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Optimal Bidding Strategy for Demand Response Aggregator in Day-Ahead Markets via Stochastic Programming and Robust Optimization

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Abstract—This paper evaluates the optimal bidding strategy for demand response (DR) aggregator in day-ahead (DA) markets. Because of constraint of minimum power quantity requirement, small-sized customers have to become indirect participants of electricity markets via the DR aggregator, who could offer various contracts accessing customers’ demand reduction capacity in advance. In day-ahead markets, DR aggregator schedules those contracts and submits accumulated DR offers to the system operator. The objective is to maximize the profit of the DR aggregator. The key element affecting the bidding decision and aggregator’s profit is the uncertain hourly DA prices. The stochastic programming adopts scenario-based approach for helping the profit-seeking DR aggregator control uncertainties. Robust optimization employs forecast values with bounded price intervals to address uncertainties while adjusting the robustness of the solution flexibly. Both scenarios can be modelled as mixed-integer linear programming (MILP) problems which could be solved by available solvers.

Index Terms—Day-ahead markets, demand response aggregator, price uncertainties, robust optimization, scenario-based stochastic programming.

I. INTRODUCTION

The advancement of smart grid technologies facilitates convenient and fast interaction among all electricity market participants and provides technical support for the participation of demand response (DR) in electricity markets [1]. The promulgation of some regulating policies, such as FERC order 719 [2] and American Recovery and Reinvestment Act [3], further promotes the role of DR as a power resource [4]. From a development perspective, DR can reduce generation cost, lower CO2 emissions, improve system efficiency and enhance social welfare [5].

Although in terms of technology and policy, DR has become an available power resource for consumer to access to markets, its application in small-sized customers still face challenges. The negligible individual load reduction capacity of small customers leaves them off from negotiating with the Independent System Operator (ISO). Meanwhile, in order to avoid a large scale of small DR owners, the ISO usually sets a minimal single DR bid requirement. For example, in Ireland and France, the thresholds are 4 MW and 25 MW respectively.

As new entities in electricity markets, DR aggregators act like brokers between DR resource owners and the ISO. From the customers’ perspective, DR aggregators are buyers who help them value the demand reduction and provide various contracts which customers can voluntarily choose whether to take part in or not [6]. From the ISO’s perspective, DR aggregators hold aggregated DR and submit offers in day-ahead (DA) markets equally as power generation companies do. The introduction of aggregators enriches the operation hierarchy of electricity market. In [7], the interaction of households, the system operator, and aggregators is investigated in a comprehensive hierarchical market structure. In [8], a noble multi-layered coupon incentive-based demand reduction scheme is proposed. But they focus on the scenario of clearing the DR dispatch through award-based mechanisms. In [9], loads are classified based on characteristics and corresponding utilities, customers can voluntarily participate in specific DR programs. For small customers, if they intend to involve more active participation in DA markets, a practice is to reach an agreement with an aggregator, and authorize the aggregator to access their DR capacity and bid in DA markets [10]. As for profit-seeking DR aggregators, the central concern is profit, which is greatly influenced by market clearing price. In existing literatures, the clearing price in DA markets could either be assumed as perfect forecasting information which has already considered influence of various market specification [10], or be regarded as uncertainties, as shown in [12]-[15].

In this paper, an optimal bidding strategy for a DR aggregator in DA markets is proposed. In the medium-run, the aggregator designs various contracts for small-sized customers. Once the aggregator reaches an agreement with small DR resource owners, the aggregator obtains the right of accessing contractual load reduction quantity through specific prices...
[10]. Then in the DA market, the aggregator compiles accessible DR and determines the optimal bid to be submitted to the ISO. The DR aggregator is to be assumed as price-taker, which implies the impact of participation of the aggregator in the market has been ignored. The objective of the proposed model is to maximize the profit (revenue from selling demand reductions to the ISO minus the cost of buying those demand reductions from contracted customers) of the DR aggregator. The cost is settled when the aggregator signs contracts with customers. Simultaneously, the aggregator makes bidding decision through integrating those contracts, and the revenue is settled at the DA market clearing price. The key factor affecting the profit is the uncertain DA prices. This paper aims to mitigate the impacts of DA price uncertainty on the final profit. Two approaches are considered to explore those uncertainties, i.e. stochastic programing and robust optimization. The scenario-based stochastic programming employs Monte Carlo simulation for maximizing expected profit for the upcoming day while considering conditional value-at-risk (CVaR) associated with uncertain DA hourly prices. Forecast prices with bounded price uncertainties are considered in the robust optimization model for maximizing revenue under the worst case while flexibly adjusting the robustness of the solution. To the best of our knowledge [11] is the only work that considers both stochastic optimization and robust optimization in DR, but in the perspective of residential appliances management.

The rest of this paper is organized as follows. In Section II, the problem of optimal bidding strategy for the price-taking DR aggregator is formulated. In Section III and Section IV, the scenario-based stochastic programming and robust optimization are applied to obtain the optimal solution respectively. Numerical results and discussion are presented in Section IV. Finally, conclusions are drawn in Section V.

II. PROBLEM STATEMENT

A. Objective of DR Aggregator

Various contracts signed between the aggregator and customers not only enroll customers in electricity markets but also qualify the aggregator to access DR capacity. Holding those contracts, the price-taking aggregator can schedule hourly demand reduction, submit offers to ISO and arbitrage in DA markets. The objective function, which aims at maximizing the DR aggregator’s profit in the DA market, is formulated as follows:

$$\max \sum_{t=1}^{24} (\pi_t^{DA} p_t^{LC} - c_t^{LC})$$

(1)

where $\pi_t^{DA}$ is the hourly wholesale electricity prices, $p_t^{LC}$ and $c_t^{LC}$ represent the hourly demand reduction quantity submitted to ISO and the cost of paying off contracted DR owners at hour $t$.

B. Demand Response Contracts

Generally, there are three DR actions: reduce electricity usage at critical periods without changing consumption patterns during other periods, shift some demands from some periods to others, and use on-site/distributed generation at critical intervals. In this paper, only the first action is considered, i.e. load curtailment (LC).

In LC contracts, the aggregator has the right to curtail the load of customers during specific periods without recovery. Each LC contract $r$ comprises the information of load curtailment price $\pi_t^{LC}$, the associated load curtailment quantity $p_t^{LC}$, minimum load curtailment duration $\text{Dur}_{t}^{LC}$, and maximum load curtailment duration $\text{Dur}_{t}^{LC}$ [10]. Contracted LC constraints are shown as follows:

$$p_t^{LC} = \sum_{r \in \text{NLC}} p_t^{LC}_{r} l_{r,t}^{LC}$$

(2)

$$c_t^{LC} = \sum_{r \in \text{NLC}} c_t^{LC}_{r} l_{r,t}^{LC}$$

(3)

$$m_{t}^{LC} + n_{t}^{LC} \leq 1 \quad \forall r, \forall t$$

(4)

$$\sum_{t=t}^{t+\text{Dur}_{r}^{LC}-1} l_{r,t}^{LC} \geq \text{Dur}_{r}^{LC} m_{t}^{LC} \quad \forall r, \forall t \in [1, 25 - \text{Dur}_{r}^{LC}]$$

(5)

$$m_{t}^{LC} = 0 \quad \forall r, \forall t \in [26 - \text{Dur}_{r}^{LC}, 24]$$

(6)

$$n_{t}^{LC} = 0 \quad \forall r, \forall t \in [1, \text{Dur}_{r}^{LC} - 1] \cup [25, 24 + \text{Dur}_{r}^{LC} - \text{Dur}_{r}^{LC}]$$

(7)

$$\sum_{t=1}^{24} l_{r,t}^{LC} \leq 1 \quad \forall r$$

(8)

$$n_{t}^{LC} - m_{t}^{LC} = l_{r,t}^{LC} - l_{r,(t-1)}^{LC} \quad \forall r, \forall t.$$ 

(10)

The total quantity of scheduled load curtailment and corresponding cost are shown in (2) and (3). $\text{NLC}$ is the set of LC contracts. $l_{r,t}^{LC}$ is a binary variable which is 1 if the $r^{th}$ contract is scheduled at hour $t$, 0 otherwise. $m_{r,t}^{LC}$ and $n_{r,t}^{LC}$ are binary variables indicating whether LC contract $r$ will start or stop at hour $t$. Constraint (4) guarantees that at most one of start and stop indicators would be 1 at hour $t$. Constraints (5)-(8) show the minimum and maximum operation duration limit. Constraint (9) enforces the LC contract can be scheduled one time at most. Constraint (10) administrates the relationship between start, stop and schedule binaries.

III. SCENARIO-BASED STOCHASTIC PROGRAMMING

It should be noted that the objective (1) is not properly formulated as DA prices $\pi_t^{DA}$ are uncertain. Auto-Regressive and Moving Average (ARMA) model and Monte Carlo simulation are adopted to generate a large number of price scenarios. Meanwhile, in order to reduce computation complexity, the scenario reduction method is used, which eliminates scenarios with very low probabilities and
aggregates those scenarios of close distances based on certain probability metric [16]. Finally, a set $\Omega$ of scenarios with specific probabilities $\rho(s)$ can be obtained. The sum of those probabilities is 1:

$$\sum_{\rho(s) \in \Omega} \rho(s) = 1 \quad (11)$$

Taking the uncertainty of DA prices into consideration, the objective (1) can be rewritten as:

$$\max \sum_{\rho(s) \in \Omega} \rho(s) \text{Prof}(s) \quad (12)$$

where the expected profit of the DR aggregator in scenario $s$ is:

$$\text{Prof}(s) = \sum_{t=1}^{24} \left[ \pi_t^{DA}(s) p_t^{LC} - C_t^{LC} \right] \quad (13)$$

The object (12) is a risk-neutral formulation, which maximizes the expected profit without considering the remaining parameters characterizing the distribution of profit. To formalize the uncertainty of DA prices, we can rewrite (12) as (13) and add a risk measure term, which is an auxiliary variable $\xi$ used to express the trade-off between expected profit and the variability of such profit [15]. Among various risk measures, conditional value at risk (CVaR) is adopted in this paper. After integrating CVaR into (12), the objective of the stochastic programming problem is finalized as:

$$\max \sum_{\rho(s) \in \Omega} \rho(s) \text{Prof}(s) + \alpha \left[ \xi - \frac{1}{1-\beta} \sum_{\rho(s) \in \Omega} \rho(s) \eta(s) \right] \quad (14)$$

where $\beta$ is the confidence level, $\xi$ is VaR, $\eta(s)$ is an auxiliary nonnegative variable which equals the difference between VaR and Profit(s) if VaR is greater than Prof(s), and equals 0 otherwise. The constraints of CVaR could be represented as:

$$\text{Prof}(s) - \xi + \eta(s) \geq 0, \quad \forall \rho(s) \in \Omega \quad (15)$$
$$\eta(s) \geq 0, \quad \forall \rho(s) \in \Omega \quad (16)$$

Besides (15) and (16), constraints associated to load curtailment (2)-(10) are also considered.

### IV. ROBUST OPTIMIZATION

The variable amount of scenario-based stochastic programming increases rapidly as the growth of the number of scenarios, possibly leading to dramatic increasing computation burden [15]. In comparison, the scale of robust optimization is small and fluctuates slightly. Rather than scenario-based stochastic programming which relies on scenarios with exact values, robust optimization models the DA price uncertainties via considering certain price intervals at specific confidence level:

$$\pi_t^{DA} \leq \pi_t^{DA} \leq \pi_t^{DA} \quad (17)$$

In robust optimization, the objective of expected profit maximization can be formulated as:

$$\max \sum_{t=1}^{24} \left[ \pi_t^{DA} p_t^{LC} - C_t^{LC} \right] + \gamma \tau + \sum_{t=1}^{24} \delta_t \quad (18)$$

subject to:

$$\gamma + \delta_t \geq \left( \pi_t^{DA} - \pi_t^{DA} \right) y_t \quad \forall t = 1, 2, \ldots, 24 \quad (19)$$
$$\delta_t \geq 0 \quad \forall t = 1, 2, \ldots, 24 \quad (20)$$
$$y_t \geq 0 \quad \forall t = 1, 2, \ldots, 24 \quad (21)$$
$$\gamma \geq 0 \quad (22)$$
$$y_t \geq p_t^{LC} \quad \forall t = 1, 2, \ldots, 24. \quad (23)$$

Robust problem (18) is obtained from the duality theory. $\gamma$ and $\delta_t$ are dual variables of corresponding deterministic problem. While $y_t$ is an auxiliary variable used to obtain equivalent linear expressions. Parameter $\Gamma$ indicates the robustness level of the model (18) reflecting the level of conservatism of the optimal solution. $\Gamma$ takes the real value between zero to 24. If $\Gamma=24$, that is the most conservative case, where all price deviations are considered, while if $\Gamma=0$, the influence of the hourly price deviations in (18) is ignored, which means it is the most optimistic case [17]. Similar to stochastic programming, in robust optimization, besides (17), (19)-(23), other constraints (2)-(10) concerning LC contracts also should be included.

### V. CASE STUDIES

In this section, based on historical data of Demark market from July 1st, 2012, to July 31st, 2012, ARMA model is used to forecast DA prices on August 1st, 2012 [18]. For stochastic programming, Monte Carlo simulation is used for scenario generation. For robust optimization, bounded price intervals are obtained using the method in [19] with a confidence level of 95%. The proposed optimization problem is mixed integer linear programming (MILP), which is solved by CPLEX 12.2 [20]. Data containing three types of LC contracts are listed in Table I. For example, the LC contract 3 allows the DR aggregator to curtail 10 MW of load for any successive 3 hour to 6 hours, and the price paid to customers is €48/MW.  

<table>
<thead>
<tr>
<th>Contract</th>
<th>Quantity $p_t^{LC}$ (MW)</th>
<th>Price $\pi_t^{LC}$ (€/MW)</th>
<th>Min LC duration Dur$_t^{LC}$ (h)</th>
<th>Max LC duration Dur$_t^{σ}$ (h)</th>
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<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>40</td>
<td>4</td>
<td>8</td>
</tr>
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<td>15</td>
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<td>3</td>
<td>15</td>
<td>48</td>
<td>3</td>
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The purpose of this study is to evaluate the participation of DR aggregator in DA markets with uncertain prices. Three cases are studied:

- **Case 1**: Scenario-based stochastic programming.
- **Case 2**: Robust optimization.
- **Case 3**: Perfect information.
A. Scenario-based stochastic programming

In this case, 500 scenarios are generated for simulating DA price uncertainties and reduced to 20 scenarios via scenario reduction technique, as depicted in Fig. 1. In general, it can be obtained that forecast DA prices in all scenarios are double-peaked, i.e. the first peak happens around hour 10 and the second peak happens around hour 19.

In Fig. 2, it shows the optimal LC schedule for a risk-neutral (\( \alpha = 0 \)) DR aggregator through stochastic programming. Although the trend of the price for the second peak (i.e. near hour 19) is higher than the first peak (i.e. near hour 10) in the majority of scenarios, prices at neighbor hours of hour 19 drop greatly. By contrast, prices at neighbor hours of hour 10 are forecasted greater than \( €\ 44/MW \) in the majority of scenarios.

B. Robust optimization

In this case, the robust optimization is employed to obtain the optimal bidding strategy for the DR aggregator. Figure 3 plots bounded price intervals and actual prices. The parameter \( \Gamma \) controls the sensitivity of the expected profit and the robustness level of the objective [15], which can be measured by the first term and the sum of last two terms respectively in (18).

Figure 4 shows the expected profit for different values of \( \Gamma \), and it can be easily observed that the optimal value of \( \Gamma \) leading to maximum expected profit is 16. In Fig. 4, at the beginning, as the increase of the value of \( \Gamma \), the expected profit increases. After the optimal \( \Gamma \), the expected profit fluctuates and gradually reaches stability. Figure 5 deficit the optimal LC schedule when \( \Gamma \) equals 16. Different from stochastic programming, contract 3 is scheduled at hour 17-20, and contract 1 and 2 are scheduled at hour 8-14 and hour 10-14.
C. Perfect Information

The DR aggregator is assumed knowing perfect information of DA prices in advance, which is the most ideal situation. With perfect information, the objective (1) becomes a deterministic problem, whose expected profit actually becomes the real profit, which is €1308.53. Following the optimal bidding strategy modelled via stochastic programming, robust optimization and perfect information, the price-taking DR aggregator submits an offer to ISO. Table II presents the actual profit for each approach when the DA market is cleared. In real operation, compared with perfect information, stochastic programming and robust optimization can help the DR aggregator earn €1291.73 and €1247.03 respectively. In term of profit, stochastic programming is closer to the ideal result. But considering the huge number of variables inherent in scenarios, robust optimization is more effective.

<table>
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<th>Stochastic Programming</th>
<th>Robust Optimization</th>
<th>Perfect Information</th>
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<tbody>
<tr>
<td>Actual Profit (€)</td>
<td>1291.73</td>
<td>1247.03</td>
<td>1308.53</td>
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</table>

VI. CONCLUSION

Because of the minimal single bid requirement in current DR programs, small customers have to sign contracts with DR aggregators to participate in electricity markets. Price-taking aggregators accumulate load reduction capacity and submit offers to ISO in day-ahead markets. The price uncertainties affect aggregators’ profit significantly. This paper evaluates two approaches to deal with the uncertainties, i.e. scenario-based stochastic programming and robust optimization. Both model MILP formulation and can be solved by state-of-art solvers. Numerical results show that both of two approaches can help DR aggregator promote the net profit. Although the decision via stochastic programming is closer to the decision under perfect information, its scenario-based characteristic may lead to a huge scale of variables. By contrast, robust optimization performs more balanced.

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