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Emission mechanism of GeV-quiet soft gamma-ray pulsars; A case for peculiar geometry?

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ABSTRACT

There is a growing new class of young spin-down powered pulsars called GeV-quiet soft gamma-ray pulsar; (1) spectral turnover appears around 10MeV, (2) the X-ray spectra of below 20 keV can be described by power law with photon index around 1.2 and (3) the light curve in X-ray/soft gamma-ray bands shows single broad pulse. Their emission properties are distinct from the normal gamma-ray pulsars, for which the spectral peak in \( \nu F_\nu \) appears in GeV energy bands and the X-ray/gamma-ray light curves show sharp and double (or more) peaks. In this paper, we discuss that X-ray/soft gamma-ray emissions of the GeV-quiet soft gamma-ray pulsars are caused by the synchrotron radiation of the electron/positron pairs, which are created by the magnetic pair-creation process near the stellar surface. In our model, the viewing geometry is crucial factor to discriminate between the normal gamma-ray pulsars and soft gamma-ray pulsars. Our model suggests that the difference between the magnetic inclination angle (\( \alpha \)) and the Earth viewing angle (\( \beta \)) of the soft gamma-ray pulsars is small, so that the synchrotron emissions from the high magnetic field region around the polar cap region dominates in the observed emissions. Furthermore, the inclination angle of the soft gamma-ray pulsar is relatively small, \( \alpha \leq 30 \) degree, and our line of sight is out of the gamma-ray beam emitted via the curvature radiation process in the outer gap. We also analysis the six year Fermi data for four soft gamma-ray pulsars to determine the upper limit of the GeV flux.

Key words:
1 INTRODUCTION

The *Fermi* gamma-ray telescope has discovered about 150 $\gamma$-ray pulsars. The *Fermi* revealed that the pulsars with high-spin down power emit the GeV gamma-rays and the typical gamma-ray spectra are described by the single power law plus exponential cut-off function with a cut-off energy $\sim \text{GeV}$. It is now widely accepted that the GeV gamma-ray emission region locates in outer magnetosphere near the light cylinder, where the co-rotation speed with the pulsar becomes the speed of light (Aliu et al. 2008; Abdo et al. 2010b).

The *soft gamma-ray pulsar* is growing new class of young spin-down powered pulsars that are observed in the non-thermal X-rays and soft gamma-ray bands (Kuiper & Hermsen, 2013, 2014). These soft gamma-ray pulsars are divided into two groups, that is, GeV-loud (e.g. Crab and Vela pulsars) and GeV-quiet. Currently, six GeV-quiet soft gamma-ray pulsars (hereafter, GeV-quiet SGPSRs) have been known; PSRs B1509-58, J1617-5055, J1811-1925, J1838-0655, J1846-02658 and J1930+1852. Figure 1 summarizes the spin down power, characteristic age and spectral characteristics of the radio pulsars (small-dots), *Fermi*-LAT pulsars (filled-boxes) and GeV-quiet SGPSRs (filled-circles). We can see in Figure 1 that GeV-quiet SGPSRs have a relatively large spin down power and small characteristics age. Furthermore, we find in Figure 1 that the spectral properties of GeV-quiet SGPSRs are distinct from those of the *Fermi*-LAT pulsars, that is, the weaker gamma-ray emissions but a stronger X-ray emissions comparing with the *Fermi*-LAT pulsars. In fact, all of GeV-quiet SGPSRs show (1) no GeV emissions and (2) a single broad light curves in X-ray/soft gamma-ray bands. The original one, PSR B1509-58, was firstly recognized as the Crab-type pulsar (Ulmer et al. 1993), since the spectral peak appears in $\sim 1\text{MeV}$ energy, which is resemble to the spectrum of the Crab pulsar (Kuiper et al. 2001). Unlike the Crab pulsar, however, the off-set of radio/X-ray peak phases is fairly large (Abdo et al. 2010a). Moreover, the *Fermi* revealed that PSR B1509-58 is not bright in GeV gamma-ray bands (Abdo et al. 2010a), which is incompatible with the spectrum of the Crab pulsar, suggesting the X-ray/gamma-ray emission mechanism of the PSR B1509-58 is different from that of the Crab pulsar. In addition to PSR B1509-58, PSRs J1617-5055 (Torii et al. 1998), J1811-1925 (Torii et al. 1997), J1838-0655 (Lin et al. 2009), J1846-0258 (Gotthelf et al. 2000) and J1930+1852 (Camilo et al. 2002) are classified as GeV-quiet soft gamma-ray pulsars. Except

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1 For updated list, see https://confluence.slac.stanford.edu/display/GLAMCOG/Public+List+of+LAT-Detected+Gamma-Ray+Pulsars
for PSR J1617-5055, all of the soft gamma-ray pulsars are found in the center of supernova remnants.

The formation of GeV-quiet soft gamma-ray spectra has not been conclusive yet. Zhang & Cheng (2000) discussed X-ray/gamma-ray emissions from PSR B1509-58 within framework of the outer gap model and considered that PSR 1509-58 is the Crab-like pulsar, that is, the X-ray/gamma-ray emissions are created by the synchrotron radiation and inverse-Compton emissions of the electron and positron pairs created in the outer magnetosphere. The calculated overall spectrum qualitatively agrees with the multi-wavelength data. However, the light cylinder radius of PSR B1509-69 is so large as to make it very difficult to attenuate all of the GeV curvature photons emitted from the outer gap, which may be inconsistent with no detection of GeV gamma-rays by Fermi.

Harding et al. (1997) proposed operation of the photon-splitting process for the formation of the soft gamma-ray spectrum of PSR B1509-58, whose spin down dipole magnetic field is \( B_s \sim 2 \times 10^{13} \text{G} \). They argued that as the stellar magnetic field approaches the critical value, \( B_c \sim 4.4 \times 10^{13} \text{G} \), the magnetic photon-splitting process plays an important role as attenuation of the gamma-rays emitted in the polar cap region. They discussed that the photon-splitting and pair-creation cascade process can explain the position of the spectral peak \( \sim 10 \text{MeV} \) of PSR B1509-58. However, this model will not explain the soft gamma-ray spectra of PSRs J1617-5055 and PSR J1811-1925, whose inferred magnetic fields are only \( B_s \sim 3 \times 10^{12} \text{G} \) and \( 2 \times 10^{12} \text{G} \), respectively, which are the typical values of the canonical gamma-ray pulsars.

Wang et al. (2013) proposed a new model for PSR B1509-58 in the framework of the outer gap accelerator model (Takata et al. 2010; Wang et al. 2010). They discussed that the Earth viewing angle measured from the rotation axis is smaller than (or close to) the inclination angle of the magnetic axis. In such a small viewing angle, the outward GeV emissions, which creates the observed spectra of the Fermi-LAT pulsars, are missed by the observer, while the inward emissions contribute to the observed emissions. Wang et al. (2013) argued furthermore that the magnetic pair-creation cascade initiated by the inward 0.1-1GeV emissions near the stellar surface eventually produces the soft spectrum of the PSR B1509-58. Lin et al. (2009) also proposed that the X-ray emissions from GeV-quiet SGPSR J1838-0655 is produced by the synchrotron radiation of the pairs, which are produced by the magnetic pair-creation process of the inward gamma-rays from the outer gap.

Main purpose of this paper is to apply the model of the inward emissions to other
GeV-quiet SGPSRs, since the member of the GeV-quiet SGPSRs is growing and since no previous studies have been discussed the emission mechanisms. In particular, we will apply our model to four GeV-quiet soft gamma-ray pulsars, PSRs J1617-5055, J1811-1925, J1846-0258 and J1930+1852, for which detailed spectral data in 10-100keV bands were found in the literature. Although no detection of the emissions above 100keV has been reported, they share some properties of the emissions with PSR B1509-58; for example, (1) their radio emissions are dim or quiet, (2) the pulse profile in X-ray/soft gamma-ray bands is described by a single broad curve, (3) there are no GeV emissions and (4) the broad band spectral shape suggests the maximum energy flux at MeV energy bands. It is likely therefore that the emission processes of those GeV-quiet SGPSRs are different from the typical gamma-ray pulsars. The spin down parameters of those soft gamma-ray pulsars are summarized in Table 1.

In the paper, we also analyze the six year Fermi data and determine the upper limit flux of the GeV emissions (section 2), because we could not find any published results. We describe theoretical model in section 3 and compare the calculated spectra and light curves in section 4. A brief summary is presented in section 5.

2 Fermi Data Analysis

We used the γ-ray data from the Fermi Large Area Telescope (LAT) to search any gamma-ray emissions from the four soft gamma-ray pulsars, PSR J1617−5055, PSR J1811−1925, PSR J1846−0258 and PSR J1930+1852. The data analysis was performed using the Fermi Science Tools package (v9r32p5) available from the Fermi Science Support Center (FSSC). The data we used here were obtained from the reprocessed Fermi Pass 7 database and the instrumental response function used was the P7REP_SOURCE_V15 version. We used the data in the period starting from 2008-08-04 15:43:37 to 2014-05-30 01:27:16 (UTC). We selected the photons carrying energy between 100 MeV and 100 GeV within 20°×20° regions of interest (ROI) centered at the positions of the pulsars. To prevent the contamination by the Earth’s albedo, the events with zenith angle greater than 100° or rocking angle greater than 52° were filtered.

Binned likelihood analysis was performed using the gtlike function. To model the background source contributions, we included all 2-year Fermi Gamma-ray LAT (2FGL) catalog

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2 http://fermi.gsfc.nasa.gov/ssc/data/analysis/software/
Figure 1. Characteristics of the canonical pulsars. Small dots, filled boxes and filled circles show the radio pulsars, Fermi-LAT pulsars and GeV-quiet soft gamma-ray pulsars, respectively. The large filled boxes located at $L_{sd} = 7 \times 10^{36}$ erg s$^{-1}$ and $4.6 \times 10^{38}$ erg s$^{-1}$ correspond to the Vela and Crab pulsars, respectively. Top left: Spin down power v.s. the characteristic age. Top right: Spin down power v.s. gamma-ray efficiency. $L_\gamma = 4\pi D^2 F_\gamma$, where $D$ is the distance and $F_\gamma$ is the observed flux above 100MeV. Bottom left: Spin down power v.s. X-ray efficiency. $L_X = 4\pi D^2 F_X$, where $F_X$ is the observed X-ray flux below 10keV. Bottom right: Spin down power v.s. $\eta_\gamma / \eta_X$. We extensively used the ATNF pulsar catalog (Manchester et al. 2005) and the Fermi second catalog of the pulsars (Abdo et al. 2013). We referred the observed X-ray flux of GeV-quiet soft gamma-ray pulsars from Becker & Aschenbach (2002) for J1617-5055, Torii et al. (1997) for J1811-1925, Lin et al. (2009) for J1838-0655, Gotthelf et al. (2000) for J1846-0258 and Camilo et al. (2002) for J1930+1852, respectively.

point sources (Nolan et al., 2012) associated with the extended source templates within 20° from the ROI center. The spectral parameters for sources greater than 10° from the pulsars were kept fixed to the values defined in the catalog. For sources between 6° and 10° away from the center of ROI, only the spectral indices were kept fixed to the catalog definitions. The galactic diffuse background (gll_iem_v05.fits) and the isotropic diffuse background (iso_source_v05.txt) were also included in the modeling. All of these background modeling resources are available from the FSSC.

Using the full energy range extracted, 100 MeV to 100 GeV, we modeled the four soft gamma-ray pulsars as point sources using the simple power law

$$\frac{dN}{dE} = N_0 \left( \frac{E}{E_0} \right)^{-\Gamma}.$$  

The spectral energy distributions (SEDs) under 1 GeV were calculated using the modeled
Table 1. GeV-quiet soft gamma-ray pulsars. The second ($P$), third ($L_{sd}$) and fourth ($B_s$) columns are rotation period, spin down power and surface dipole magnetic field, respectively. The fifth ($D$) is the distance to the source and is used to estimate the observed luminosity in Figure 1. The sixth ($\alpha$), seventh ($\beta$) and eight ($f_{gap}$) are the magnetic inclination angle, Earth viewing angle and the fractional gap thickness, respectively, inferred from the fitting of the observations. The fitting result for PSR B1509-68 was taken from Wang et al. (2013). We did not fit PSR J1838-0655, since no point of the spectral data for the hard X-ray emissions (>10keV) from the pulsar have not been published.


3 THEORETICAL MODEL

In our model, we suggest the emissions from the GeV-quiet soft gamma-ray pulsars are produced via synchrotron radiation of the pairs, which are created by the interaction of the inward gamma-rays and the strong magnetic field near the polar cap region.
shows schematic picture for the inward gamma-rays emissions and subsequent pair-creation process and synchrotron radiation process. We emphasize that the viewing geometry is a crucial factor to differentiate between the typical gamma-ray pulsars and soft gamma-ray pulsars. Our model expects that millisecond pulsar does not show GeV-quiet soft gamma-ray spectrum, since its dipole magnetic field $B_s \sim 10^8$–$10^9$G is too small to operate the magnetic pair-creation process (except for very close to stellar surface, where the stronger multi-pole magnetic field may dominate the dipole field). Since a detail method of the calculation was described by Wang et al. (2013), we briefly mention the guideline of the model.
Figure 3. TS-map for PSR J1617−5055 in energy range from 100 MeV to 100 GeV.

Figure 4. Schematic view of the inward emissions from the outer gap accelerator. This figure is from Wang et al. (2013). Primary particles accelerated in the gap emit the gamma-ray photons via the curvature radiation process in the direction of its motion, which comprises of the motion along the magnetic field line and the co-rotation motion. Incoming particles emit the 100MeV-1GeV gamma-rays below the null charge surface. The 100MeV-1GeV gamma-rays emitted near the stellar surface interacts with the strong magnetic field and produce new pairs. The gyration motion of the pairs with an infinite pitch angle produces hard X-ray/soft gamma-rays via the synchrotron radiation, which covers a wider sky area than the curvature radiation of the primary particles.

3.1 Inward emissions of the outer gap

In the outer gap model, the charged particles are accelerated by the electric field parallel to the magnetic field, and emit the GeV gamma-rays via the curvature radiation process. Takata et al. (2008) argued that the outer gap accelerator produces outward and inward gamma-rays, which are produced by the outgoing particles and incoming particles acceler-
ated in the gap; for the magnetic inclination angle smaller than 90 degree, the outgoing and incoming particles are positrons and electrons, respectively. In the outer gap, since (i) the strong acceleration region extends between the null charge surface of the Goldreich-Julian charge density and the light cylinder and (ii) most of pairs are produced around the null charge surface (Cheng et al. 2000), the outgoing particles are accelerated by almost full potential drop in the gap, while the incoming particles feel only potential drop between the inner boundary and the pair-creation position. Hence, it is expected that the luminosity of the outward propagating gamma-rays are about one order of magnitude larger than that of inward propagating gamma-rays, suggesting the Fermi has preferentially detected the outward emissions of the outer gap. Within the framework of the outer gap model, the gamma-ray luminosity can be written as

\[ L_\gamma \sim I_{\text{gap}} V_{\text{gap}}, \]  

(2)

where \( I_{\text{gap}} \) is total current in the outer gap and \( V_{\text{gap}} \) is electric potential drop along the magnetic field line. As aforementioned pair-creation region in the outer gap accelerator, the outgoing particles are accelerated by almost full potential drop in the gap, which can be estimated as

\[ V_{\text{out}}^{\text{gap}} \sim f_{\text{gap}}^2 B_{lc} R_{lc}, \]  

(3)

where \( R_{lc} = Pc/2\pi \) is the light cylinder radius, \( B_{lc} \) is the magnetic field at the light cylinder, and \( f_{\text{gap}} \), which takes a value of \( \sim 0.2 - 0.3 \), is the ratio of the gap thickness and the light cylinder radius at the light cylinder. For the inward emissions, the incoming particles are accelerated with a potential of \( V_{\text{in}}^{\text{gap}} \sim 0.1V_{\text{out}}^{\text{gap}} \).

In the outer gap magnetosphere, the charge particles are accelerated by the electric field along the magnetic field line \( E_{||} \sim V_{\text{gap}}/R_{lc} \) and emit gamma-rays through the curvature radiation process. Assuming balance between the electric force and radiation drag force, the saturated Lorentz factor is proportional to \( \Gamma \propto V_{\text{gap}}^{1/4} \). As a result, the typical energy of the curvature radiation is proportional to \( E_c \propto \Gamma^3 \propto V_{\text{gap}}^{3/4} \). Since \( V_{\text{in}}^{\text{gap}} \sim 0.1V_{\text{out}}^{\text{gap}} \), the energy of the curvature radiation of inward emissions is a factor of \( \sim 5 \) smaller than that of the outward emissions and it typically becomes 0.1-1GeV.

For the outer gap accelerator, the strong acceleration region extends beyond the null charge surface, which is defined by surface of \( \Omega \cdot B = 0 \). It has been proposed that the active outer gap with the electric current can be extended “below” null charge surface (Takata et al. 2004; Hirotani 2006)), but the accelerating electric field below the null charge
surface is significantly reduced by the electron and positron pairs with a very weak field. We approximate the electric structure below the null charge surface as

\[ E_{\parallel}(r < r_{\text{null}}) = \frac{(r/r_{\text{in}})^2 - 1}{(r_{\text{null}}/r_{\text{in}})^2 - 1} E_{\parallel,\text{null}}, \tag{4} \]

where \( E_{\parallel,\text{null}} \) is the electric field strength at the null charge surface and is given by our three-dimensional two-layer structure model (Wang et al. 2011), and \( r_{\text{null}} \) is the radial distance to the null charge surface, which is a function of the inclination angle and azimuth angle. In addition, \( r_{\text{in}} \) is the radial distance to the inner boundary of the outer gap and is set at 20 stellar radius.

Near and below the inner boundary, the incoming particles lose their energy via the curvature radiation process. When the Lorentz factor of the incoming particles drops low enough, the curvature energy loss time scale becomes comparable to the time scale of the particle’s movement to the stellar surface. In such a case, we can show that the energy of the curvature photon is \( 9m_{e}c^2/8\alpha_f \sim 100 \text{ MeV} \), where \( \alpha \) is the fine structure constant (Takata et al. 2010). Hence, we expect that the incoming particles emit 0.1-1GeV photons between the null charge surface and the stellar surface. The gamma-ray photons emitted below the null charge surface may pass through the strong magnetic field region near the stellar surface and may initiate magnetic pair-creation cascade.

### 3.2 Magnetic pair-creation cascade

The typical cut-off energy 0.1-1GeV in the spectrum of the inward emissions will be still higher than the spectral cut-off energy (\( \sim 1\text{-}10\text{MeV} \)) of the GeV-quiet SGPSRs; for example, the original soft gamma-ray pulsar, PSR B1509-58, shows a spectral cut-off at \( \sim 5\text{MeV} \). To explain the position of the spectral cut-off of PSR B1509-58, we simulate the pair-creation cascades of the inward gamma-ray emissions (Wang et al. 2013). If the inward propagating gamma-rays emitted below the null charge surface pass through near the stellar surface, they may be absorbed by the magnetic field and be converted into electron and positron pairs (magnetic pair-creation process). The mean free path of the magnetic pair-creation may be written as (Erber 1966)

\[ \ell = \frac{4.4}{(e^2/\hbar c)} \frac{\hbar}{m_e c B_{\perp}} \exp \left( \frac{4}{3\chi} \right), \tag{5} \]

where \( \chi = \hbar \omega B_{\perp}/(2m_e c^2 B_c) \) and \( B_{\perp} = B \sin \theta_p \) with \( \theta_p \) being the angle between the magnetic field direction and propagating direction of the photon and \( B_c = 4.4 \times 10^{13}\text{G} \). We calculate
the optical depth \( \tau_{\text{opt}}(s_i) = \int_{s_{i-1}}^{s_i} ds/\ell(s) \) (\( i = 1, 2, 3.. \)), where \( s_0 = 0 \) corresponds to the position of the emitted point. We determine the pair-creation position \( s_i \) from the condition \( \tau(s_{i+1}) - \tau(s_i) = 0.1 \) and calculate the number of created pairs from the equation \( \delta N(s_i) = N_0 \{ \exp[-\tau(s_{i-1})] - \exp[-\tau(s_i)] \} \), where \( N_0 \) is the emitted gamma-rays in the gap. We also taken into account the pair-creation process of the gamma-rays with the X-rays.

### 3.3 Synchrotron emissions from new pairs

The created pairs have a pitch angle \( \theta_p \) and loose their-energy via the synchrotron radiation. We solve the evolution of the Lorentz factor \( (\gamma) \) of the pairs with the equations of

\[
\frac{dP_{||}}{dt} = -\frac{2e^4B^2\gamma^2\sin^2\theta_p}{3m_e^2c^4}\cos\theta_p
\]

(6)

and

\[
\frac{dP_{\perp}}{dt} = -\frac{2e^4B^2\gamma^2\sin^2\theta_p}{3m_e^2c^4}\sin\theta_p,
\]

(7)

where \( P_{||} = m_e\gamma\cos\theta_p \) and \( P_{\perp} = m_e\gamma\sin\theta_p \). Since the magnetic field and Lorentz factor of the particle at the pair-creation position are \( B_{\perp} = 2m_e^2B_c/(\chi E_\gamma) \) and \( \gamma = E_\gamma/2m_e^2 \), respectively, the maximum energy of the synchrotron radiation of the new born pairs becomes as

\[
E_{\text{syn, max}} \sim \frac{3h\gamma^2eB_\perp}{2m_e c} \sim \frac{3E_\gamma}{4\chi} \sim 38 \left( \frac{E_\gamma}{0.5\text{GeV}} \right) \left( \frac{\chi}{0.1} \right)^{-1}\text{MeV},
\]

(8)

suggesting the spectrum of the synchrotron radiations of the pairs, which are produced by the magnetic pair-creation process, has a spectral turn over around 10MeV. The position of this spectral turnover can explain that of the GeV-quiet SGPSRs. Therefore, we suggest that the observed high-energy emissions from GeV-quiet SGPSRs are produced through the synchrotron radiation occurred near the stellar surface. We also take into account the magnetic pair-creation process of the synchrotron photons, which was ignored in Wang et al. (2013).

We take into account the effects of the pitch angle and the gyration motion on the emission direction of synchrotron radiation. The particle motion is expressed by sum of the motion along the magnetic field line, gyration motion and co-rotation motion. Taking z-axis along the rotation axis, the particle motion is calculated from (Takata et al. 2007; Wang et al. 2013)

\[
v = \lambda v_{\text{syn}}/|v_{\text{syn}}'| + \Omega \times r,
\]

(9)
where $v_p$ is calculated from the condition $|v| = c$ and $v'_\text{syn}$ is given by

\[
\begin{align*}
v'_{\text{syn},x} &= \hat{B}_x + \tan \theta_p (u_x \cos T + v_x \sin T), \\
v'_{\text{syn},y} &= \hat{B}_y + \tan \theta_p (u_y \cos T + v_y \sin T), \\
v'_{\text{syn},z} &= \hat{B}_z + \tan \theta_p (u_z \cos T + v_z \sin T)
\end{align*}
\] (10)

where $\theta_p$ is the pitch angle of the created pairs and $T$ is the phase of the gyration motion. In the equation above, $\hat{B} = B/|B|$, $u = [B_y/(B_x^2 + B_y^2)]^{1/2}, -B_x/(B_x^2 + B_y^2)^{1/2}, 0)$ and $v = (B \times u)/|B \times u|$. In addition, $\lambda$ represents the direction of the particle motion projected to the magnetic field line, it takes $\lambda = 1$ for $\theta_p \leq 90^\circ$ and $\lambda = -1$ for $\theta_p > 90^\circ$.

The Earth viewing angle ($\beta$) measured from the rotation axis and the pulse phase $\psi$ for a synchrotron (or curvature) photon can be calculated from

\[
\cos \beta = \frac{v_z}{v}
\] (11)

and

\[
\psi = -\cos^{-1}(v_x/\sqrt{v_x^2 + v_y^2}) - \frac{\mathbf{r} \cdot \mathbf{v}}{v R_{lc}},
\] (12)

respectively, where $\mathbf{r}$ is the vector to the radiation point. For each viewing angle $\beta$, we calculate the phase-averaged spectrum and compare the result with the observations (section 4).

The synchrotron emissions from the pairs ($\theta_p \neq 0$) with the gyration motion covers a wider sky area than the curvature radiation of the primary particles ($\theta_p = 0$), which is emitted along the magnetic field line. Wang et al. (2013) discussed the evolution of the X-ray/gamma-ray spectrum for the different viewing geometry: Fixing magnetic inclination angle at $\alpha = 20^\circ$, the outward curvature emissions dominate the inward emissions and the spectrum extends up to several GeV if the viewing angle is $\beta \sim 70 - 90^\circ$. For mildly viewing angle $\beta \sim 50^\circ$, the inward curvature emissions and subsequent synchrotron radiation of the pairs can contribute to the spectrum. For small inclination angle $\beta \sim \alpha$ or $\beta < \alpha$, only synchrotron radiation of the pairs created by the magnetic pair-creation contributes to the observations, and the spectral peak in $\nu F_\nu$ appears at around $\sim 1\text{MeV}$, which can explain the spectral properties of GeV-quiet SGPSRs. Hence, our model suggests that the GeV-quiet SGPSR is a peculiar case of the viewing geometry and it has a relatively small viewing angle and inclination angle comparing with the normal gamma-ray pulsars.

In the present calculation, we apply the rotating dipole magnetic field in vacuum (Cheng et al. 2000). Force-free magnetosphere has been investigated for magnetic field and current structure in the pulsar magnetosphere (Contopoulos et al. 1999; Spitkovsky 2006), and
provides a distinct GeV pulse profile from the vacuum dipole field (Bai & Spitkovsky 2010). More realistic pulsar magnetosphere will be between the vacuum dipole field and the force-free field (e.g. Li et al. 2012; Kalapotharakos et al. 2012). In the present calculation, however, since the emission regions in X-ray/soft gamma-ray bands are near the neutron star surface, where the force-free field and vacuum dipole field may be close to each other, the rotating vacuum dipole field may provide a good approximation to discuss the pulse profile.

4 RESULTS

To fit the observed spectrum, main model parameters are the fractional gap thickness $f_{\text{gap}}$, the magnetic inclination angle $\alpha$ and the Earth viewing angle $\beta$. The inclination angle and the viewing angle of a pulsar is sometimes constrained by rotating vector model, which fits the observations of the radio polarization (Radhakrishnan & Cooke 1969). For instance, the fitting of the original soft gamma-ray pulsar, PSR B1509-58, suggests the inclination angle of $\alpha < 60$ degree (Crawford et al. 2001). The geometrical model of the pulsar wind nebula is also used to constrain the viewing geometry of the pulsar (Gaensler et al., 2002; Ng & Romani 2008). Using CHANDRA data, for example, Lu et al. (2002) found clear torus structure of the pulsar wind nebula surrounding the soft gamma-ray pulsar PSR B1930+1852 and suggested the Earth viewing angle of $\beta = 41$ degree. We emphasize that within the framework of our model, the viewing geometry that explains (1) the softness of the spectra, (2) it’s flux level and (3) the single peak in the light curve of the GeV-quiet soft gamma-ray pulsars are constrained in a narrow range of the parameters. In Table 1, we tabulate the best fitting parameters of the inclination angle and the Earth viewing angle for four GeV-quiet SGPSRs.

4.1 J1617-5055

The 69 ms spin-down powered pulsar PSR J1617-5055 was discovered by the X-ray observations (Torii et al. 1998, Garmire et al. 1999; Becker & Aschenbach 20002; Kargaltsev et al. 2009) with the radio pulsation founded shortly afterwards (Kaspi et al. 1998). Soft gamma-ray emissions at $\sim 100$keV bands were discovered by INTEGRAL (Landi et al. 2007). The timing analyses show that the spin down dipole magnetic field is $B_s \sim 3 \times 10^{18}$Gauss and the characteristic age is $\tau_a \sim 8.1$kyr. The dispersion measure gives the distance to the pulsar of $d \sim 6.1-6.9$kpc. The X-ray spectrum below $\sim 10$keV is fitted by a single power law function with a photon index of $p \sim 1.4$, and there is a spectral break around 10keV (Torii et al.
Figure 5. The spectrum (left) and X-ray light curve of PSR J1617-5055. The solid line in the left panel and the grey histogram in the right panel show the calculated spectrum and the light curve, respectively. The results are for the inclination angle of $\alpha = 15$ degree and the Earth viewing angle of $\beta = 25$ degree. For the observed flux, the data were taken from Landie et al. (2007) for BeppoSAX and INTEGRAL and from Kuiper and Hermsen (2014) for RXTE. The upper limit of Fermi data were determined by this study. For the light curve, the data was taken from Becker and Aschenback (2002).

The inferred X-ray conversion efficiency in 0.5-10 keV bands is $L_X / \dot{E} \sim 1.4 \times 10^{-3}$ for a distance of $d = 6$ kpc (Becker and Aschenback 2002). These X-ray properties are common among the GeV-quiet soft gamma-ray pulsar. Both the X-ray and radio pulse profiles of PSR J1617-5055 show a single peak, but absolute phase difference between the X-ray and radio peaks has not been known.

Figure 5 compares the calculated spectrum (left panel, solid line) and X-ray light curve (right panel, grey histogram) with the observations; the phase 0 (and 0.5) in Figure 5 corresponds to the phase at which the magnetic axis points towards the Earth. We assumed the inclination angle of $\alpha = 15$ degree and the Earth viewing angle measured from the rotation axis of $\beta = 25$ degree. In the present scenario, we have argued that the emissions from the GeV-quiet SGPSRs are created by the synchrotron radiation process of the pairs, which are produced by the interaction of the inwardly emitted GeV gamma-rays and the strong magnetic field near the pulsar. Since the pairs are mainly produced above the polar cap, the pulse peak with a strong synchrotron emission appears if the line of sight cuts near the polar cap region. Hence it is required the condition that the earth viewing angle is not significantly shifted from the magnetic inclination angle. Furthermore, a small inclination angle is required to avoid the detection of curvature radiation (GeV emissions) from the outer gap.
4.2 J1811-1925

The 65-ms pulsar PSR J1811-1925 at the center of G11.2-0.3 was discovered by ASCA observations (Torii et al. 1997; Kaspi et al. 2001). This pulsar has not been detected in the radio band (Crawford et al. 1998). The X-ray timing analysis suggests that the spin down dipole magnetic field is $B_s \sim 2 \times 10^{12}$ G and spin-down age is $\tau \sim 24$ kyr (Torii et al. 1999). However, the CHANDRA observation combined with Very Large Array observations (Roberts et al. 2003) suggests that the reverse shock of SNR has not yet reached the PWN, which indicates that the system is about 2000 years old, which is consistent with the historical record of supernova in A.D. 386 (Clark & Stephenson 1977). The distance to the pulsar is $d \sim 5$ kpc as inferred from HI measurements (Becker et al. 1985; Green et al. 1988). Figure 6 compares the calculated spectrum (left panel, solid line) and X-ray light curve (right panel, grey histogram) with the observations. We assumed the inclination angle of $\alpha = 10$ degree and the Earth viewing angle measured from the rotation axis of $\beta = 35$ degree.

4.3 J1846-0258

The 326 ms pulsar PSR J1846-0258 (also known as AX J1846.4-0258) was discovered by Gotthelf et al. (2000) in the X-ray bands, and is at the center of SNR Kes 75 (c.f. Kesteven 1968; Helfand et al. 2003; Molkov et al. 2004; Bird et al. 2007; Kumar & Safi-Harb 2008; McBride et al. 2008; Ng et al. 2008). No radio emission has been observed from PSR J1846-0258.
Figure 7. Same as with Figure 5, but for PSR J1846-0258. The model calculation is result for $\alpha = 10^{\text{degree}}$ and $\beta = 35^{\text{degree}}$, respectively. For the observed flux and light curve in X-ray/soft gamma-ray bands, the data were taken from Kuiper and Hermsen (2014).
**4.4 J1930+1852**

The 136ms radio pulsar PSR J1930+1852 was found in SNR G54.1+0.3 by Camilo et al. (2002). Both the X-ray and radio pulse profiles of PSR J1930+1852 show a single peak, but absolute phase difference between the X-ray and radio peaks has not been known. The timing analyses show that the spin down age of this pulsar is 2.9kyr, and the surface dipole magnetic field is $B_s \sim 1 \times 10^{13}$G. Lu et al. (2002) suggested that the distance of SNR G54.1+0.3 is about $d \sim 5$ kpc by measuring the X-ray absorption column density. PSR J1930+1852 is surrounded by a PWN, which has clear torus and jet structure. The ratio between the observed semi-major and semi-minor axes of the torus of PWN suggests that the Earth viewing angle inferred from the geometrical model is $\beta \sim 41$ degree (Lu et al. 2002), which is used in the calculation. Figure 8 compares the calculated spectrum (left panel, solid line) and X-ray light curve (right panel, grey histogram) with the observations. We assumed the inclination angle of $\alpha = 20$ degree and the Earth viewing angle measured from the rotation axis of $\beta = 41$ degree.

5 DISCUSSION AND SUMMARY

The present model suggests that the observed X-ray/soft gamma-rays are the synchrotron emissions from the high magnetic field region near the stellar surface, and the difference between the magnetic inclination angle and the Earth viewing angle is small, say $|\alpha - \beta| \leq \ldots$
30 degree. To avoid the GeV emissions from the outer gap, furthermore, the inclination angle is required to be small, say $\alpha \leq 30$ degree. We emphasize that the GeV-quit soft gamma-ray pulsars actually emit outgoing GeV gamma-rays from the outer gap, which make gamma-ray spectra of \textit{Fermi}-LAT pulsars, but our line of sight is out of emission cone due to the smaller magnetic inclination and a smaller Earth viewing angle. For the Earth viewing angle is $\beta \sim 40 - 50$ degree, the synchrotron emissions from the incoming pairs and the GeV emissions of outgoing particles can be observed, as we described in section 3. This may be the case for GeV-loud soft gamma-ray pulsars PSRs J0205+6449 and J2229+6114, which show a very soft GeV spectra and the smaller ratio of the GeV fluxes and X-ray fluxes comparing with the typical gamma-ray pulsars (e.g. Vela pulsar, Kuiper and Hermsen 2013, 2014). For the viewing angle of $\beta \sim 70 - 90$ degree, the outward GeV emissions makes a spectral peak in $\nu F_\nu$ at several GeV. Within the framework of our scenario, therefore, the viewing geometry is crucial factor to discriminate between the normal gamma-ray pulsars and soft gamma-ray pulsars, and the GeV-quiet SGPSR is peculiar case of the viewing geometry.

The GeV-quiet SGPSRs are relatively young and have higher-spin down powers compared with \textit{Fermi}-LAT pulsars, as Figure 1 shows; the typical characteristic age and the spin down power of the GeV-quiet SGPSRs are $\tau_s \sim 10^{3-4}$yr and $L_{sd} \sim 0.5 - 1 \times 10^{37}$erg s$^{-1}$. With the current study, it is not obvious the reason why GeV-quiet SGPSRs with the characteristic age of $\tau_s > 10^4$yr and $L_{sd} < 5 \times 10^{36}$ergs$^{-1}$ have not yet found, while many typical gamma-ray pulsars with those spin down parameters have been found by the \textit{Fermi}. However, we expect that with the appropriate viewing geometry, the pulsars with higher-surface magnetic field and/or high-spin down pulsars are preferentially detected as the GeV-quiet SGPSRs. For the future study, therefore, we will study the evolution of spectrum in X-ray/gamma-ray bands with the viewing geometry, spin down parameters etc. and will discuss the population of the GeV-quiet soft gamma-ray pulsar, GeV-loud soft gamma-ray pulsars and typical gamma-ray pulsars. The linking among these three group of the gamma-ray pulsars with provide us a comprehensive picture of the high-energy pulsars.

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