

Mathematics: What Has It To Do With Me?¹

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Synopsis

Mathematics teachers must have encountered the following question raised by students: “What is the use of mathematics?” Although the value of mathematics is not to be determined solely by its applications, to the general public this is a more important and more convincing facet of the subject. Nevertheless, this also brings up the corresponding query: is this subject being properly used? Does mathematics play a role in moral education?

Most people would not deny the importance of mathematics in scientific research and technological development, including the fields of social sciences and business management. The irony lies in that whenever they see accomplishments in these areas many tend to forget the role played by mathematics. Allen Hammond, an editor of *Science*, once described mathematics as “our invisible culture” [3]. The American mathematician Paul Richard Halmos (1916-2006) even complained in a lecture titled “Mathematics as a creative art” delivered at the University of Illinois in 1967 [2]: “It saddens me that educated people don’t even know that my subject exists.” Certainly that does not really mean the majority of educated people do not know that there is something called mathematics; it simply shows that most people do not understand what mathematics is about. Most educated people will know what

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sculptures, paintings are without being artists; they will know what songs, melodies are without being musicians; they will know what poetry, literature are without being men of letters; they will know what nuclear energy, protein, microbes, constellations are without being scientists. However, how many will know what functions, axiomatic approach, commutative groups, manifolds are without being mathematicians? Furthermore, many people feel ashamed in admitting that they are ignorant in arts, music, literature, science; yet feel totally unashamed in saying that they know nothing about mathematics, and even think that it is just natural to be ignorant in it. Even if they do not appear to be delighted when admitting frankly their ignorance in mathematics, they do so with full complacency.

In fact, we are basically living in a world of mathematics. Mathematics is “present in our surroundings, affecting our daily lives directly or indirectly. Its past, present, or, we can even predict, its future, will always be fascinating and exciting” [6]. People’s misunderstanding of mathematics can roughly be described in several dichotomies: some think mathematics is boring and dull or tedious and incomprehensible, while others think it is a fascinating game for the mind; some think mathematics is a study which has nothing to do with the real world, while others think it is a panacea that can tackle all sorts of problems; some think that the kind of mathematics which has no direct application is merely a hair-splitting game, while others think that only pure theory is worthy of the good name of mathematics. What do you think about these views?

Mathematics teachers must have encountered the following question raised by students: “What is the use of mathematics?” Mathematics is used and applied to areas which are vast as well as deep. The Chinese mathematician Hua Luogeng (Hua Lo-Keng 1910-1985) offered his answer in a nice way in an article published in the Chinese newspaper *Peoples’ Daily* (dated May 28, 1959):

The immenseness of the universe, the minuteness of elementary particles, the swiftness of rockets, the ingenuity of chemical engineering, the changes of the earth, the mystery of living things, the variety of uses in daily life, all these involve the use of mathematics.

In an article published in 2003 I wrote the following passage [7]:

Indeed, the value of mathematics is not to be determined solely by its applications in daily life. However, to the general public this is a more important and more convincing facet of mathematics. Nevertheless, this also brings up the corresponding query: is this subject being properly used? The English mathematician Godfrey Harold Hardy (1877-1947) wrote some years before his passing the book A Mathematician's Apology that was published in 1940. In between lines one senses the desolate mood of the author who felt he was past his prime. On the other hand, he was pleased with himself in having chosen a very 'pure' subject, the theory of numbers, as his study. He said, "Real mathematics has no effects on war. No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems very unlikely that anyone will do so for many years. [...] So a real mathematician has his conscience clear; there is nothing to be set against any value his work may have; mathematics is, as I said at Oxford, a 'harmless and innocent' occupation." However, we should not acquire hastily an idea that Hardy was proud of the uselessness of mathematics, or even thought that any application of mathematics will tarnish the purity of it. If you want to understand better what Hardy meant, you should read his book, and should read with particular care sections 21 and 22 [4]. Furthermore, we should also note the historical context of the period when this book was written. Europe was at war; the tragic Second World War had begun not very long ago. Had Hardy lived thirty more years, he would have known that the theory of numbers is not so 'harmless and innocent'—cryptography used for military purpose makes use of the seemingly useless knowledge of the theory of numbers! Like all other scientists, mathematicians have no way to stay aloof; like all scientific achievements, mathematical achievements can bring benefit as well as harm to human beings.

Cryptography can be used not only for military purposes but also for civilian purposes. Today, cryptography is used by banks and other financial institutions in electronic transactions. Thus, Hardy did not need to harbour excessive guilt. Still mathematics is not always used properly, and can even be said to be used in an immoral way. I read the following on a local Chinese newspaper *Ming Pao* (dated July 11, 1999) some years ago.

The American car manufacturer General Motors Corporation once produced a car model which had its fuel tank installed near to the rear bumper. When the car was hit an explosion could easily result. In 1993 there was a car accident which led to an explosion with six passengers seriously burnt. Later, a victim brought the case to court and the verdict was that General Motors should pay 4.9 billion dollars as indemnity. The prosecutor pointed out that in fact the car manufacturer knew well beforehand that there was a flaw in the design. However, a confidential unannounced cost-benefit analysis of the company gave the following result: if the company retrieved all the five million defected cars from the market for modification, it had to pay 8.59 dollars for each car; if it did not fix the problem, even if there was an accident, the expected indemnity for each car was estimated to be 2.40 dollars. To save the difference of approximately 6 dollars for each car the company did not retrieve the cars for modification. Eventually, an accident happened and the company had to pay a sum equivalent to 980 dollars for each car as indemnity! The main point does not lie in the fact that in order to save 6 dollars the company had to pay 980 dollars. The important point lies in that human life is so precious that the value cannot be calculated nonchalantly as an expected value in probability theory.

This reminds me of the English novel *Hard Times* written in 1854 by Charles Dickens. There is an episode of the little girl Sissy who complained to her mistress Louisa that she could not learn well the subject of mathematics [1, page 45]:

‘[...] and that’s another of my mistakes—of accidents upon the sea. And I find (Mr. M’Choakumchild said) that in a given time a hundred thousand persons went to sea on long voyages, and only five hundred of them were drowned or burnt to death. What is the percentage? And I said Miss;’ here Sissy fairly sobbed as confessing with extreme contrition to her greatest error; *‘I said it was nothing.’*

‘Nothing, Sissy?’

‘Nothing, Miss—to the relations and friends of the people who were killed. I shall never learn,’ said Sissy. *‘And the worst of all is, that although my poor father wished me so much to learn, and although I am so anxious to learn because he wished me to, I am afraid I don’t like it.’*

To many mathematics teachers, mathematics is a “neutral” school subject, regarded only as a useful tool. Unlike other subjects such as literature, social studies, economics etc., mathematics does not seem to involve any moral value. Perhaps many may think in this way, but we need to listen to what pupils say in their hearts.

In the above we mentioned the situation when mathematics is not used properly. However, mathematics can also contribute to moral education imperceptibly. The scholar official Xu Guang-Qi (徐光啟 1562-1633) of the Ming Dynasty in China, together with the Italian Jesuit Matteo Ricci (利瑪竇 1552-1610), translated the first six books of Euclid’s *Elements*, which was published in 1607. Xu had the following opinion on the value of the book [9]:

The benefit derived from studying this book is many. It can dispel shallowness of those who learn the theory and improve their concentration. It can supply fixed methods for those who apply to practice and kindle their creative thinking. Therefore everyone in this world should study this book.

He continued to say [9]:

*Five categories of personality will not learn from this book [the *Elements*]: those who are impetuous, those who are thoughtless, those who are complacent, those who are envious, and those who are arrogant. Thus to learn from this book one not only strengthens ones intellectual capacity but also builds a moral base.*

The Russian mathematics educator Igor Fedorovich Sharygin (1937-2004), who was very fond of geometry, once said [8]:

Geometry is a phenomenon of the human culture. [...] Geometry, as well as mathematics in general, helps in moral and ethical education of children. [...] Geometry develops mathematical intuition, introduces a person to independent mathematical creativity. [...] Geometry is a point of minimum for the distance between school mathematics and the mathematics of high level.

He also said [5]:

Learning mathematics builds up our virtues, sharpens our sense of justice and our dignity, strengthens our innate honesty and our principles. The life of mathematical society is based on the idea of proof, one of the most highly moral ideas in the world.

Nowadays, how many mathematics teachers still hold on to such belief when they teach?

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