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Cosimulation of Electromagnetics–Circuit Systems Exploiting DGTD and MNA

Ping Li, Student Member, IEEE, Li Jun Jiang, Senior Member, IEEE, and Hakan Bağı, Member, IEEE

Abstract—A hybrid electromagnetics (EM)–circuit simulator exploiting the discontinuous Galerkin time domain (DGTD) method and the modified nodal analysis (MNA) algorithm is developed for analyzing hybrid distributive and nonlinear multiport lumped circuit systems. The computational domain is split into two subsystems. One is the EM subsystem that is analyzed by DGTD, while the other is the circuit subsystem that is solved by the MNA method. The coupling between the EM and circuit subsystems is enforced at the lumped port where related field and circuit unknowns are coupled via the use of numerical flux, port voltages, and current sources. Since the spatial operations of DGTD are localized, thanks to the use of numerical flux, coupling matrices between EM and circuit subsystems are small and are directly inverted. To handle nonlinear devices within the circuit subsystem, the standard Newton–Raphson method is applied to the nonlinear coupling matrix system. In addition, a local time-stepping scheme is applied to improve the efficiency of the hybrid solver. Numerical examples including single and multiport linear/nonlinear circuit networks are presented to validate the proposed solver.

Index Terms—Discontinuous Galerkin time domain (DGTD) method, hybrid EM–circuit solver, local time stepping (LTS), modified nodal analysis (MNA), multiport circuit networks, Newton–Raphson method, nonlinear elements, transient analysis.

I. INTRODUCTION

WITH increasing operating frequencies of electronic circuits, minimization of chip packaging, and integration of multifunctional capabilities, any simulation tool developed for circuit-system modeling must consider unintentional emissions and couplings between the distributive and lumped circuit networks [1]. This requirement can be fulfilled by a hybrid approach that solves Maxwell and circuit equations simultaneously. Since interactions between electromagnetic fields and active devices have significant impacts on the system’s performance, incorporation of nonlinear lumped element modeling within the simulation tool is also required [2]–[11], [34] and is receiving more attention. The presence of nonlinear interactions renders the system response highly susceptible to small changes in the EM and circuit subsystems. Therefore, utmost accuracy that can only be obtained by coupled solution, which considers all physical interactions between the two subsystems, is highly desired.

While the EM–circuit system can be analyzed both in frequency and time domain, time-domain methods have inherent merits since it can produce broadband results through a single simulation. Furthermore, it allows the analysis of nonlinear physical phenomenon, such as the harmonic generation and intermodulation without resorting to the harmonic balance or port-extraction method [2] when nonlinear circuit elements are present.

Among various available full-wave methods, the finite difference time domain (FDTD) is a popular choice for solving Maxwell equations since its formulation and implementation are rather straightforward [17]. Recently, FDTD has been extended to model lumped circuit networks [3]–[9]. Lumped elements are treated through a direct stamping technique that assigns each lumped element into an edge of the FDTD grid [3], [4], using an equivalent source concept [5]–[7], or by transforming the admittance matrix from the Laplace domain to the time domain with recursive convolution method [8], [9].

The time-domain finite-element method (TDFEM) [18] is another Maxwell-equation solver; it is preferred over FDTD since it can model arbitrarily shaped geometries more accurately and allows for high-order discretization schemes. Similar to FDTD, TDFEM has been used in solving radiation, electrical packaging, and circuit problems [10]–[16]. In [11], an FDTD-like direct stamping method based on the basic $I–V$ relationship is incorporated into the FEM matrix. Although this method is straightforward, it lacks the capability of modeling complex networks. Later, an equivalent source generator approach, similar to the equivalent source concept in FDTD, is introduced into TDFEM [12], [13]. Recently, TDFEM combined with modified nodal analysis (MNA) [19] is developed to accurately consider arbitrarily complex circuit networks [14]–[16]. These hybrid approaches split the computational domain into two parts. One is the EM part, and another is the circuit domain. The EM part is solved by TDFEM, while the circuit subsystem is analyzed via MNA. The interaction...
between the EM and circuit subsystems realized at the lumped port residing over FEM-based mesh edges. Although these methods are well developed, the deficiencies are that the final assembled global mass and stiffness matrices have to be altered due to additional terms arising from circuit networks. When the circuit network includes nonlinear elements, the FEM-based approach becomes very computationally expensive because the entire system matrix has to be factorized and solved at each time step, as the system is time dependent and nonlinear. To overcome this deficiency, special extraction technique is employed to construct a relatively smaller time-dependent matrix [14]. In [15] and [16], orthogonal vector basis functions are used to construct small matrices. The computational domain is solved layer by layer with the reduction-recovery method. Also based on MNA, hybrid EM–circuit simulation methods based on the time-domain integral equation (TDIE) method are proposed in [20]–[22]. In [22], another transmission line solver is integrated together with TDIE and MNA method.

DGTD method [23] is recently extended to solve Maxwell equations. Unlike FEM, all spatial operations of DGTD are localized and solutions are allowed to be discontinuous across boundaries between neighboring elements. The information exchange between the elements is facilitated via numerical flux. The use of numerical flux allows for generation of mass and stiffness matrices without an assembling operation that involves all neighboring elements. The resulting matrices are block diagonal; dimension of each block is equal to the number of degrees of freedom in each element. Mass-matrix blocks are inverted and stored before time marching which produces a very compact and efficient solver when combined with an explicit time integration scheme. In [24], DGTD is applied to study the transient behavior of interconnect structures with a single linear lumped element. Each of the lumped elements is treated via a direct stamping approach that assigns each of them into a rectangular port surface. At each port surface, the basic $V$–$I$ relationship is enforced for linear $R$, $L$, and $C$ lumped elements. This direct stamping method lacks the flexibility to model arbitrary complex networks and nonlinear devices. In [25], single lumped port networks are modeled by a direct call for the SPICE software. The EM subsystem and the circuit subsystem are solved iteratively, but the convergence of these iterations is not guaranteed since the two systems are not coupled rigorously. Especially in the presence of highly nonlinear lumped elements, the stability of the iterative scheme is expected to be jeopardized. In addition, the compatibility of the adopted Runge–Kutta time-marching scheme with the leap-frog-based SPICE commercial simulators is an issue. Finally, extra time is required for the interface communication.

The inherent advantages of DGTD, i.e., explicitness of the time marching and localized spatial operations, render it very suitable for analyzing nonlinear circuit networks. In [30], the DGTD method is employed to analyze single port circuit networks. To generalize this method, the aim of this paper is to develop a hybrid EM–circuit simulator to model distributive complex multiport circuit networks including both linear/nonlinear elements. Inspired by the works in [14]–[16] and [31], the entire computational system is divided into two subsystems: one is the EM subsystem, another is the circuit subsystem. The EM subsystem is analyzed by solving Maxwell equations via DGTD, and the circuit subsystem is modeled by MNA based on Kirchhoff’s circuit laws. The lumped port residing over an impedance surface is defined at the interface between the EM and circuit subsystems. The coupling from the EM subsystem to the circuit subsystem is achieved by introducing a port current source calculated through the circuit solver. The advantages of the proposed hybrid simulator are: first, the introduction of lumped circuit networks does not alter the mass and stiffness matrices since the coupling from circuit networks is considered through the numerical flux with proper boundary conditions. The dimension of the coupling matrix (as shown in Section II-C) is equal to the number of degrees of freedom for $E$-field in that mesh element plus the number of nonreference node voltages and branch currents flowing through voltage sources in the circuit network. This property is very important for circuit networks especially when nonlinear elements are included since only a small time-dependent matrix need to be inverted at each step. Second, the proposed solver compared with the method in [24] can consider modeling arbitrary complex multiport circuit networks including both linear and nonlinear lumped elements. Third, the establishment of the EM–circuit coupling matrix can efficiently handle nonlinear circuit networks while no hybrid coupling matrix system is considered in [25]. In addition, the proposed hybrid method enables equations in EM-subsystem and circuit networks to be integrated into a unified solver instead of referring to external commercial softwares. Finally, local time-stepping (LTS) strategy [29] can be easily integrated into our proposed algorithm to increase the efficiency of proposed solver in the presence of multiscale meshes.

The remainder of this paper is organized as follows. Section II first describes the equation underlying the DGTD and MNA. Then, the method used for coupling the two sets of equations is presented. Finally, the LTS scheme used to increase the efficiency of proposed solver is briefly reviewed. Section III presents numerical results that demonstrate the accuracy, efficiency, and applicability of the proposed solver. Conclusions are presented at the end of this paper.

II. THEORY AND FORMULATION

A. DGTD Formulation

Let $\Omega$ represents the interested computation domain and bounded by boundary $\partial \Omega$. The domain $\Omega$ is discretized into a set of nonoverlapping subdomains $\Omega_i$ bounded by a surface $\partial \Omega_i$, where $\Omega = \bigcup \Omega_i$. Applying the discontinuous Galerkin testing to the Maxwell curl equations in $\Omega_i$ yields

$$\int_{\Omega_i} \Phi^{(i)}_{\ell}(\epsilon \partial \Phi - \nabla \times \mathbf{H}) dV = \int_{\partial \Omega_i} \Phi^{(i)}_{\ell}(\mathbf{n} \times (\mathbf{H}^* - \mathbf{H})) dS \quad (1)$$

$$\int_{\Omega_i} \Psi^{(i)}_{\ell}(\mu \partial \Psi + \nabla \times \mathbf{E}) dV = -\int_{\partial \Omega_i} \Psi^{(i)}_{\ell} \mathbf{n} \times (\mathbf{E}^* - \mathbf{E}) dS \quad (2)$$
where $\Phi_l^{(i)}$ and $\Psi_l^{(i)}$ denote the $i$th and $l$th vector basis functions for the electric field $E^{(i)}$ and $H^{(i)}$, respectively. $\hat{n}$ is the unit outward normal vector of $i$th subdomain; $\hat{n} \times \mathbf{E}^*$ and $\hat{n} \times \mathbf{E}^t$ are the numerical fluxes used for information exchange between adjacent elements. At the lumped port, a surface electric current density $J_{\text{CKT}}$ provided by the circuit subsystem exists. The following boundary condition must be satisfied [31]:

$$
\hat{n} \times (H^{(j)} - H^{(i)}) = J_{\text{CKT}}
$$

(3)

$$
\hat{n} \times (E^{(j)} - E^{(i)}) = 0
$$

(4)

where the superscripts $i$ and $j$ represent local and neighboring elements, respectively. With this boundary condition, a general expression of the numerical flux can be derived as

$$
\hat{n} \times \mathbf{E}^* = \frac{\hat{n} \times (Y^{(i)} E^{(i)} + Y^{(j)} E^{(j)})}{Y^{(i)} + Y^{(j)}} - \alpha \frac{\hat{n} \times (\hat{n} \times (H^{(i)} - H^{(j)}) + J_{\text{CKT}})}{Y^{(i)} + Y^{(j)}}
$$

(5)

$$
\hat{n} \times \mathbf{H}^* = \frac{\hat{n} \times (Z^{(i)} H^{(i)} + Z^{(j)} H^{(j)}) - Z^{(j)} J_{\text{CKT}}}{Z^{(i)} + Z^{(j)}} + \alpha \frac{\hat{n} \times [\hat{n} \times (E^{(i)} - E^{(j)})]}{Z^{(i)} + Z^{(j)}}
$$

(6)

where $Z^{(i)} = (\mu^{(i)} / \varepsilon^{(i)})^{1/2}$ and $Y^{(i)} = 1/Z^{(i)}$ are the characteristic wave impedance and admittance of the local and neighboring elements, respectively. The choice of the numerical flux depends on parameter $\alpha$. $\alpha = 0$ corresponds to the center flux, while $\alpha = 1$ corresponds to fully penalized upwind flux, and others correspond to partially penalized flux. In this paper, upwind flux is employed which can be derived according to the Rankine–Hugoniot condition. At the normal boundary surfaces where no lumped port exists, the coupled upwind flux, and others correspond to partially penalized flux. Consequently, the fully-discrete local system of equations can be obtained from the semidiscrete system in (7) and (8) as

$$
M^{(i)} \mathbf{e}^{(i)}_{n+1} + \Delta t \bar{J}_{n+1}^{(i)} = M^{(i)} \mathbf{e}^{(i)}_n + \Delta t \left[ \bar{S}^{(i)} \mathbf{h}^{(i)}_{n+1/2} - \bar{F}^{(i)} \right] / 2
$$

(11)

$$
M^{(i)} \mathbf{h}^{(i)}_{n+1/2} = M^{(i)} \mathbf{h}^{(i)}_n - \Delta t \left[ \bar{S}^{(i)} \mathbf{e}^{(i)}_{n+1} - \bar{j}^{(i)} + \bar{F}^{(i)} \mathbf{h}^{(i)}_n + \bar{F}^{(i)} \right]
$$

(12)

Note that apart from unknowns $\mathbf{e}^{(i)}_{n+1}$ and $\mathbf{h}^{(i)}_{n+1/2}$ in (11) and (12), a third unknown $\mathbf{h}^{(i)}_{\text{CKT},n+1}$ is introduced from the circuit network. These unknowns are obtained by solving the coupled system of DGTD and MNA equations.

B. MNA for Nonlinear Multiport Circuit Networks Modeling

To model the nonlinear multiport circuits, time domain MNA is employed. In the MNA process, Kirchhoff’s current law is enforced at all nonreference nodes, and Kirchhoff’s voltage law is applied to independent loops. The resultant circuit matrix equation at time $t = (n + 1) \Delta t$ is

$$
\begin{bmatrix}
  [G] & [B_1] \\
  [B_2] & [D]
\end{bmatrix}
\begin{bmatrix}
  \mathbf{V}^{\text{CKT}}_{n+1} \\
  \mathbf{I}^{\text{CKT}}_{n+1}
\end{bmatrix}
+ \Delta t \begin{bmatrix}
  \mathbf{V}^{\text{CKT}}_{n+1} \\
  \mathbf{I}^{\text{CKT}}_{n+1}
\end{bmatrix}
= \begin{bmatrix}
  \mathbf{I}^{\text{CP}}_n + \mathbf{I}^{\text{Ind}}_{n+1} \\
  \mathbf{V}^{\text{Port}}_{n+1} + \mathbf{V}^{\text{Ind}}_{n+1}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{e}^{(i)} \mathbf{h}^{(i)}_{\text{CKT},n+1}
\end{bmatrix}.
\end{bmatrix}
\end{array}
$$

(13)

Here, the admittance matrix $[G]$ is determined by interconnections between circuit elements. $[B_1]$ and $[B_2]$ are determined by the connection of supplied voltage sources with only 0, 1, and $-1$ elements. When there are no dependent sources, the $[B_2]$ matrix is the transpose of the $[B_1]$ matrix. $[D]$ is equal to zero if there are no controlled sources. $\mathbf{V}^{\text{CKT}}_{n+1}$ denotes the unknown node voltages. $\mathbf{I}^{\text{CKT}}_{n+1}$ denotes the unknown currents through voltage sources. $\mathbf{I}^{\text{CKT}}_{n+1}$ represents currents through branches containing nonlinear elements. $\mathbf{I}^{\text{CP}}_n$ comprises current sources at $t = n \Delta t$ derived from companion models of inductors and capacitors based on the trapezoidal integration rule used in this paper.
\( \mathbf{I}_{n+1}^{\text{ind}} \) denotes the independent current sources like the Norton current source. \( \mathbf{V}_{n+1}^{\text{Port}} \) holds the values of supplied voltage sources coupled from the EM part, while the \( \mathbf{V}_{n+1}^{\text{ind}} \) represents independent voltage sources in the circuit subsystem like Thevenin voltage source. The overall dimension of the circuit subsystem in (28), denoted as \( N_{\text{CKT}} \), is equal to the number of voltage nodes plus the number of voltage sources.

### C. Coupling Between the EM and Multiport Circuit Subsystems

We assume that there are \( F \) independent lumped circuit networks in total. For multiport circuits, unlike single port network [30], the port-to-port coupling has to be considered. To include the port-to-port interaction, each port voltage is extracted and put them into the vector term \( \mathbf{V}_{n+1}^{\text{Port}} \) in (13). For general case, we further suppose that each of the circuit networks has \( K_f \) \( (f = 1, 2, \ldots, F) \) ports. For the \( f \)th network, a rectangular lumped port is introduced at each interface between the EM region and this circuit network.

Since the electrical size of lumped ports is small compared with the wavelength, quasi-static approximation is assumed with constant electric and magnetic fields over the lumped ports. Since there could be more than one element adjacent to the port \( q \) \((q = 1, 2, \ldots, K_f)\), we assume that the element \( i_q \) is one of the elements adjacent to the \( q \)th port. At the time \( t = (n + 1)\Delta t \), the supplied voltage at the \( q \)th lumped port can be calculated by the line integral of the \( E \) field in the \( i_q \) element along this lumped port. That is,

\[
\mathbf{V}_{n+1,q,f}^{\text{Port}} = - \sum_{k=1}^{n_f} \mathbf{e}_{n+1,k}^{(l_q,f)} \int \mathbf{\Phi}_k^{(l_q,f)} \cdot \mathbf{i}_{q,f}^{(l_q,f)} dl
\]

\[= - [C]^{(l_q,f)} \mathbf{e}_{n+1,f}^{(l_q,f)}
\]

where \( \mathbf{i}_{q,f} \) is the unit reference vector along the direction from the reference potential (reference ground) to the desired potential points at the \( q \)th port.

The locally coupled EM–circuit system equation can be established by combining (11), (13), and (14):

\[
\mathbf{F}_f(x_{n+1,f}) = \mathbf{b}^f
\]

where

\[
x_{n+1,f} = \begin{bmatrix} [e_{n+1}^{(f)}]^T & [\mathbf{V}_{n+1,f}^{\text{EM}}]^T & [\mathbf{V}_{n+1,f}^{\text{CKT}}]^T \end{bmatrix}^T
\]

\[
e_{n+1}^{(f)} = \begin{bmatrix} [e_{n+1}^{(1,f)}]^T & [e_{n+1}^{(2,f)}]^T & \ldots & [e_{n+1}^{(K_f,f)}]^T \end{bmatrix}^T
\]

\( \mathbf{V}_{n+1,f}^{\text{EM}} \) comprises voltages at the lumped port and other node voltages (the first \( K_f \) voltages are the port voltages). \( \mathbf{V}_{n+1,f}^{\text{CKT}} \) contains the amplitude of currents through port voltages and other independent voltage sources in the circuit subsystem (the first \( K_f \) current sources are those through the lumped ports).

\[
\mathbf{F}_f(x_{n+1,f}) = \begin{bmatrix} [M^{(f)}] & 0 & \Delta t[T^{(f)}]/2 \end{bmatrix} \begin{bmatrix} e_{n+1}^{(f)} \end{bmatrix} + \begin{bmatrix} 0 & [G^{(f)}] & [B^{(f)}] \end{bmatrix} \begin{bmatrix} V_{n+1,f}^{\text{EM}}^{\text{CKT}} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}
\]

\[
\mathbf{b}^f = \begin{bmatrix} b_{\text{EM}}^f \\
\mathbf{I}^{CP}_{n+1,f} \\
0 \end{bmatrix}
\]

where \([M^{(f)}],[T^{(f)}],[C^{(f)}]\) are diagonal matrices, and \( b_{\text{EM}}^f \) and \( \mathbf{I}^{CP}_{n+1,f} \) are column vectors.

\[
[M^{(f)}]_{i,i} = \begin{bmatrix} M^{(i_1,f)}, M^{(i_2,f)}, \ldots, M^{(i_{K_f},f)} \end{bmatrix}
\]

\[
[T^{(f)}]_{ij} = [T^{(i_1,f)}, T^{(i_2,f)}, \ldots, T^{(i_{K_f},f)}]
\]

\[
[C^{(f)}]_{ij} = C^{(i_1,f)}, C^{(i_2,f)}, \ldots, C^{(i_{K_f},f)}
\]

\[
\mathbf{b}^f_{\text{EM}} = \mathbf{M}^f_{e} \mathbf{e}_{n}^{(l_q,f)} + \Delta t \begin{bmatrix} \mathbf{S}_e^{(l_q,f)} \mathbf{h}_{i}^{(l_q,f)} - \frac{\mathbf{I}^{\text{CKT}}_{n,q,f}(T^{(l_q,f)})}{2} \\
\mathbf{F}_e^{(l_q,f)} \mathbf{e}_{n}^{(l_q,f)} - \mathbf{F}_e^{(l_q,f)} \mathbf{e}_{n}^{(l_q,f)} + \mathbf{F}_e^{(l_q,f)} \mathbf{h}_{n+\frac{1}{2}}^{(l_q,f)} + \mathbf{F}_e^{(l_q,f)} \mathbf{h}_{n+\frac{1}{2}}^{(l_q,f)} \end{bmatrix}
\]

The overall dimension of the coupled matrix in (19) is equal to \( n_e^{(l_1,f)} + n_e^{(l_2,f)} + \ldots + n_e^{(l_{K_f},f)} + N_f^{\text{CKT}} \). In this paper, the EM domain is meshed into tetrahedrons. Each cell is assigned six vector edge basis functions. Thus, the dimension of the \( f \)th coupled matrix is equal to \( 6K_f + N_f^{\text{EM}} \).

\( \mathbf{V}_{n+1}^{\text{EM}} \) contains the EM domain system, which is typically cost due to these locally coupled system matrices.

### D. Stability Analysis and LTI

The resultant marching scheme for this hybrid EM–circuit system is explicit and conditionally stable. Following the energy conservation technique, the stability condition can be obtained:

\[
\Delta t_1 \left< \frac{2\alpha c_i + \beta j c_i + 4\beta j i \frac{\gamma j}{\epsilon_i}}{\epsilon_i} \right> < 4V_i \quad 4P_i
\]

\[
\Delta t_2 \left< \frac{2\alpha j c_j + \beta j i c_j + 4\beta j i \frac{\gamma j}{\epsilon_j}}{\epsilon_j} \right> < 4V_j \quad 4P_j
\]

\[
\Delta t_3 \left< \frac{2\alpha c_i + \beta j c_i + 4\beta j i \frac{Z_j}{\mu_j}}{\mu_i} \right> < 4V_i \quad 4P_i
\]

\[
\Delta t_4 \left< \frac{2\alpha j c_j + \beta j i c_j + 4\beta j i \frac{Z_j}{\mu_j}}{\mu_i} \right> < 4V_j \quad 4P_j
\]
where \( j \in N(i) \) represents the neighboring elements of the \( i \)th element, \( P_i \) is the total area of four facets, \( V_i \) is the volume of element \( i \), and parameters \( \alpha_{ij} \), \( \beta_{ij} \), \( \gamma_{ij} \), and \( \varepsilon_{ij} \) are described in [28], [32], and [33]. To ensure stability, the time-step size must be chosen as \( \Delta t = \min\{\Delta t_1, \Delta t_2, \Delta t_3, \Delta t_4\} \).

The ratio of \( 4V_i/P_i \) effectively represents the diameter \( h_i \) of the finite element. As the ratio of \( 4V_i/P_i \) becomes smaller, the time-stepping size will also reduce, which will increase the computational time significantly. To improve the efficiency of the hybrid EM–circuit simulation method, the LTS method developed in [29] is employed. This strategy regroups the elements according to the local time-step size. For the \( k \)th group, its time-step size is \( \Delta t_k = (2m + 1)^{k-1}\Delta t_{\text{min}} \), where \( \Delta t_{\text{min}} = \min(\Delta t_i) \) \( (i = 1, 2, \ldots, K) \) denotes the minimum global time-stepping size, \( m \) is a strictly positive integer. In our case, \( m = 1 \) is chosen, which means that there is a factor 3 between the time steps of consecutive classes. For mesh cells that are not located at the interface between different groups, the classical leap-frog is applied. Otherwise, the terms coming from the neighboring elements acquire the recently updated field values from the corresponding adjacent elements. For illustration, we give the operations proceed in a step of the multiclass leap-frog method for \( K = 2 \), as shown in Fig. 1.

III. NUMERICAL RESULTS

In this section, the proposed hybrid EM–circuit simulator is applied to several numerical examples to validate and verify its feasibility and accuracy. In these examples, circuit networks include linear and nonlinear active devices. To mitigate the very small time step size caused by unstructured meshes, the LTS technique is employed for the last two examples.

A. Circuit Network Comprised Linear \( R, L, \) and \( C \) Elements

In this example, an air-filled lossless parallel-plate waveguide structure loaded by a linear circuit network in Fig. 2 is benchmarked. The width (along \( y \)), height (along \( z \)), and length (along \( x \)) of this waveguide are 3, 3, and 51.5 mm, respectively. The two plates parallel to \( xOy \) plane are perfect electrical conductor, and the two side plates parallel to the \( xOz \) plane are perfect magnetic conductor. A TEM wave is launched on the incident plane at \( x = 0 \). The incident wave is a first-order differential Gaussian pulse. The two ends of this waveguide are terminated by the first-order absorbing boundary conditions. The dimension of the coupled matrix is \( 14 \times 14 \) with six unknowns in the EM subsystem and eight unknowns in the circuit subsystem. Fig. 3 shows the computed

---

**Fig. 1.** Steps for multiclass step scheme with two classes [29]. The time step size of Class 2 is denoted as \( \Delta t \).

**Fig. 2.** Circuit network comprised of only linear \( R, L, \) and \( C \) lumped elements. The circuit parameters are: \( R_1 = 150 \Omega \), \( R_2 = 10 \Omega \), \( R_3 = 100 \Omega \), \( R_4 = 10 \Omega \), \( R_5 = 50 \Omega \), \( R_6 = 377 \Omega \), \( L_1 = 10 \) nH, \( L_2 = 1 \) nH, \( L_3 = 0.1 \) nH, \( L_4 = 0.1 \) nH, \( C_1 = 0.01 \) pF, and \( C_2 = 0.1 \) pF.

**Fig. 3.** Comparison of the \( S \)-parameter and the input impedance with those obtained from ADS. (a) The magnitude of \( S_{11} \). (b) The magnitude of \( S_{21} \). (c) The phase of \( S_{11} \) and \( S_{21} \). (d) The real and imaginary parts of input impedance \( Z_{11} \).
Fig. 4. Schematic structure of the full-wave rectifier under study.

Fig. 5. Currents at the input and output ports versus the time.

Fig. 6. Voltages at the input and output ports versus the time.

input impedance, scattering parameters $S_{11}$ and $S_{21}$ from dc to 10 GHz. The results calculated from Agilent ADS are also shown for comparison. It can be clearly observed that good agreements are achieved.

B. Full-Wave Rectifier

This example is a full-wave rectifier (shown in Fig. 4) containing four silicon diodes driven by a 2.5 GHz Thevenin sinusoidal voltage source through a microstrip transmission line. The basic voltage–current relationship of the silicon diode is $i_D(t) = I_0[\exp(v_D(t)/V_0) - 1]$, where $I_0 = 1.0 \times 10^{-14}$ A and $V_0 = 0.026$ V. The relative permittivity of the microstrip substrate is 4.2 with the height equal to 0.51 mm, the width and length of the microstrip are 1 and 5 mm, respectively. In the positive half-period, the current flows along the path denoted by red arrows; while in the negative half-period, the current flows along the path denoted by blue arrows. Due to the existence of nonlinear elements, the iterative Newton–Raphson method is applied to solve the coupled matrix system in (16). In this example, the number of iteration is 10 to ensure stability. Figs. 5 and 6 show the current and voltage at the input and output ports of the rectifier, respectively. It is noted that the period of the output signal is halved compared with the input signal. It is further noted that the difference of the input and output voltages is about 1.4 V, as shown in Fig. 6. This agrees with the physical principle of silicon diodes whose forward voltage is about 0.7 V.

C. Active Devices

In this section, two different transistor models are investigated. One is the small-signal model for JFET, another is the
TABLE I
ELEMENT PARTITIONING BY CLASSES FOR THE JS8851-AS FET

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<th>Class</th>
<th>#Elements</th>
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<tr>
<td>1</td>
<td>64</td>
<td>8.55e-15</td>
</tr>
<tr>
<td>2</td>
<td>3935</td>
<td>2.563e-14</td>
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<td>3</td>
<td>1796</td>
<td>7.695e-14</td>
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TABLE II
COMPUTATIONAL TIME OF LTS VERSUS THE STANDARD LEAP-FROG SCHEME

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<td>Solution time without LTS</td>
<td>694.1 s</td>
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<td>Computational gain with LTS</td>
<td>3.76</td>
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</tbody>
</table>

large signal model for MESFET. In addition, the LTS strategy is employed in these two examples.

1) Small-Signal MESFET: The small-signal equivalent circuit of a microwave wave MESFET amplifier is simulated by the proposed EM–circuit simulator. The circuit consists of a common-source configured JS8851-AS FET mounted over a gap [8], [9], and [13]. The dimension of the circuit structure and the equivalent circuit model are presented in Fig. 7. This circuit configuration has four lumped ports. One is the Thevenin voltage source port (Port 1), and another two are connected to the gate (Port 2) and drain (Port 3) of the FET amplifier, and the last one is the load port (Port 4). These four rectangular ports have same size of 0.254 \( \times \) 0.79 mm\(^2\). The dimension of the coupling matrix between the EM and microwave FET amplifier circuit subsystems is 23 \( \times \) 23,

including 12 field unknowns and 11 unknowns introduced from the circuit subsystem. The total number of generated unstructured mesh is 5795. Based on the partition strategy of LTS, the meshes are grouped into three classes, as shown in Table I.

To study the broadband characteristics, a differential Gaussian resistive voltage source [25] is applied at Port 1. The CPU time for 101 376 time steps based on the smallest time step size is listed in Table II. The CPU gain with LTS to the standard leap-frog marching scheme is 3.76. The calculated S-parameters by the proposed algorithm are shown in Fig. 8. Simulations by ADS are also performed to provide comparison. Very good agreements are observed. The deviation from ADS is primarily due to the lack of full-wave capability of ADS circuit simulation.

2) MESFET Microwave Power Amplifier: To further validate the proposed full-wave simulator, a nonlinear microwave amplifier circuit is studied. The microstrip matching networks and the large-signal equivalent circuit model are shown in Fig. 9. This example has been studied by FDTD [6], TDFEM [14], and TDIE [21] before. The circuit model includes one nonlinear voltage-controlled capacitor

Fig. 8. (a) Magnitude of the \( S_{11} \) of the amplifier circuit. (b) Magnitude of the \( S_{21} \) of the amplifier circuit.

Fig. 9. (a) Circuit structure and the microstrip line matching networks. (b) Large-signal equivalent circuit model.

TABLE III
ELEMENT PARTITIONING BY CLASSES FOR THE LARGE-SIGNAL POWER AMPLIFIER

<table>
<thead>
<tr>
<th>Class</th>
<th>#Elements</th>
<th>( \Delta t(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5084</td>
<td>7.28e-14</td>
</tr>
<tr>
<td>2</td>
<td>10836</td>
<td>2.18e-13</td>
</tr>
</tbody>
</table>
and one nonlinear voltage-controlled current source. In the model

\[
\begin{align*}
C_{gs} &= \frac{3}{\sqrt{1 - \frac{V_c}{0.35}}} \text{ pF} & V_c < 0.35 \text{V} \\
C_{gs} &= 3\sqrt{2(0.5 + \frac{V_c}{0.7})} \text{ pF} & V_c \geq 0.35 \text{V}
\end{align*}
\]  
\tag{28}
\]

and

\[
I_{ds} = \tanh(V_{ds})(A_0 + A_1V_{gs} - A_2V_{gs}^2 - A_3V_{gs}^3)
\tag{29}
\]

where \(A_0 = 0.5304\), \(A_1 = 0.2595\), \(A_2 = -0.0542\), and \(A_3 = -0.0305\). The relative dielectric constant of the substrate is 2.33 with height equal to 0.7874 mm. Totally, four lumped ports are defined. The gate and drain of the amplifier are connected to Ports 2 and 3, respectively. The size of coupled matrix related to the microwave amplifier is 23. The number of Newton–Raphson method is fifteen. The total number of unstructured meshes is 15 920. According to the LTS scheme, they are grouped into two classes, as shown in Table III.

First, the dc operation characteristics with different biasing conditions are analyzed. The biasing of gate (\(V_{GG}\)) is added at Port 1, and the biasing of drain (\(V_{DD}\)) is enforced at Port 4. The channel current from drain to source is dependent on both gate biasing and drain biasing. The channel current is plotted as a function of applied biasing of gate and drain in Fig. 10. The saturation and linear regions can be clearly observed. The result agrees with the simulation in [6].

Next, the nonlinear behavior of this amplifier is studied. In this case, the chosen biasing conditions are \(V_{GS} = -0.81 \text{V}\) and \(V_{DS} = 6.4 \text{V}\). Apart from the dc biasing, a single-tone period signal operating at 6 GHz is added at Port 1. The corresponding input is 5.95 dBm. The voltage waveforms obtained from the transient analysis are shown in Fig. 11.

The power dissipated in the load is calculated from the Fourier transform of the steady-state voltage minus the
dc-voltage at the output. Due to the nonlinear capacitor and voltage-controlled current source, harmonics of the base signal arise. They can be observed from the power spectrum shown in Fig. 12. The output power versus different input power is plotted in Fig. 13. It is noted that the gain of the output power depressed when the input power is larger than 15 dBm. The 1-dB compression point happens around 26 dBm (output power).

To study the broadband behavior of this FET amplifier, a small Gaussian voltage source is added on the gate biasing (output power).

IV. CONCLUSION

In this paper, a hybrid EM–circuit simulation method based on DGTD and MNA is developed to simulate the distributive part and lumped circuit networks together. By introducing lumped port at the interface between the EM region and circuit subsystem, the interaction of these two subsystems is properly captured. The coupling from the EM to circuit is facilitated by the port voltage obtained by the line integral of E-field along the lumped port, while the coupling from the circuit to EM is realized by the port current calculated from circuit equations. Due to the local property of the DGTD, the resultant coupled EM–circuit system is very small. Hence, it can be solved with high efficiency even when nonlinear elements are included in the lumped network. The computational cost is further decreased by the LTS strategy when highly unstructured meshes are generated. To suppress the instability issue introduced by nonlinear elements, the standard iterative Newton–Raphson method is used to solve the coupled nonlinear system matrix. The proposed algorithm is validated by numerical benchmarks.

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REFERENCES


