Ambiguity and the Value of Hedging *

Kit Pong Wong †

University of Hong Kong

March 2014

This paper examines the optimal production and hedging decisions of the competitive firm under price uncertainty when the firm's preferences exhibit smooth ambiguity aversion and an unbiased forward hedging opportunity is available. Ambiguity is modeled by a second-order probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. Ambiguity preferences are modeled by the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk. Within this framework, the separation and full-hedging theorems remain intact. Banning the firm from trading its output forward at the unbiased forward price has adverse effect on the firm's production decision. The firm finds the unbiased forward hedging opportunity more valuable in the presence than in the absence of ambiguity. Furthermore, the value of hedging increases when the firm's beliefs are more ambiguous, or when the firm becomes more ambiguity averse.

JEL classification: D21; D24; D81

Keywords: Ambiguity; Ambiguity aversion; Hedging; Production

*The author thanks Udo Broll, Bob Webb (the editor), and an anonymous referee for their helpful comments and suggestions. The usual disclaimer applies.

†Correspondence author, School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong. Tel: +852-2859-1044, Fax: +852-2548-1152, e-mail: kpwong@econ.hku.hk

Kit Pong Wong is a Professor of Finance and the Director of the School of Economics and Finance at the University of Hong Kong in Hong Kong.

Ambiguity and the Value of Hedging

This paper examines the optimal production and hedging decisions of the competitive firm under price uncertainty when the firm's preferences exhibit smooth ambiguity aversion and an unbiased forward hedging opportunity is available. Ambiguity is modeled by a second-order probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. Ambiguity preferences are modeled by the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk. Within this framework, the separation and full-hedging theorems remain intact. Banning the firm from trading its output forward at the unbiased forward price has adverse effect on the firm's production decision. The firm finds the unbiased forward hedging opportunity more valuable in the presence than in the absence of ambiguity. Furthermore, the value of hedging increases when the firm's beliefs are more ambiguous, or when the firm becomes more ambiguity averse.

INTRODUCTION

Since the seminal work of Holthausen (1979), there has been a large body of research on the production and hedging decisions of the competitive firm under price uncertainty *à la* Sandmo (1971). Two notable results emanate from this literature (Broll, 1992; Broll and Wong, 1999; Broll and Zilcha, 1992; Danthine, 1978; Feder et al., 1980; Wong, 2004, 2012, 2013). First, the separation theorem states that the firm's optimal output level depends neither on the risk attitude of the firm, nor on the incidence of the underlying price uncertainty should the firm be able to trade its output forward. Second, the full-hedging theorem states that the firm should fully hedge against its exposure to the price risk provided that the forward price is unbiased.[1]

Most of the extant models in the literature assume that the firm's preferences admit the standard von Neumann-Morgenstern expected utility representation. Such a modeling

[1] The full-hedging theorem is analogous to a well-known result in the insurance literature that a risk-averse individual fully insures at an actuarially fair price (Mossin, 1968).

approach rules out the possibility that the firm is unable to unambiguously assign a probability distribution that uniquely describes the price risk, which gives rise to ambiguity, or uncertainty in the sense of Knight (1921). Beginning with Ellsberg (1961), it has been well documented that ambiguity could lead to the violation of the independence axiom, which is responsible for the decision criterion being linear in the outcome probabilities. Individuals seem to prefer gambles with known rather than unknown probabilities, suggesting that they might be ambiguity averse. Indeed, ambiguity aversion has been confirmed in a variety of experimental settings (Chow and Sarin, 2001; Einhorn and Hogarth, 1986; Sarin and Weber, 1993), and in surveys of business owners and managers (Chesson and Viscusi, 2003; Viscusi and Chesson, 1999).

The purpose of this paper is to incorporate ambiguity into the model of the competitive firm under price uncertainty. Klibanoff et al. (2005) have recently developed a powerful decision criterion known as "smooth ambiguity aversion" that is compatible with ambiguity averse preferences under uncertainty (hereafter referred to as the KMM model). The KMM model features the recursive structure that is far more tractable in comparison to other models of ambiguity such as the pioneering maxmin expected utility (or multiple-prior) model of Gilboa and Schmeidler (1989).[2] Specifically, the KMM model represents ambiguity by a second-order probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. The KMM model then measures the firm's expected utility under ambiguity by taking the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk.[3] This recursive structure creates a crisp separation between ambiguity and ambiguity aversion, i.e., between beliefs and tastes, thereby making the conventional techniques used in the decision theory under uncertainty applicable in the context of ambiguity (Alary et al., 2013; Gollier, 2011; Snow, 2010, 2011; Taboga, 2005;

[2] See Lien (2000) and Lien and Wang (2003) who adopt the maxmin expected utility model of Gilboa and Schmeidler (1989) to examine the competitive firm's production and hedging decisions.

[3] Skiadas (2013) shows that smooth ambiguity preferences can be approximated by preferences admitting an expected utility representation in continuous-time or high-frequency models under Brownian or Poisson uncertainty.

Treich, 2010).

Within the context of the KMM model, the separation and full-hedging theorems are shown to be robust to the incorporation of ambiguity and ambiguity preferences. Banning the ambiguity-averse firm from trading its output forward at the unbiased forward price has the usual adverse effect on the firm's production decision. The unbiased forward hedging opportunity is shown to have higher value in the presence than in the absence of ambiguity, with more ambiguous beliefs, and with greater ambiguity aversion. The value of hedging as such increases when ambiguity and ambiguity preferences prevail.

The rest of this paper is organized as follows. The next section develops the KMM model of the competitive firm under price uncertainty when the firm can trade its output forward at the unbiased forward price. The subsequent section characterizes the production and hedging decisions of the ambiguity-averse firm. The penultimate section examines how the value of hedging is affected by the presence of ambiguity, by more ambiguous beliefs, and by greater ambiguity aversion. The final section concludes.

THE MODEL

Consider the competitive firm of Sandmo (1971) within the context of the KMM model. There is one period with two dates, 0 and 1. To begin, the firm produces a single commodity according to a deterministic cost function, $C(Q)$, where $Q \geq 0$ is the output level, and $C(Q)$ is compounded to date 1. The firm's production technology exhibits decreasing returns to scale so that $C(0) = C'(0) = 0$, and $C'(Q) > 0$ and $C''(Q) > 0$ for all $Q > 0$.

At date 1, the firm sells its entire output, Q, at the then prevailing per-unit price, $\tilde{P}$, which is not known ex ante.[4] The price risk, $\tilde{P}$, is distributed according to an objective cumulative distribution function, $H(P)$, over support $[\underline{P}, \overline{P}]$, where $0 < \underline{P} < \overline{P}$. The firm, however, is uncertain about $H(P)$ and thus faces ambiguity. Let $F(P|\theta)$ be the

[4]Throughout the paper, random variables have a tilde ($\tilde{}$) while their realizations do not.

firm's subjective cumulative distribution function of $\tilde{P}$ over support $[\underline{P}, \overline{P}]$, where θ is the realization of an unknown parameter, $\tilde{\theta}$. The KMM model represents ambiguity by a second-order subjective cumulative distribution function of $\tilde{\theta}$, $G(\theta)$, over support $[\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} < \overline{\theta}$, which captures the firm's uncertainty about which of the subjective cumulative distribution function, $F(P|\theta)$, governs the price risk, $\tilde{P}$. As in Snow (2010, 2011), the firm's ambiguous beliefs are assumed to be unbiased in the sense that the expected price risk is equal to the objective price risk:

$$\int_{\underline{\theta}}^{\overline{\theta}} F(P|\theta)\mathrm{d}G(\theta) = H(P), \tag{1}$$

for all $P \in [\underline{P}, \overline{P}]$.[5]

To hedge against the price risk, $\tilde{P}$, the firm can trade the commodity forward at the unbiased per-unit forward price, P^f, determined at date 0:

$$P^f = \int_{\underline{P}}^{\overline{P}} P\mathrm{d}H(P) = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{P}}^{\overline{P}} P\mathrm{d}F(P|\theta)\mathrm{d}G(\theta), \tag{2}$$

where the second equality follows from Equation (1). Let X be the number of units of the commodity sold (purchased if negative) forward by the firm at date 0. The firm's profit at date 1 is, therefore, given by $\tilde{\Pi} = \tilde{P}Q + (P^f - \tilde{P})X - C(Q)$. The firm possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its profit at date 1, Π, with $U'(\Pi) > 0$ and $U''(\Pi) < 0$, indicating the presence of risk aversion.

The recursive structure of the KMM model implies that the firm's expected utility under ambiguity can be computed in three steps. First, the firm's expected utility for each subjective cumulative distribution function of $\tilde{P}$ is calculated. Second, each (first-order) expected utility obtained in the first step is transformed by an increasing function, $\varphi(u)$, where u is the firm's utility level. Finally, the (second-order) expectation of the transformed expected utility obtained in the second step is taken with respect to the second-

[5]The assumption that the expected price risk is equal to the objective price risk is motivated by the premise that the behavior of an ambiguity-neutral decision maker should be unaffected by the introduction of, or changes in, ambiguity.

order subjective cumulative distribution function of $\tilde{\theta}$. The firm's ex-ante decision problem as such can be stated as

$$\max_{Q\geq 0, X} \int_{\underline{\theta}}^{\overline{\theta}} \varphi\bigg\{ \int_{\underline{P}}^{\overline{P}} U[PQ + (P^f - P)X - C(Q)]\mathrm{d}F(P|\theta)\bigg\}\mathrm{d}G(\theta). \tag{3}$$

Inspection of the objective function of program (3) reveals that the effect of ambiguity, represented by the cumulative distribution functions, $F(P|\theta)$ and $G(\theta)$, and the effect of ambiguity preferences, represented by the shape of the ambiguity function, $\varphi(u)$, can be separated and thus studied independently.

The firm is said to be ambiguity averse if, for any given pair of output level and forward position, (Q, X), the objective function of program (3) decreases when the firm's ambiguous beliefs, specified by $G(\theta)$, change in a way that induces a mean-preserving spread in the distribution of the firm's first-order expected utility. According to this definition, Klibanoff et al. (2005) show that ambiguity aversion implies concavity for $\varphi(u)$, and that a concave transformation of $\varphi(u)$ results in greater ambiguity aversion.[6] Throughout the paper, $\varphi(u)$ is assumed to satisfy that $\varphi'(u) > 0$ and $\varphi''(u) < 0$ so that the firm is ambiguity averse.

The first-order conditions for program (3) are given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{P}}^{\overline{P}} \varphi'\bigg\{ \int_{\underline{P}}^{\overline{P}} U[PQ^* + (P^f - P)X^* - C(Q^*)]\mathrm{d}F(P|\theta)\bigg\}$$

$$\times U'[PQ^* + (P^f - P)X^* - C(Q^*)][P - C'(Q^*)]\mathrm{d}F(P|\theta)\mathrm{d}G(\theta) = 0, \tag{4}$$

and

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{P}}^{\overline{P}} \varphi'\bigg\{ \int_{\underline{P}}^{\overline{P}} U[PQ^* + (P^f - P)X^* - C(Q^*)]\mathrm{d}F(P|\theta)\bigg\}$$

$$\times U'[PQ^* + (P^f - P)X^* - C(Q^*)](P^f - P)\mathrm{d}F(P|\theta)\mathrm{d}G(\theta) = 0, \tag{5}$$

[6]When $\varphi(u) = [1 - \exp(-\alpha u)]/\alpha$, Klibanoff et al. (2005) show that the maxmin expected utility model of Gilboa and Schmeidler (1989) is the limiting case as the constant absolute ambiguity aversion, α, approaches infinity under some conditions.

where Q^* and X^* are the firm's optimal output level and forward position, respectively. The second-order conditions for program (3) are satisfied given that $U''(\Pi) < 0$, $C''(Q) > 0$, and $\varphi''(u) < 0$.

THE EFFECT OF HEDGING

To solve Equations (4) and (5) simultaneously, suppose that the firm adopts a full-hedge, i.e., $X^* = Q^*$. Then, Equation (5) becomes

$$\varphi'\{U[P^f Q^* - C(Q^*)]\}U'[P^f Q^* - C(Q^*)]\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{P}}^{\overline{P}}(P^f - P)\mathrm{d}F(P|\theta)\mathrm{d}G(\theta) = 0, \qquad (6)$$

which holds given Equation (2). Hence, Equation (6) implies that $X^* = Q^*$ is indeed a solution to program (3). Substituting $X^* = Q^*$ into Equation (4) yields

$$\varphi'\{U[P^f Q^* - C(Q^*)]\}U'[P^f Q^* - C(Q^*)][P^f - C'(Q^*)] = 0, \qquad (7)$$

which follows from Equation (2). Hence, Equation (7) implies that Q^* solves $C'(Q^*) = P^f$. The following proposition summarizes these results.

Proposition 1. If the ambiguity-averse competitive firm can trade the commodity forward at the unbiased per-unit forward price, P^f, the firm's optimal output level, Q^* solves $C'(Q^*) = P^f$, and its optimal forward position is a full-hedge, i.e., $X^* = Q^*$.

The intuition for Proposition 1 is as follows. Since the firm can always sell the last unit of its output forward at the forward price, P^f, the usual optimality condition applies in that the marginal cost of production, $C'(Q^*)$, must be equated to the known marginal revenue, P^f, which determines the optimal output level, Q^*. Since P^f is unbiased, the firm, being risk averse, indeed finds it optimal to adopt a full-hedge, i.e., $X^* = Q^*$, that completely eliminates the price risk, $\tilde{P}$. Proposition 1 as such extends the celebrated separation and full-hedging theorems to the case of smooth ambiguity preferences.

Since the separation theorem holds, the firm's output level is fixed at Q^*. In this case, the firm's hedging problem is akin to a standard portfolio choice problem wherein the investment in the risky asset is equal to the unhedged position, $Q^* - X$, while the investment in the riskless asset is equal to the forward position, X. Gollier (2011) examines the comparative statics of ambiguity aversion on portfolio choices and asset prices. He shows that the demand for the risky asset under ambiguity aversion is positive, zero, or negative, depending on whether the equity premium is positive, zero, or negative, respectively. In the hedging context, the equity premium is simply equal to the expected output price net of the forward price. It follows immediately from the results of Gollier (2011) that the firm optimally opts for an under-hedge ($Q^* - X^* > 0$), a full-hedge ($Q^* - X^* = 0$), or an over-hedger ($Q^* - X^* < 0$), if the expected output price is greater than, equal to, or less than the forward price, respectively, which are consistent with the results of Proposition 1. Indeed, the comparative static results of Gollier (2011) are directly applicable to the firm's optimal forward position when the forward price is biased, as is shown in Iwaki and Osaki (2012).

To examine the effect of hedging on production, suppose that the firm is banned from trading the commodity forward so that $X \equiv 0$. In this case, the firm's optimal output level, Q°, solves the following first-order condition:

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{P}}^{\overline{P}} \varphi' \Big\{ \int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta) \Big\}$$

$$\times U'[PQ^\circ - C(Q^\circ)][P - C'(Q^\circ)]\mathrm{d}F(P|\theta)\mathrm{d}G(\theta) = 0. \tag{8}$$

The following proposition shows that $Q^\circ < Q^*$.

Proposition 2. Banning the ambiguity-averse competitive firm from trading the commodity forward at the unbiased per-unit forward price, P^f, results in a reduction in the optimal output level, i.e., $Q^\circ < Q^*$.

Proof. Partially differentiating the objective function of program (3) with respect to Q, and evaluating the resulting derivative to zero, yields the optimal output level, $Q(X)$, for a given forward position, X:

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{P}}^{\overline{P}} \varphi'\Big\{ \int_{\underline{P}}^{\overline{P}} U\{PQ(X) + (P^f - P)X - C[Q(X)]\}\mathrm{d}F(P|\theta)\Big\}$$

$$\times U'\{PQ(X) + (P^f - P)X - C[Q(X)]\}\{P - C'[Q(X)]\}\mathrm{d}F(P|\theta)\mathrm{d}G(\theta) = 0. \tag{9}$$

Equations (4) and (9) imply that $Q^* = Q(X^*)$, and Equations (8) and (9) imply that $Q^\circ = Q(0)$. Using Equation (9) to totally differentiate the objective function of program (3) with respect to X, and evaluating the resulting derivative at $X = 0$ so that $Q(0) = Q^\circ$, yields

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{P}}^{\overline{P}} \varphi'\Big\{ \int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\Big\}$$

$$\times U'[PQ^\circ - C(Q^\circ)](P^f - P)\mathrm{d}F(P|\theta)\mathrm{d}G(\theta) > 0, \tag{10}$$

where the inequality follows from the fact that $X^* = Q^* > 0$, Equation (5), and the second-order conditions for program (3). Substituting Equation (8) into Equation (10) yields $C'(Q^\circ) < P^f$. It then follows from $C''(Q) > 0$ that $Q^\circ < Q^*$. □

To see the intuition for Proposition 2, recast Equation (8) as[7]

$$C'(Q^\circ) = P^f + \frac{\mathrm{Cov}\Big\{\varphi'\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\tilde{\theta})\Big\}U'[\tilde{P}Q^\circ - C(Q^\circ)], \tilde{P}\Big\}}{\mathrm{E}\Big\{\varphi'\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\tilde{\theta})\Big\}U'[\tilde{P}Q^\circ - C(Q^\circ)]\Big\}}, \tag{11}$$

where P^f is given by Equation (2), and $\mathrm{E}(\cdot)$ and $\mathrm{Cov}(\cdot,\cdot)$ are the expectation and covariance operators with respect to the joint cumulative distribution function of $\tilde{\theta}$ and $\tilde{P}$. Equation (11) states that the firm's optimal output level, Q°, is the one that equates the marginal cost of production, $C'(Q^\circ)$, to the certainty equivalent output price that takes the firm's

[7]For any two random variables, $\tilde{X}$ and $\tilde{Y}$, it is true that $\mathrm{Cov}(\tilde{X},\tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

smooth ambiguity preferences into account. Indeed, the second term on the right-hand side of Equation (11) captures the price risk premium, which must be negative since the firm optimally sells its output forward, i.e., $X^* > 0$, at the unbiased forward price, P^f, thereby rendering $C'(Q^\circ) < P^f$. It then follows from the strict convexity of the cost function that $Q^\circ < Q^*$.

THE VALUE OF HEDGING

If the firm can trade the commodity forward at the unbiased per-unit forward price, P^f, Proposition 1 shows that the firm optimally opts for a full-hedge and thus its profit at date 1 is certain at $P^fQ^* - C(Q^*)$. Since $X^* > 0$, the firm must attain a higher value of its objective function of program (3) than in the case that trading the commodity forward at the unbiased forward price, P^f, is prohibited:

$$\varphi\{U[P^fQ^* - C(Q^*)]\} > \int_{\underline{\theta}}^{\overline{\theta}} \varphi\bigg\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\bigg\}\mathrm{d}G(\theta). \tag{12}$$

Let W° be the solution to

$$\varphi\{U[P^fQ^* - C(Q^*) - W^\circ]\} = \int_{\underline{\theta}}^{\overline{\theta}} \varphi\bigg\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\bigg\}\mathrm{d}G(\theta). \tag{13}$$

Equations (12) and (13) imply that $W^\circ > 0$, which captures the firm's willingness to pay to possess the unbiased forward hedging opportunity.

When the firm faces no ambiguity, i.e., $F(P|\theta) = H(P)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and $P \in [\underline{P}, \overline{P}]$, Proposition 1 remains intact so that the firm's profit at date 1 is certain at $P^fQ^* - C(Q^*)$. Let $Q^\dagger$ be the firm's optimal output level in the absence of ambiguity and forward hedging opportunity. The firm's willingness to pay to possess the unbiased forward hedging opportunity in this case, $W^\dagger > 0$, is the solution to

$$\varphi\{U[P^fQ^* - C(Q^*) - W^\dagger]\} = \varphi\bigg\{\int_{\underline{P}}^{\overline{P}} U[PQ^\dagger - C(Q^\dagger)]\mathrm{d}H(P)\bigg\}. \tag{14}$$

Comparing W° and $W^\dagger$ yields the following proposition.

Proposition 3. The presence of ambiguity raises the ambiguity-averse competitive firm's willingness to pay to possess the unbiased forward hedging opportunity, i.e., $W^\circ > W^\dagger$.

Proof. Since $\varphi''(u) < 0$, Jensen's inequality implies that

$$\int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta)$$

$$< \varphi\Big\{\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\mathrm{d}G(\theta)\Big\}$$

$$= \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}H(P)\Big\}, \tag{15}$$

where the equality follows from Equation (1). Since $Q^\dagger$ is the optimal output level in the absence of ambiguity and unbiased forward hedging opportunity, it must be true that

$$\varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\dagger - C(Q^\dagger)]\mathrm{d}H(P)\Big\} > \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}H(P)\Big\}. \tag{16}$$

Hence, Equations (15) and (16) imply that

$$\int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta) < \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\dagger - C(Q^\dagger)]\mathrm{d}H(P)\Big\}. \tag{17}$$

It then follows from Equations (13), (14), and (17) that $W^\circ > W^\dagger$. $\square$

The intuition for Proposition 3 is as follows. The presence of the unbiased forward hedging opportunity allows the firm to lock in the price risk, $\tilde{P}$, at the forward price, P^f, thereby completely eliminating the price risk. Ambiguity as such is also eliminated, which creates additional benefit to the ambiguity-averse firm that faces ambiguity. Hence, the firm finds it more valuable to possess the unbiased forward hedging opportunity in the presence than in the absence of ambiguity, rendering that $W^\circ > W^\dagger$.

According to Jewitt and Mukerji (2014), the firm's beliefs are said to be more ambiguous if the firm is made worse off (indifferent) as a result of the change in beliefs when the firm is ambiguity averse (neutral). This definition of greater ambiguity is akin to the Rothschild-Stiglitz (1970) notion of mean-preserving-spread increases in risk. Let $\hat{F}(P|\theta)$ and $\hat{G}(\theta)$ be the cumulative distribution functions when the firm's beliefs are more ambiguous in the sense of Jewitt and Mukerji (2014). Let $\hat{Q}^{\circ}$ be the firm's optimal output level with more ambiguous beliefs and with no hedging. In this case, the firm's willingness to pay to possess the unbiased forward hedging opportunity, $\hat{W}^{\circ} > 0$, is the solution to

$$\varphi\{U[P^f Q^* - C(Q^*) - \hat{W}^{\circ}]\} = \int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[P\hat{Q}^{\circ} - C(\hat{Q}^{\circ})]\mathrm{d}\hat{F}(P|\theta)\Big\}\mathrm{d}\hat{G}(\theta), \tag{18}$$

since Proposition 1 remains intact. Comparing W° and $\hat{W}^{\circ}$ yields the following proposition.

Proposition 4. Greater ambiguity raises the ambiguity-averse competitive firm's willingness to pay to possess the unbiased forward hedging opportunity, i.e., $\hat{W}^{\circ} > W^{\circ}$.

Proof. Note that

$$\begin{aligned}
&\int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^{\circ} - C(Q^{\circ})]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta) \\
&> \int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[P\hat{Q}^{\circ} - C(\hat{Q}^{\circ})]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta) \\
&> \int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[P\hat{Q}^{\circ} - C(\hat{Q}^{\circ})]\mathrm{d}\hat{F}(P|\theta)\Big\}\mathrm{d}\hat{G}(\theta),
\end{aligned} \tag{19}$$

where the first inequality follows from the fact that Q° is the firm's optimal output level with the initial ambiguous beliefs and with no hedging, and the second inequality follows from the fact that $\hat{F}(P|\theta)$ and $\hat{G}(\theta)$ are the cumulative distribution functions when the firm's beliefs are more ambiguous in the sense of Jewitt and Mukerji (2014). Hence, Equations (13), (18), and (19) imply that $\hat{W}^{\circ} > W^{\circ}$. $\square$

Propositions 3 and 4 show that the global effect and the marginal effect of ambiguity on the firm's willingness to pay to possess the unbiased forward hedging opportunity are consistent with each other.

Klibanoff et al. (2005) show that the firm becomes more ambiguity averse when $\varphi(u)$ is replaced by $K[\varphi(u)]$ in the objective function of program (3), where $K(\cdot)$ satisfies that $K'(\cdot) > 0$ and $K''(\cdot) < 0$. Let $Q^\diamond$ be the optimal output level when the firm's smooth ambiguity preferences are represented by $K[\varphi(u)]$ and the firm is banned from trading the commodity forward at the unbiased forward price, P^f. In this case, the willingness to pay by this more ambiguity-averse firm to possess the unbiased forward hedging opportunity, $W^\diamond > 0$, is the solution to

$$K\Big\{\varphi\{U[P^f Q^* - C(Q^*) - W^\diamond]\}\Big\}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} K\Big\{\varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\diamond - C(Q^\diamond)]\mathrm{d}F(P|\theta)\Big\}\Big\}\mathrm{d}G(\theta), \tag{20}$$

since Proposition 1 remains intact. Comparing W° and $W^\diamond$ yields the following proposition.

Proposition 5. Greater ambiguity aversion raises the ambiguity-averse competitive firm's willingness to pay to possess the unbiased forward hedging opportunity, i.e., $W^\diamond > W^\circ$.

Proof. Since $K''(\cdot) < 0$, Jensen's inequality implies that

$$\int_{\underline{\theta}}^{\overline{\theta}} K\Big\{\varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\diamond - C(Q^\diamond)]\mathrm{d}F(P|\theta)\Big\}\Big\}\mathrm{d}G(\theta)$$

$$< K\Big\{\int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\diamond - C(Q^\diamond)]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta)\Big\}. \tag{21}$$

Since Q° is the optimal output level of the less ambiguity-averse firm in the absence of forward hedging opportunity, it must be true that

$$\int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{\int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta)$$

$$> \int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{ \int_{\underline{P}}^{\overline{P}} U[PQ^\diamond - C(Q^\diamond)]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta). \tag{22}$$

Hence, Equations (21) and (22) imply that

$$\int_{\underline{\theta}}^{\overline{\theta}} K\Big\{\varphi\Big\{ \int_{\underline{P}}^{\overline{P}} U[PQ^\diamond - C(Q^\diamond)]\mathrm{d}F(P|\theta)\Big\}\Big\}\mathrm{d}G(\theta)$$

$$< K\Big\{ \int_{\underline{\theta}}^{\overline{\theta}} \varphi\Big\{ \int_{\underline{P}}^{\overline{P}} U[PQ^\circ - C(Q^\circ)]\mathrm{d}F(P|\theta)\Big\}\mathrm{d}G(\theta)\Big\}. \tag{23}$$

It then follows from Equations (13), (20), and (23) that $W^\diamond > W^\circ$. $\square$

The intuition for Proposition 5 is as follows. When the firm is more ambiguity averse, the incentive to remove ambiguity becomes stronger. The firm's willingness to pay to possess the unbiased forward hedging opportunity as such increases in a systematic manner with greater ambiguity aversion.

CONCLUSION

This paper examines the production and hedging decisions of the competitive firm under price uncertainty *à la* Sandmo (1971) when the firm's preferences exhibit smooth ambiguity aversion developed by Klibanoff et al. (2005), and an unbiased forward hedging opportunity is available. According to Klibanoff et al. (2005), ambiguity is modeled by a second-order probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. On the other hand, ambiguity preferences are modeled by the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk. Within this framework, the separation and full-hedging theorems are shown to be robust to the incorporation of ambiguity and ambiguity preferences. Furthermore, the ambiguity-averse firm is shown to produce more with than without the unbiased forward hedging opportunity.

The firm also values the unbiased forward hedging opportunity more in the presence than in the absence of ambiguity, with more ambiguous beliefs, and with greater ambiguity aversion. The value of hedging as such increases when ambiguity and ambiguity preferences prevail.

BIBLIOGRAPHY

Alary, D., Gollier, C., & Treich, N. (2013). The effect of ambiguity aversion on insurance and self-production. Economic Journal, 123, 1188–1202.

Broll, U. (1992). The effect of forward markets on multinational firms. Bulletin of Economic Research, 44, 233–240.

Broll, U., & Wong, K. P. (1999). Hedging with mismatched currencies. The Journal of Futures Markets, 19, 859–875.

Broll, U., & Zilcha, I. (1992). Exchange rate uncertainty, futures markets and the multinational firm. European Economic Review, 36, 815–826.

Chesson, H. W., & Viscusi, W. K. (2003). Commonalities in time and ambiguity aversion for long-term risks. Journal of Risk and Uncertainty, 24, 57–71.

Chow, C. C., & Sarin, R. K. (2001). Comparative ignorance and the Ellsberg Paradox. Journal of Risk and Uncertainty, 22, 129–139.

Danthine, J.-P. (1978). Information, futures prices, and stabilizing speculation. Journal of Economic Theory, 17, 79–98.

Einhorn, H. J., & Hogarth, R. M. (1986). Decision making under ambiguity. Journal of Business, 59, S225–S250.

Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. Quarterly Journal of Economics, 75, 643–669.

Feder, G., Just, R. E., & Schmitz, A. (1980). Futures markets and the theory of the firm under price uncertainty. Quarterly Journal of Economics, 94, 317–328.

Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with a non-unique prior. Journal of Mathematical Economics, 18, 141–153.

Gollier, C. (2011). Portfolio choices and asset prices: the comparative statics of ambiguity aversion. Review of Economic Studies, 78, 1329–1344.

Holthausen, D. M. (1979). Hedging and the competitive firm under price uncertainty. American Economic Review, 69, 989–995.

Iwaki, H., & Osaki, Y. (2012). Production and hedging decisions under smooth ambiguity aversion. Working paper, Faculty of Business Administration, Kyoto Sangyo University, Japan.

Jewitt, I., & Mukerji, S. (2014). Ordering ambiguous acts. Working paper, Department of Economics, Oxford University, UK.

Klibanoff, P., Marinacci, M., & Mukerji, S. (2005). A smooth model of decision making under ambiguity. Econometrica, 73, 1849–1892.

Knight, F. H. (1921). Risk, uncertainty and profit. Boston: Houghton Mifflin.

Lien, D. (2000). Production and hedging under Knightian uncertainty. The Journal of Futures Markets, 20, 397–404.

Lien, D., & Wang, Y. (2003). Futures market equilibrium under Knightian uncertainty. The Journal of Futures Markets, 23, 701–718.

Mossin, J. (1968). Aspects of rational insurance purchasing. Journal of Political Economy, 76, 553–568.

Rothschild, M., & Stiglitz, J. E. (1970). Increasing risk: I. a definition. Journal of Economic Theory, 2, 225–243.

Sandmo, A. (1971). On the theory of the competitive firm under price uncertainty. American Economic Review, 61, 65–73.

Sarin, R. K., & Weber, M. (1993). Effects of ambiguity in market experiments. Management Science, 39, 602–615.

Skiadas, C. (2013). Smooth ambiguity aversion toward small risks and continuous-time recursive utility. Journal of Political Economy, 121, 775–792.

Snow, A. (2010). Ambiguity and the value of information. Journal of Risk and Uncertainty, 40, 133–145.

Snow, A. (2011). Ambiguity aversion and the propensities for self-insurance and self-protection. Journal of Risk and Uncertainty, 42, 27–43.

Taboga, M. (2005). Portfolio selection with two-stage preferences. Finance Research Letters, 2, 152–164.

Treich, N. (2010). The value of a statistical life under ambiguity aversion. Journal of Environmental Economics and Management, 59, 15–26.

Viscusi, W. K., & Chesson, H. W. (1999). Hopes and fears: the conflicting effects of risk ambiguity. Theory and Decision, 47, 153–178.

Wong, K. P. (2004). Hedging, liquidity, and the competitive firm under price uncertainty. The Journal of Futures Markets, 24, 697–706.

Wong, K. P. (2012). Production and hedging under state-dependent preferences. The Journal of Futures Markets, 32, 945–963.

Wong, K. P. (2013). Cross hedging with currency forward contracts. The Journal of Futures Markets, 33, 653–674.