whose obliquity increases with decreasing  $|s_8|$ .
- Jupiter's obliquity is only  $3^\circ$ , but its spin precession rate is close to nodal regression rate of Uranus's orbit (eigenfrequency  $-s_7$ ) (Ward & Canup 2006).

Cassini states 1 & 4



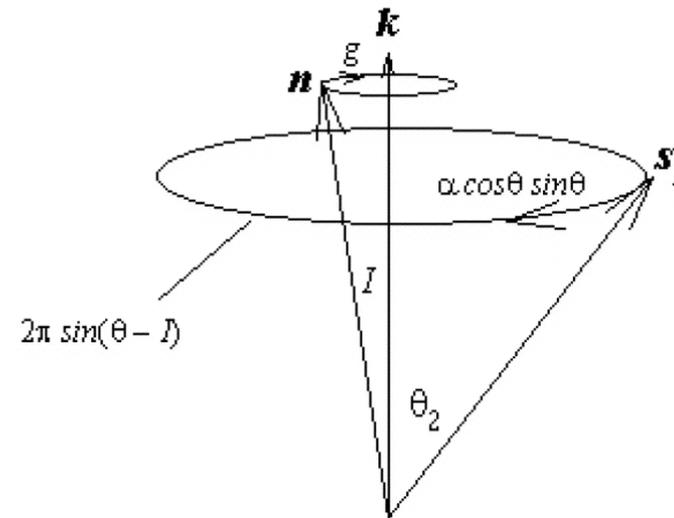
$$P = \frac{2\pi \sin(\theta + I)}{\alpha \cos\theta \sin\theta}$$

$$= 2\pi/g =$$

$$P = \frac{2\pi \sin(\theta - I)}{\alpha \cos\theta \sin\theta}$$

(Ward & Hamilton 2004)

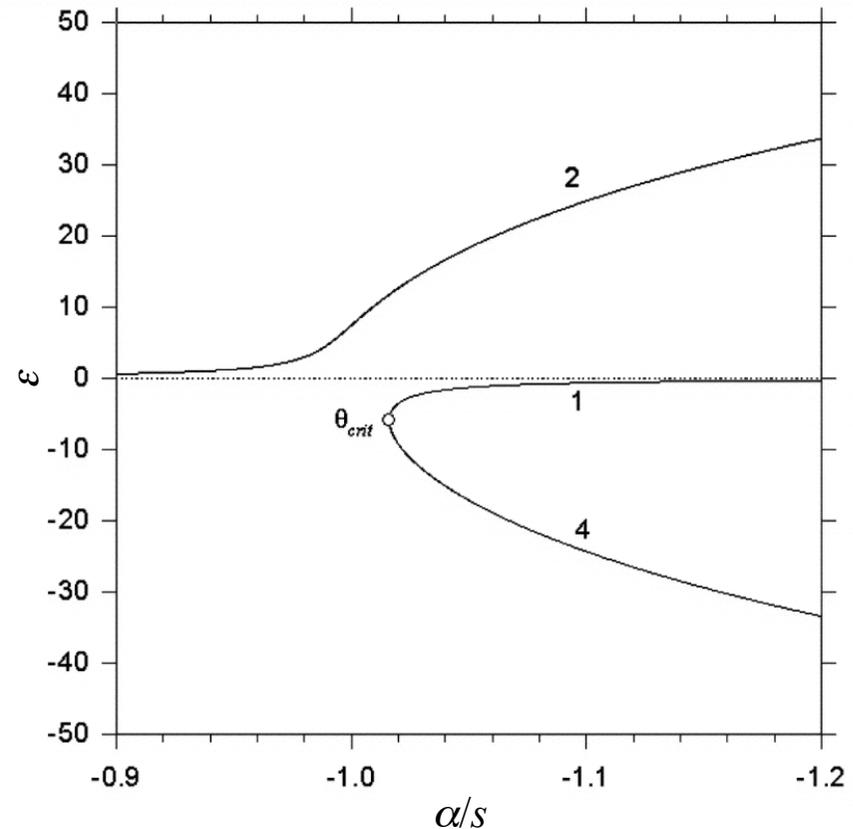
Cassini state 2

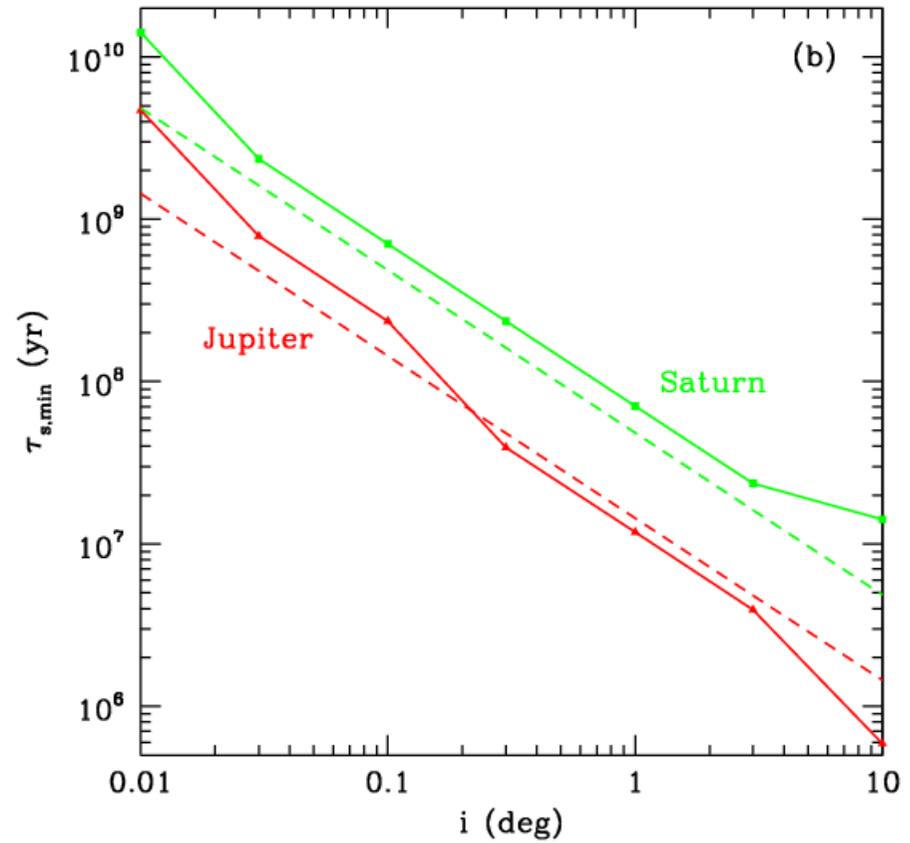
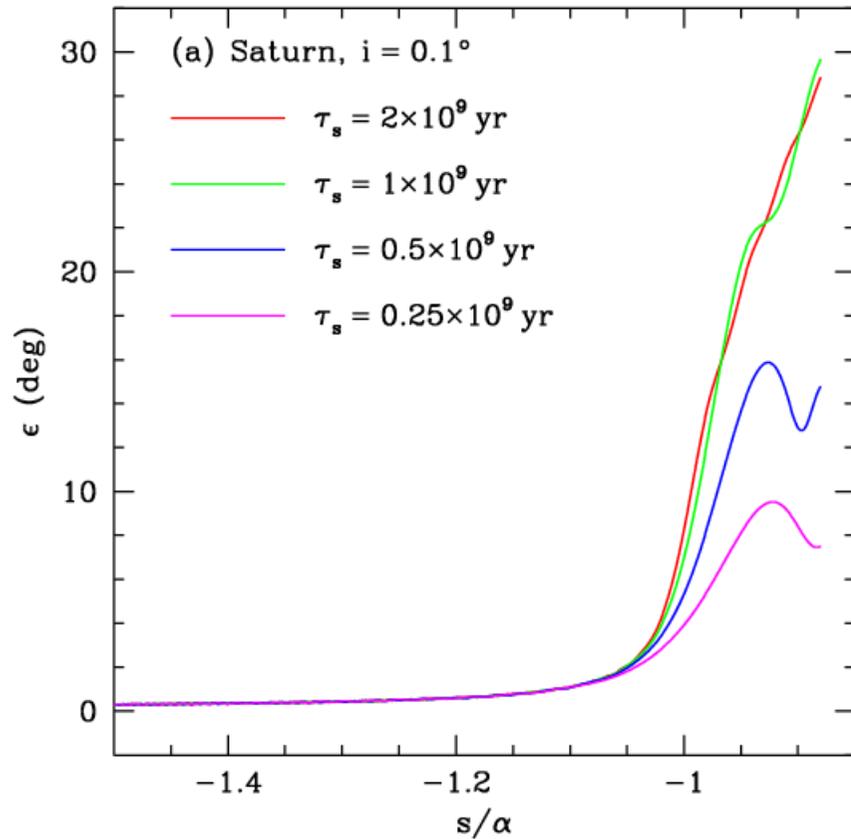


- $\mathbf{k}$  = normal to invariable plane
- $\mathbf{n}$  = orbit normal, precessing about  $\mathbf{k}$  at rate  $s$
- $\mathbf{s}$  = unit vector of spin direction
- **Cassini states**:  $\mathbf{s}$  coplanar with  $\mathbf{n}$  and  $\mathbf{k}$ , and  $\mathbf{s}$  and  $\mathbf{n}$  co-precess at the same rate  $s$ .

(Ward & Hamilton 2004)

- Resonance capture:
  - If spin axis starts near Cassini state 2 with  $|\alpha/s| \ll 1$  and  $|\alpha/s|$  increases, obliquity can become large.
  - Decrease in  $|s_8|$  due to depletion of planetesimals and/or outward migration of Neptune.
  - Timescale for change in  $s_8$  must be longer than libration period to stay in resonance.





- For Jupiter and Saturn on inclined orbit precessing at frequency  $s$  that changes on timescale  $\tau_s$ , we find

$$\tau_{s,min} \sim 1/i.$$

# Numerical Methods

## Secular Simulations

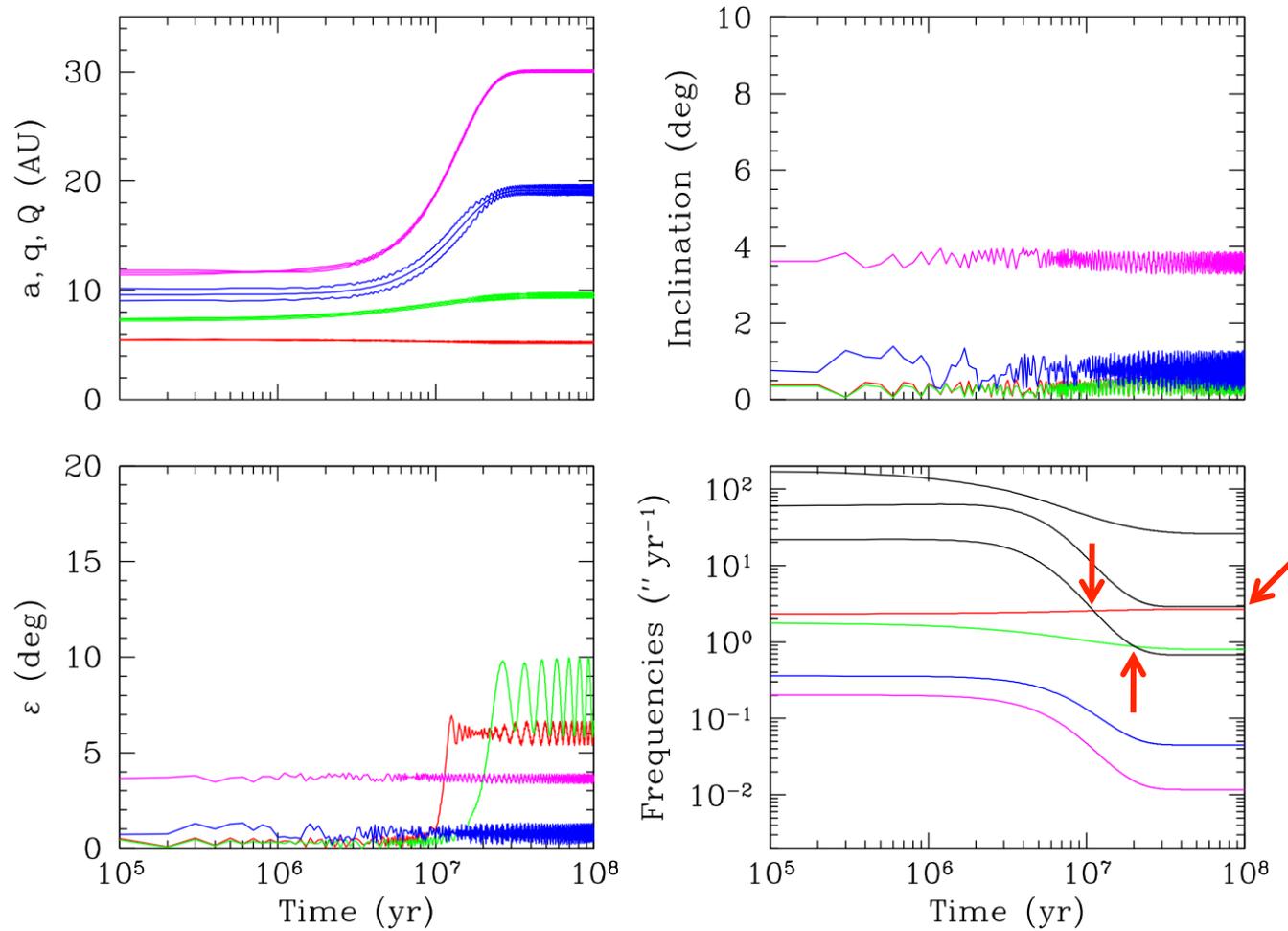
- Imposed migration of planets on timescale  $\tau$ .
- Planets interact according to secular Laplace-Lagrange equations.
- Spin evolution due to torques from the Sun and mutual planetary interactions.

## ***N*-body Simulations**

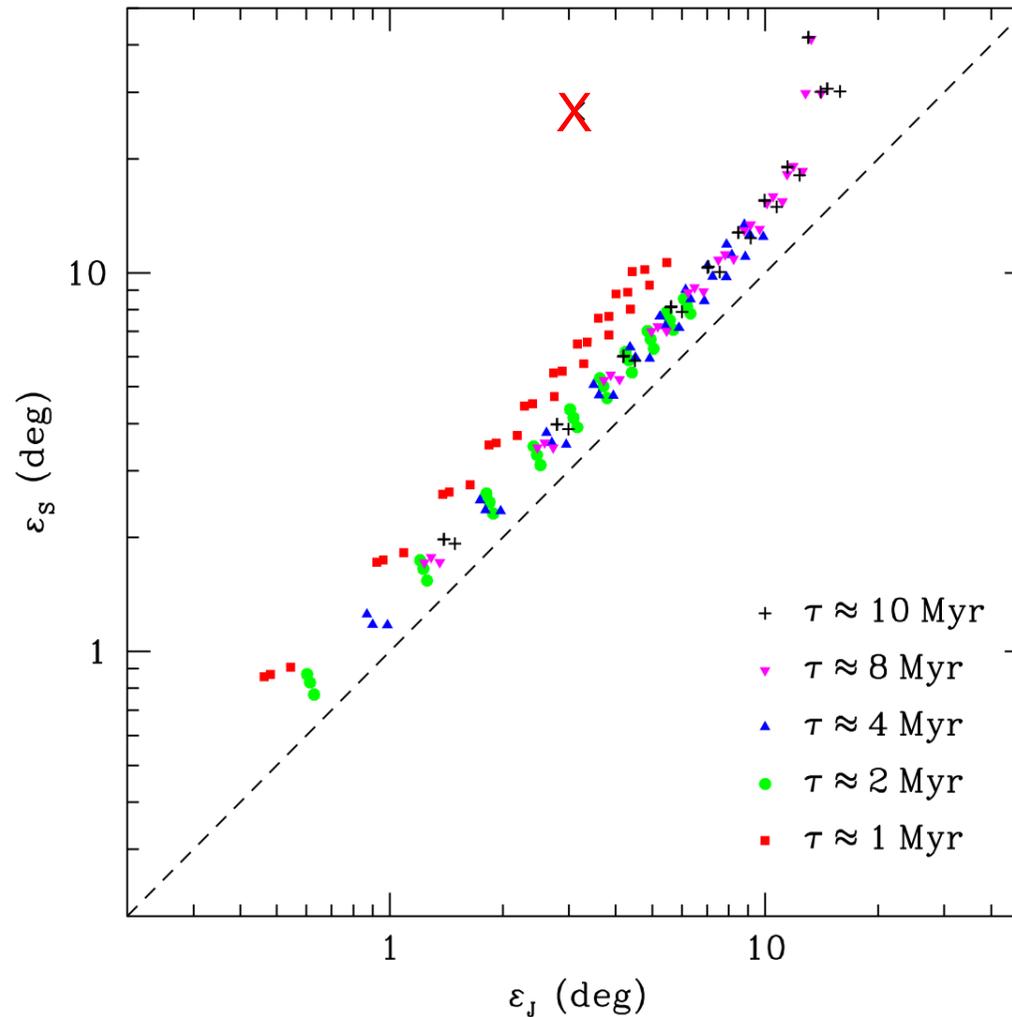
- *N*-body code SyMBA modified to include spin evolution due to torques from the Sun and mutual planetary interactions (Lee et al. 2007).
- Recursively subdivided timestep used by SyMBA to handle close encounters between the planets also implemented for spin evolution due to planetary torques.
- 2000 planetesimals in a disk outside the outermost ice giant.
- 1280 simulations for each migration model.

# Secular Simulations of Resonant Nice Model

$$\tau = 10\text{Myr}, i_N = 4^\circ$$



$\varepsilon_S$  vs  $\varepsilon_J$  from simulations with  $\tau = 1-10$  Myr and  $i_N = 1-10^\circ$

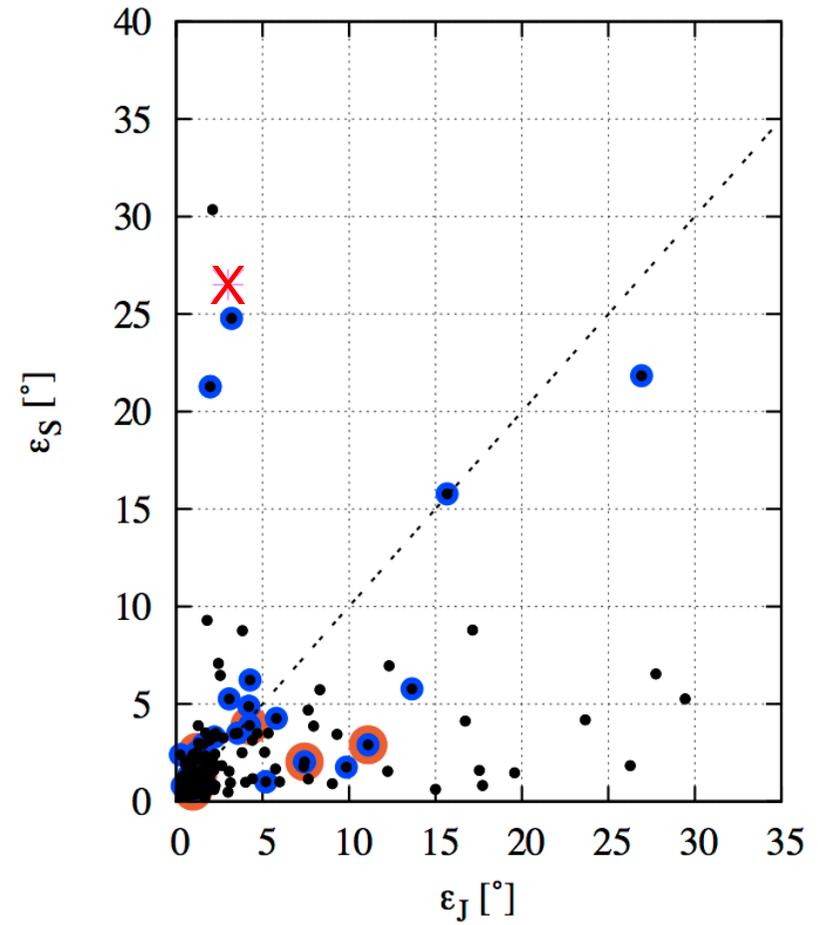
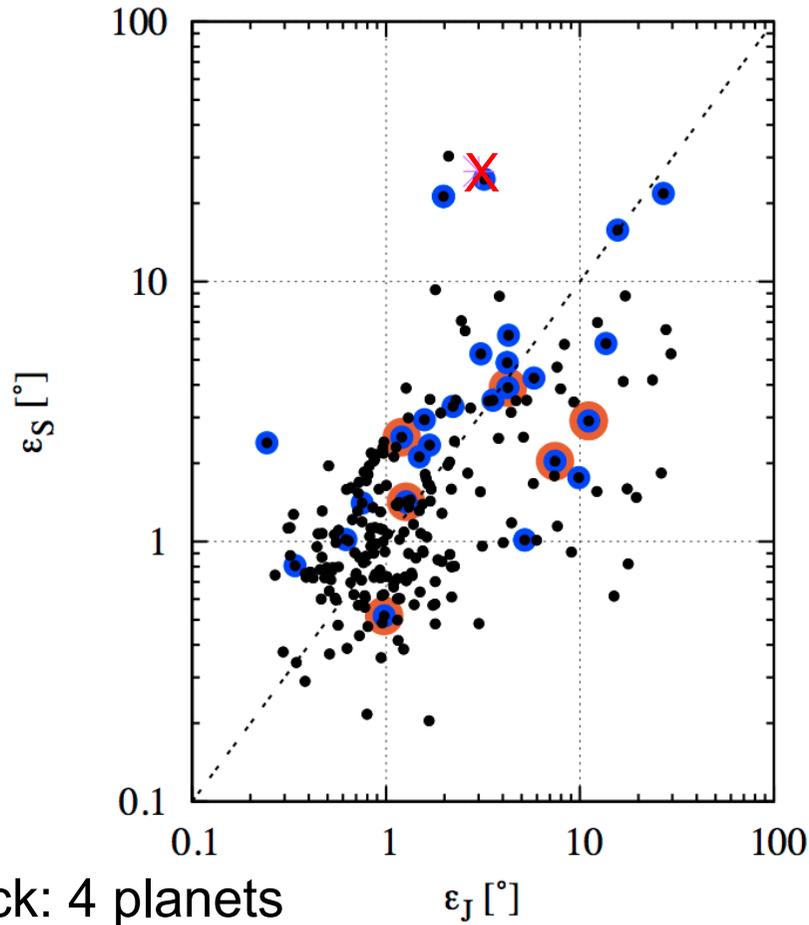


- $|s_8|$  must cross Jupiter's spin precession frequency to reach Saturn's and  $\varepsilon_J \sim \varepsilon_S$ .

- Secular model leaves out many relevant dynamical effects.
- Could it be that migration during the encounter phase was fast enough to avoid tilting Jupiter while a late very slow migration of Neptune to its final location tilted Saturn?

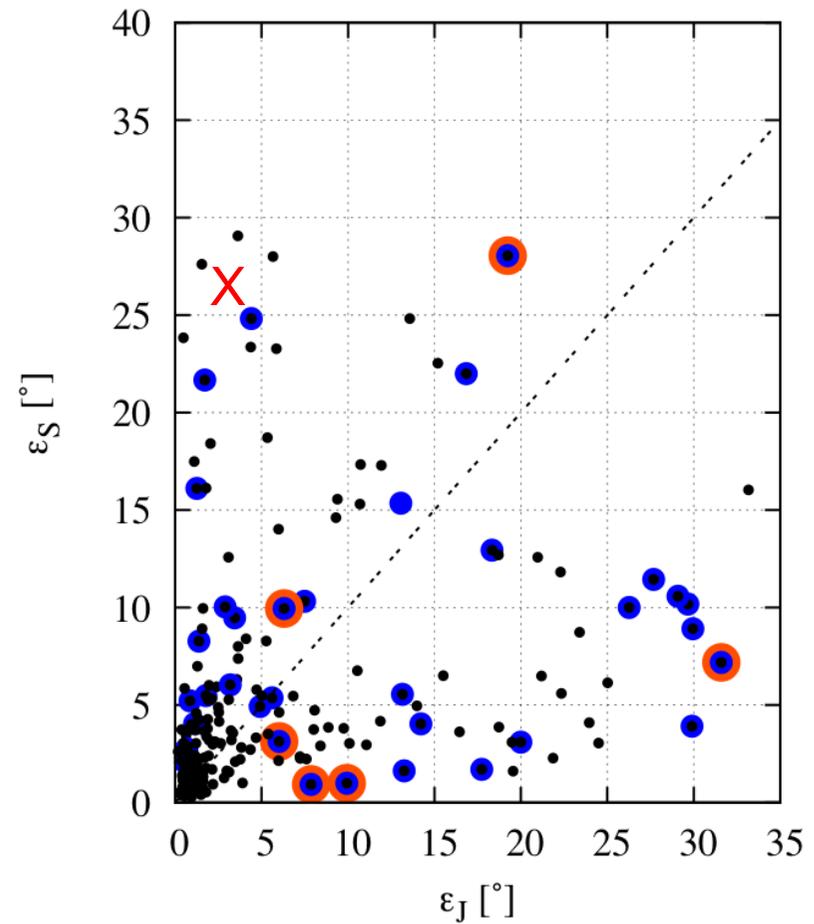
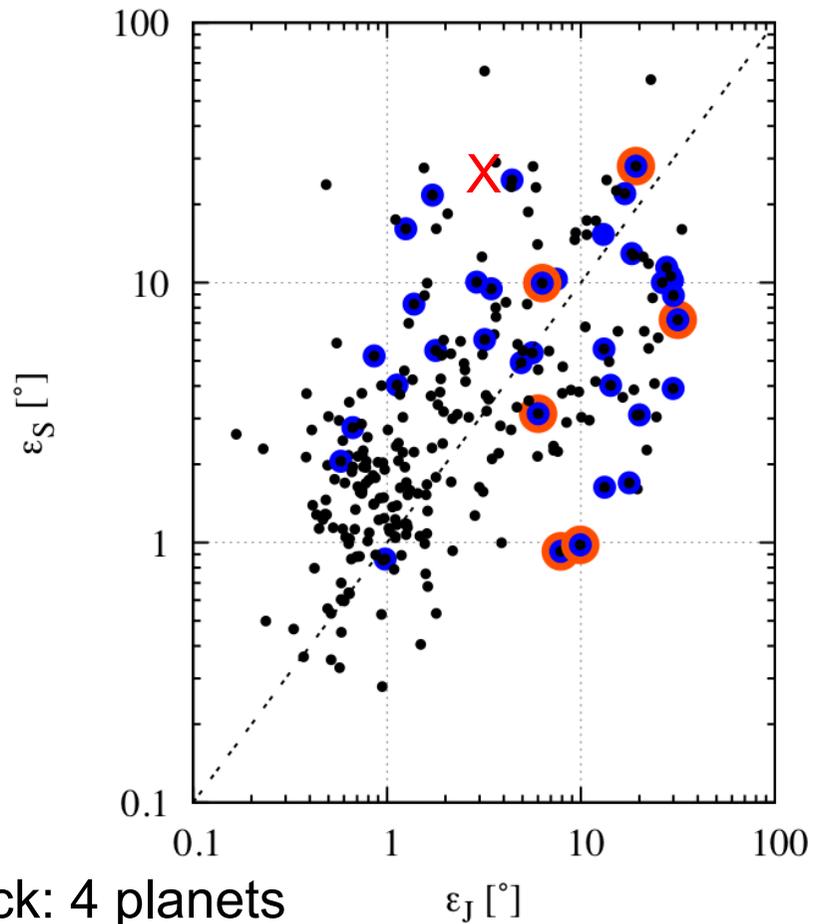
# N-body Simulations

## Resonant Nice Model with $50 M_{\oplus}$ Disk



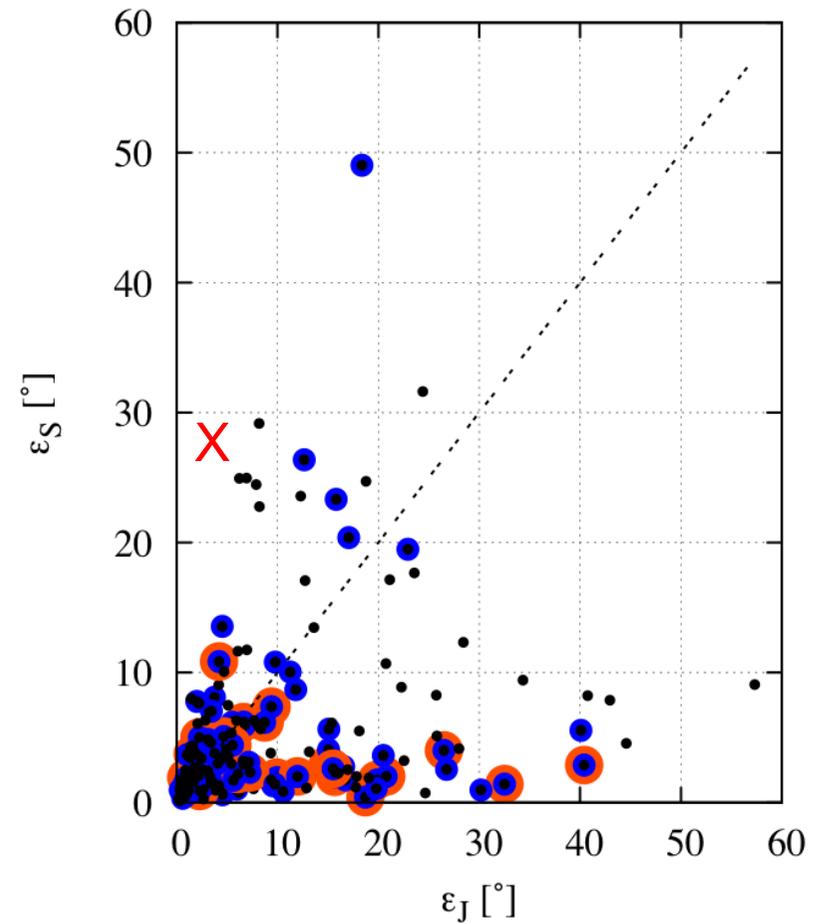
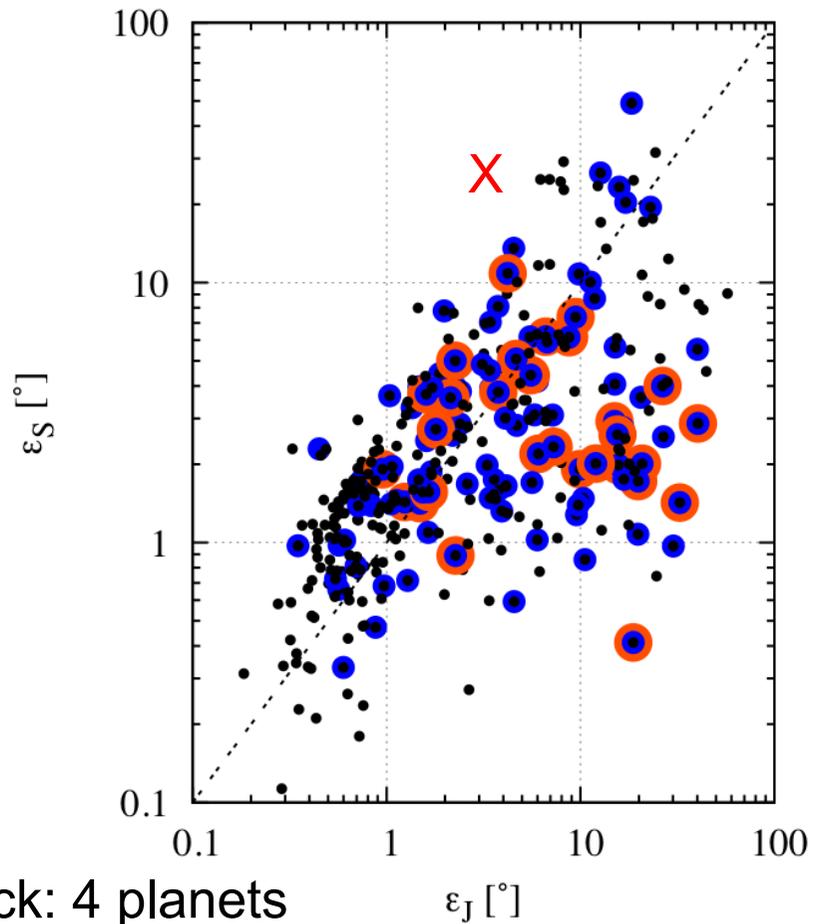
- Black: 4 planets
- Blue:  $a$  within 10%
- Red: All orbital constraints

## Compact 5-Planet Model with $35 M_{\oplus}$ Disk



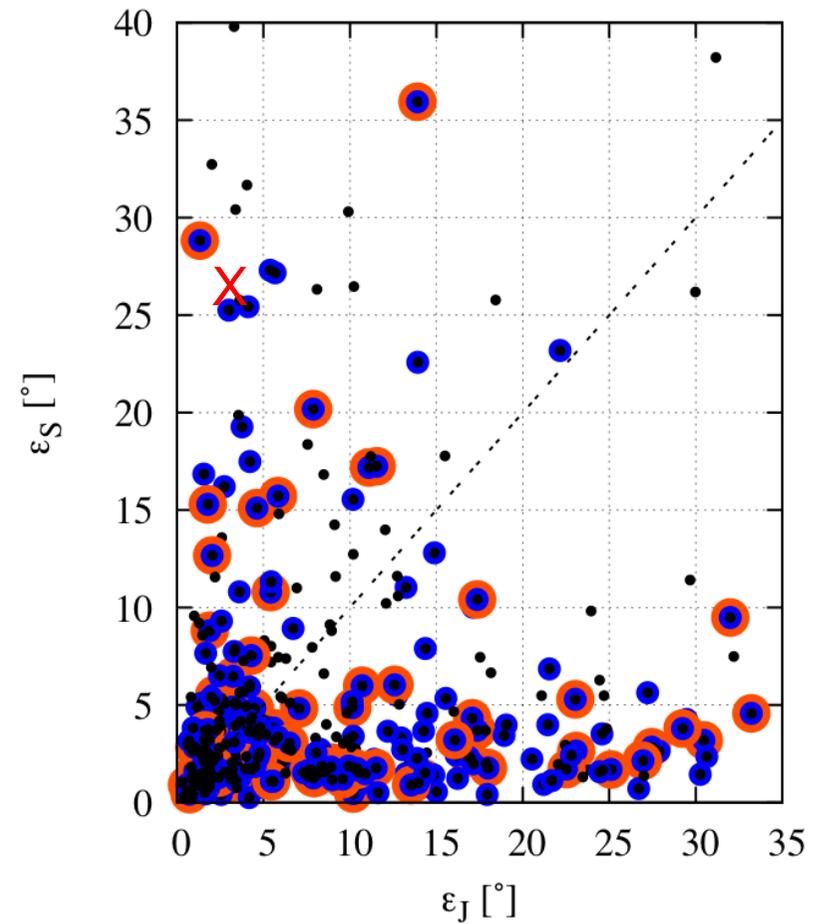
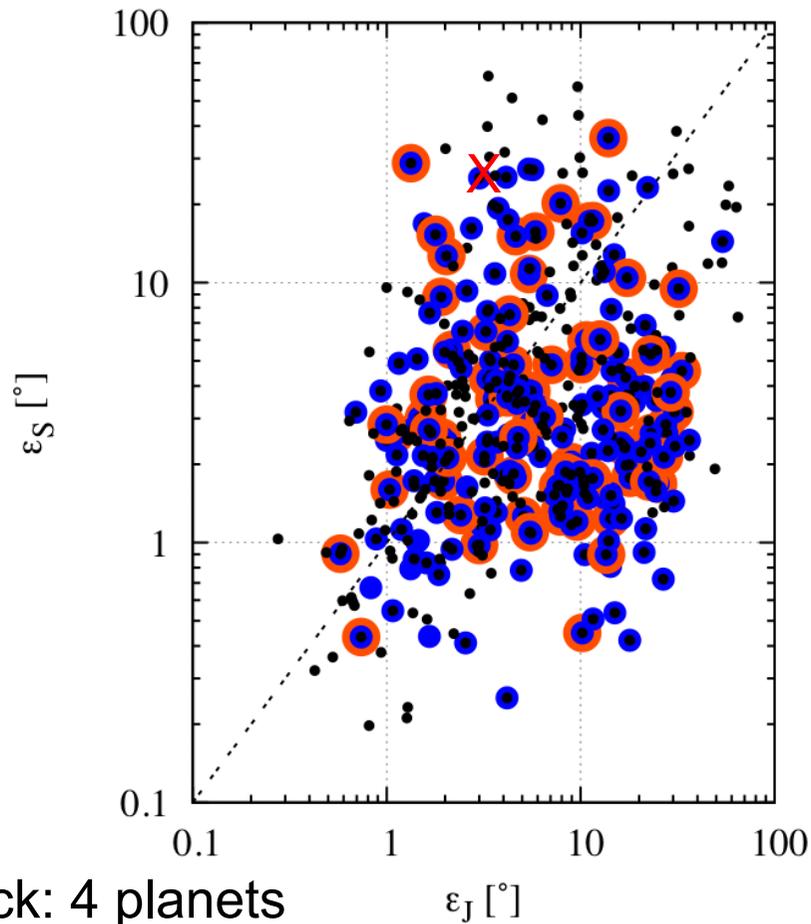
- Black: 4 planets
- Blue:  $a$  within 10%
- Red: All orbital constraints

## Loose 5-Planet Model with $35 M_{\oplus}$ Disk



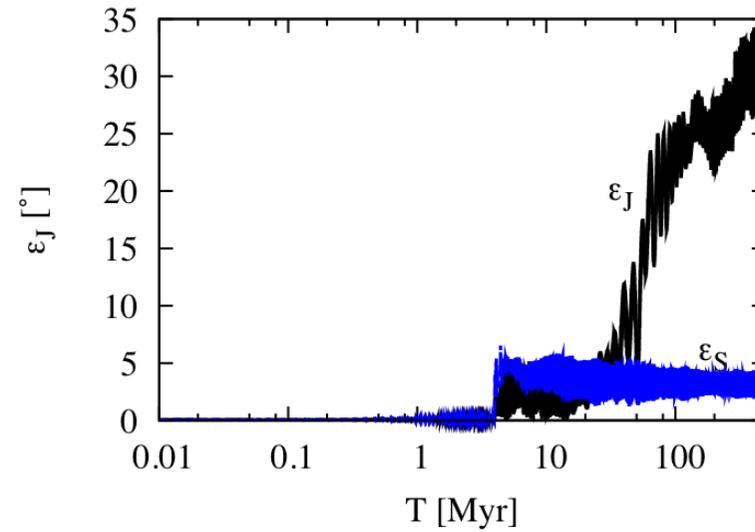
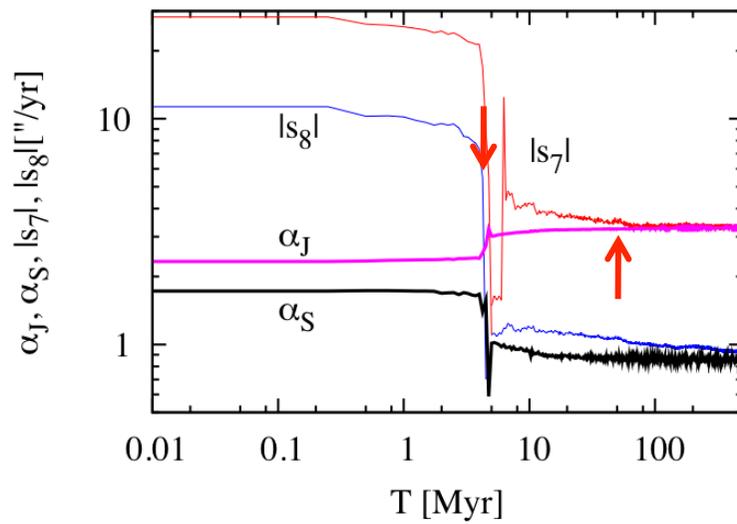
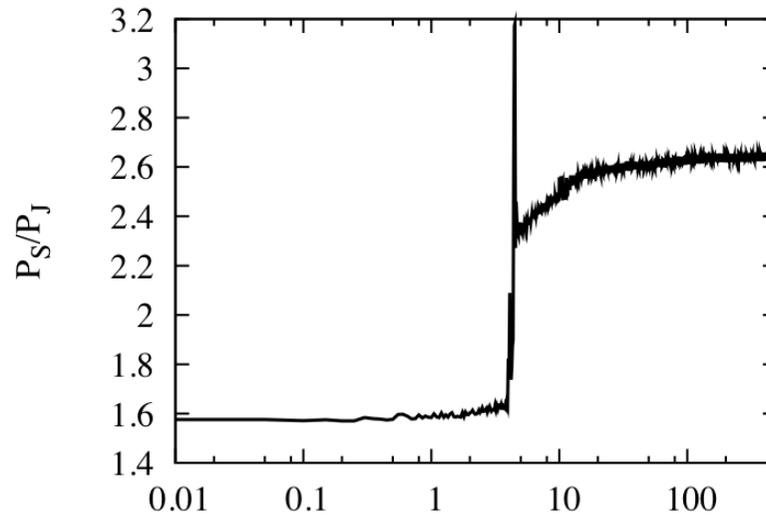
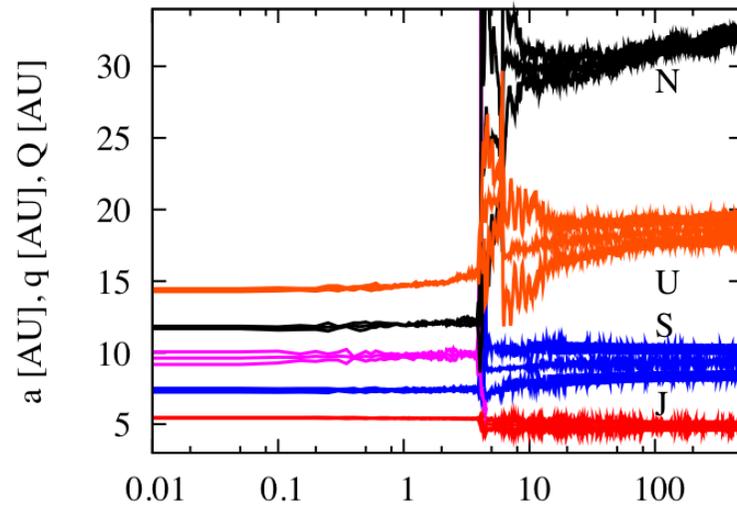
- Black: 4 planets
- Blue:  $a$  within 10%
- Red: All orbital constraints

## Compact 5-Planet Model with $50 M_{\oplus}$ Disk



- Black: 4 planets
- Blue:  $a$  within 10%
- Red: All orbital constraints

- Unsuccessful Example:



- Successful Example:

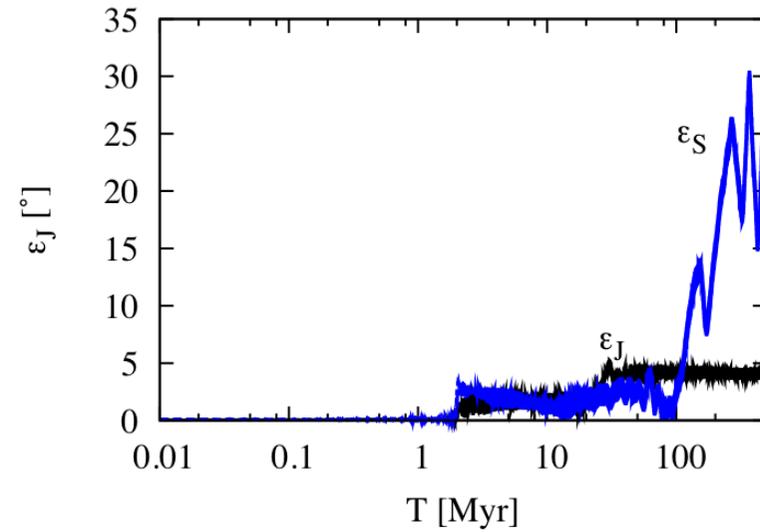
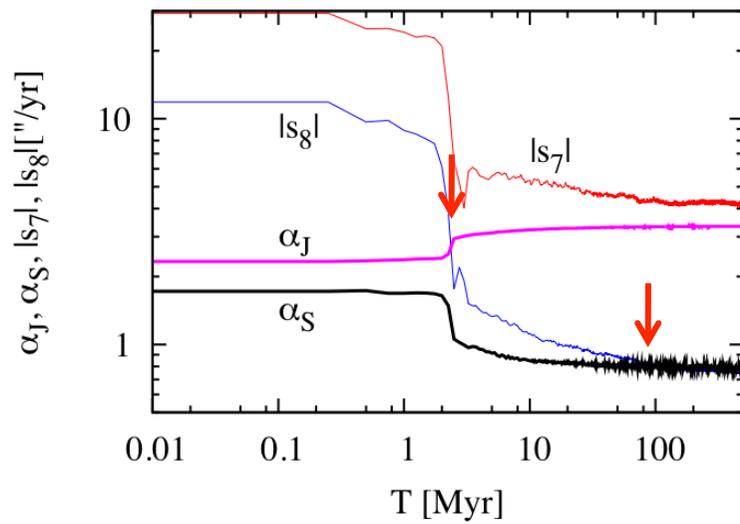
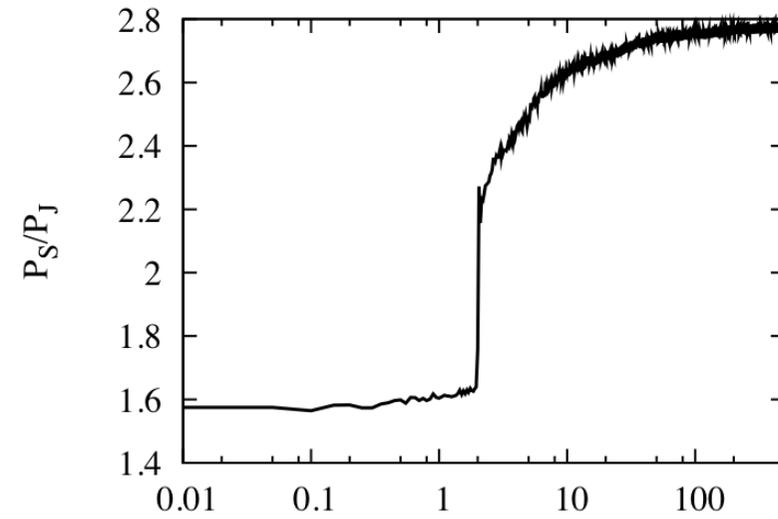
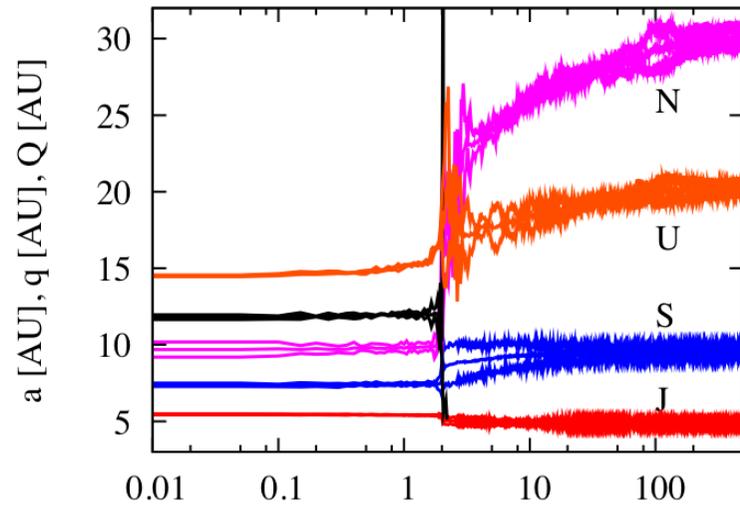


Table 1: Statistics of  $N$ -body Simulation Results

Model	$M_d$	$a_N$	$f_4$	$f_a$	$f_\varpi$	$f_\epsilon$
3:2 3:2 4:3	50 $M_\oplus$	11.6 AU	34%	2.3%	0.5%	<0.08%
3:2 3:2 4:3 4:3	35 $M_\oplus$	14.2 AU	33%	2.8%	0.46%	<0.08%
3:2 3:2 2:1 3:2	20 $M_\oplus$	22.2 AU	25%	8.3%	2.3%	<0.08%
3:2 3:2 4:3 4:3	50 $M_\oplus$	14.2 AU	33%	17%	6.6%	0.4%

# Summary

- The high probability of ejecting an ice giant in the Nice model has led to the resonant 5-planet model.
- The obliquities of Jupiter and Saturn provide constraints on the migration and encounter history of the giant planets.
- Both the resonant Nice and nominal 5-planet models fail to simultaneously reproduce the orbital and spin properties of the giant planets.
- Only a compact 5-planet model with a heavier disk has a low probability of matching both constraints, but it is still more likely to produce a Jovian obliquity that is too large.