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Risk-limiting Dispatch with Operation Constraints

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Abstract—As an extension of the current theory of risk-limiting dispatch for a system with large-scale renewable integration, this paper presents a model for risk-limiting dispatch with operation constraints, such as generation limitation and network constraint. By proposing and solving four interrelated models, the problem for risk-limiting dispatch with network constraint is finally solved by using sequential optimization. Through the analysis of the model, the paper points out the feasible procedure of dispatch decision, including determining the optimal output and the generators needed to be scheduled. With this dispatch approach, the lowest dispatch cost of the whole dispatch process can be obtained.

Index Terms—Risk-limiting dispatch, operation constraint, network constraint, sequential optimization.

I. INTRODUCTION

Nowadays, the earth has been suffering from severe global energy crisis and environmental problem. They cannot be solved without a significant contribution from electricity sector, which is one of the objectives of the smart grid. In the smart grid, the renewable energy generation serves as the most crucial auxiliary power supply.

However, we enjoying a plethora of advantages and convenience of the renewable energy generation, the large scale of integration of renewable generation brings about a great deal of challenges. The uncertainty, raised from the randomness of the renewable energy resources, has highly increased, so the traditional dispatch approaches of power grid is not so feasible as before. In other words, if the conventional approaches for dispatch are still used, the efficiency of smart grid will be greatly reduced and the full potential of smart grid will not be realized. It is because the traditional dispatch approaches of power grid are based on highly controllable output of generators and highly predictable load demand. Nonetheless, with the penetration of renewable energy generation in smart grid, the output of generators is stochastic and the load demand is no longer passive. Therefore, it is indispensable to propose a new compatible approach for smart dispatch with regard to the smart grid.

In the beginning, deterministic dispatch or worst-case dispatch was used to meet the increased uncertainty by acquiring large capacity reserves, which increase the energy cost and emissions [1]. Thus, in order to obtain more economic benefits and to decrease the emissions, some new smart dispatch paradigms have been proposed recently. First of all, based on the model predictive control (MPC) approach [2-3], [4-5] has proposed an MPC algorithm, called the Look-ahead Dispatch, to schedule all available resources including intermittent energy. Moreover, [6-8] has improved this approach by including more operation constraints, such as ramping rates constraint and network constraint, and by utilizing it in different practical scenarios. The revised algorithm has been proved to be capable of dispatching the power with lower cost and higher efficiency. Thus far, the Look-ahead dispatch mainly focuses on the economic dispatch (ED). But in the time horizon of the whole dispatch process, before the operation of ED, the unit commitment (UC) must be done. In order to solve the smart dispatch problem completely, it is necessary to take account of the dispatch process for the whole time horizon. Therefore, a global smart dispatch approach, called the risk-limiting dispatch, has been put forward [9-10]. In [9], the framework and basic concept of risk-limiting dispatch for smart grid has been shown, and the model for risk-limiting dispatch in three decision stages has been established and completely solved. Furthermore, the risk-limiting dispatch in multiple stages has been presented in [11]. [9-11] considered the power balance as the constraint and neglected the network constraint. In [12], a preliminary discussion in respect of network constraint has been given out, though the network considered was just a congested network of two buses, somewhat lack of generality and the specific output of each generator was not presented.

In conclusion, among the various dispatch approaches dealing with the penetration of renewable generation, the conventional dispatch approaches have comparatively low economic efficiency and high environmental pollution. In respect of the new smart dispatch approaches, the Look-ahead Dispatch based on MPC has fully developed and solved the ED problem well. The risk-limiting dispatch aims to handle the dispatch problem for the whole time horizon including UC and ED, which is more realistic in smart grid. Among the operation constraints, the network constraint is especially crucial, but the model considering network constraints has not been solved perfectly.

Therefore, this paper aims to make further efforts to solve the risk-limiting dispatch with network constraint by solving 4...
interrelated models. The remainder paper is organized as follows. Section II sets up 4 interrelated models and explains the reasons. Section III presents the solution of the models and analyzes the process of risk-limiting dispatch in real network. Section IV is an illustrated case and the conclusions have been shown in Section V.

II. FOUR INTERRELATED MODELS

In general, the framework of risk-limiting dispatch process is proposed in [9]. The power generated and consumed at real-time \( t \) is influenced by three decisions taken before in time sequence, shown in Fig. 1. In each decision stage, by the forecast information of renewable generation, the operators determine the power needed to be scheduled in order to satisfy the risk constraints and to obtain the lowest dispatch cost. Specifically, as the time approaches to the real-time, the prediction information will be more precise, whereas the dispatch cost will be more expensive. In this paper, we consider that there are three decision stages, the same with [9].

![Time line of dispatch](image)

In spite of the current achievements in studying the risk-limiting dispatch, the risk-limiting dispatch can be utilized in real power grid only if the realistic power flow constraint is taken account of. Up to now, only [12] has obtained some conclusions in respect of the problem, although the network considered was just a congested network of two buses, somewhat lack of generality and the specific output of each generation was not presented. To have a further move on solving the risk-limiting dispatch model with network constraint, we set up 4 interrelated models and solve them in sequence.

The reason of this taxonomy is that Model 1 proposes the basic concept of risk-limiting dispatch. Based on Model 1, the practical problem of risk-limiting dispatch with network constraint can be formulated as a combination of Model 2 and Model 3. We use Model 4 to describe this combination. Because Model 2 and Model 3 give enlightenment to the solution of Model 4, the research order is Model 1 at first, then Model 2 and Model 3, and Model 4 at last.

The following notation is used throughout the paper:

- \( c_k \) unit cost in stage \( k \)
- \( s_k \) power needed to be scheduled in stage \( k \)
- \( s^*_{k} \) the optimal solution in stage \( k \)
- \( s_{k}^{\max} \) upper limitation of output for generator \( k \)
- \( D(t) \) load demand at time \( t \)
- \( W(t) \) output of renewable generation at time \( t \)
- \( Y_{k} \) forecast information in stage \( k \)
- \( c_{i}^{'} \) unit cost for generator \( i \) in stage \( k \)
- \( s_{i}^{'} \) power to be scheduled for generator \( i \) in stage \( k \)
- \( s_{i}^{\max} \) upper limitation of output for generator \( i \)
- \( M \) the total cost of the whole dispatch process

A. Model 1

Model 1 is actually the model presented in [9]. Here we put emphasis on the physical meaning of Model 1 in power grid. Model 1 represents the situation that there is only one generator, without output limitation, can be scheduled. Model 1 is formulated in (1):

\[
\begin{align*}
\min & \ E\{\sum_{k=1}^{3} c_{k} s_{k}^{*}\} \\
\text{s.t.} & \ s_{k}^{*} \geq 0, \\
& \ PI\{\sum_{k=1}^{3} s_{k}^{*} - D(t) + W(t)\} = 1
\end{align*}
\]

This model illustrates the framework of risk-limiting dispatch in three stages, which has been completely solved in [9-10]. However, in realistic power grid, it is impossible that the number of generator to be scheduled is only one. Also, in realistic power grid, the output limitation of the generators scheduled can never be infinite due to the upper limitation of line capacity, namely the network transmission constraint. Thus, in view of the two shortcomings, we propose Model 2 and Model 3.

B. Model 2

In this model, there are totally \( n \) generators without upper limitation that can be scheduled. So the Model 1 can be revised as (2):

\[
\begin{align*}
\min & \ E\{\sum_{k=1}^{3} \sum_{i=1}^{n} c_{i}^{'} s_{i}^{*}\} \\
\text{s.t.} & \ s_{i}^{*} \geq 0, \\
& \ PI\{\sum_{k=1}^{3} \sum_{i=1}^{n} s_{i}^{*} - D(t) + W(t) Y_{i}^{*}\} = 1
\end{align*}
\]

In each stage, in order to obtain the lowest scheduling cost, some generators may not be turned on, denoted by \( s_{i}^{*} = 0 \). In other words, the UC process has been actually included. Thus this model is able to handle dispatch problem for the whole time horizon.

C. Model 3

In previous models, we assume that the output of generators is infinite. However, due to the upper limitation of line capacity, the output of generators must be constrained. If the line capacity is infinite, the solution in each stage must be the same with Model 1, denoted by \( s_{k}^{*} \). But, if the network power flow constraint must be taken account of, there must be an equivalent upper limitation \( s_{k}^{\max} \) of the output for the generator. If \( s_{k} > s_{k}^{\max} \), the actual optimal solution of the model has to be changed to \( s_{k}^{\max} \). Thus, Model 3 can be formulated as (3):

\[
\begin{align*}
\min & \ E\{\sum_{k=1}^{3} \sum_{i=1}^{n} c_{i}^{'} s_{i}^{*}\} \\
\text{s.t.} & \ s_{i}^{*} \geq 0, \\
& \ PI\{\sum_{k=1}^{3} \sum_{i=1}^{n} s_{i}^{*} - D(t) + W(t) Y_{i}^{*}\} = 1
\end{align*}
\]
\[
\min E\{\sum_{k=1}^{\frac{1}{1}} c_i s_i \}
\]
\[
s.t. \quad 0 \leq s_i \leq s_{i,\text{max}},
\]
\[
P\{\sum_{k=1}^{\frac{1}{1}} s_i - D(t) + W(t)\left| Y_i \right.\} = 1
\]

D. Model 4

Through the analysis of the physical meanings of the models above, we know that the practical problem of risk-limiting dispatch with network constraint can be formulated as a combination of Model 2 and Model 3. We use Model 4 to describe this combination, shown in (4):

\[
\min E\{\sum_{k=1}^{n} c_k s_k \}
\]
\[
s.t. \quad 0 \leq s_k \leq s_{k,\text{max}},
\]
\[
P\{\sum_{k=1}^{n} s_k - D(t) + W(t)\left| Y_i \right.\} = 1
\]

Finally, we have constructed the model for the risk-limiting dispatch with network constraint (Model 4), which will be solved in next section.

III. SOLUTION OF THE MODELS

To achieve the ultimate goal of solving Model 4, we need to solve Model 1 to Model 3 at first, for the reason that the solution of Model 1 to Model 3 provides guidance to Model 4. We present the solution respectively.

A. Solution for Model 1

Model 1 has been completely solved in [9]. Here just simply enumerating the results.

The calculation order is from stage 3 to stage 1 by sequential rolling optimization, whereas the practical decision order is from stage 1 to stage 3. The solution is shown in (5)-(7):

Stage 3:
\[
s_i^* = [d - s_i - s_{j,\text{seq}}],
\]
where \( d \) is the power imbalance and equals to \( D(t) - W(t) \), and \( \lfloor x \rfloor \) is \( \max(x, 0) \).

Stage 2:
\[
s_i^* = [\theta(\pi, Y_i) - s_i],
\]
where \( P[d \geq \theta(\pi, Y_i)\left| Y_i \right.]) = \pi, \pi = c_2 / c_3 \).

Stage 1:
\[
\begin{align*}
if: & \quad P[\theta(\pi, Y_i) \geq 0\left| Y_i \right.] + \frac{c_3}{c_2} P[d \geq \theta(\pi, Y_i)\left| Y_i \right.] \leq \frac{c_3}{c_2} s_i^* = 0, \\
otherwise: & \quad P[\theta(\pi, Y_i) \geq s_i^*\left| Y_i \right.) + \frac{c_3}{c_2} P[d \geq s_i^* \geq \theta(\pi, Y_i)\left| Y_i \right.] = \frac{c_3}{c_2}
\end{align*}
\]

B. Solution for Model 2

Similarly, the solution procedure of Model 2 is from stage 3 to stage 1 by using sequential rolling optimization, whereas the actual decision process is the opposite.

Stage 3: Since \( s_j, s_2, \) and \( Y_3 \) are known, to minimize the final cost, it is undoubtedly that we need to choose the cheapest unit cost generator, which is:
\[
c_j^* = \min(c_j^*, c_j^*, ..., c_j^*)
\]
The determination of \( c_j^* \) actually gives out the information of which generator needed to be scheduled. Then we can calculate \( s_j^* \) by (9):
\[
s_j^* = [d - s_i - s_j],
\]
Stage 2: Since \( s_j \) and \( Y_2 \) are known, the optimal solution \( s_2^* \) and \( c_2^* \) are given by (10):
\[
\min E\{\sum_{j=1}^{n} (c_j s_j^*) + c_j^* s_j^*\left| Y_2 \right.\}
\]
\[
s.t. \quad s_2^* > 0
\]

Because the objective function is linear, it can be proved that the optimization problem is convex and hence we can find only one minimum value. The optimal solution of stage 2 is given by (11), (12):
\[
c_2^* = \min(c_2^*, c_2^*, ..., c_j^*)
\]
\[
s_2^* = [\theta(\pi, Y_2) - s_i],
\]
The solution means that at first we should choose the generator with the cheapest unit cost as the one to be scheduled, and then the problem will be degenerated into Model 1.

Stage 1: Similarly, we need to select the generator with the cheapest unit cost as the one to be scheduled, and then the problem will be degenerated into Model 1. The result is shown by (13):
\[
c_j^* = \min(c_j^*, c_j^*, ..., c_j^*),
\]
\[
if: \quad P[\theta(\pi, Y_2) \geq 0\left| Y_i \right.] + \frac{c_3}{c_2} P[d \geq \theta(\pi, Y_2)\left| Y_i \right.] \leq \frac{c_3}{c_2} s_i^* = 0, \quad (13)
\]
\[
otherwise: \quad P[\theta(\pi, Y_2) \geq s_i^*\left| Y_i \right.) + \frac{c_3}{c_2} P[d \geq s_i^* \geq \theta(\pi, Y_2)\left| Y_i \right.] = \frac{c_3}{c_2}
\]

In realistic dispatch process, according to the calculation results above, the dispatch decisions are made from stage 1 to stage 3, and the global optimal solution can be obtained.

C. Solution for Model 3

Also, we solve the model from stage 3 to stage 1 by using sequential rolling optimization:

Stage 3: Since \( s_j, s_2, \) and \( Y_3 \) are known, the optimal solution is given by (14):
if : \([d - s_i - s_j] \leq s_{3\text{max}}, s_j = [d - s_i - s_j]\),
otherwise: \(s_j = s_{3\text{max}}\) \hspace{1cm} (14)

Stage 2: Since \(s_i\) and \(Y_2\) are known, \(s_j^*\) is given by (15):

\[
\min E\{c_i s_i + c_j s_j^* Y_j\} \\
\text{s.t.} \quad 0 \leq s_i \leq s_{2\text{max}}
\]

Through substitution of the variable \(s_j\), the optimization problem is also convex and hence we can find only one minimum value. The optimal solution of stage 2 is given by (16):

\[
\begin{align*}
\text{if: } & 0 \leq s_i \leq \theta(\pi, Y_2) - s_{2\text{max}}, s_j^* = s_{2\text{max}} \\
\text{if: } & \theta(\pi, Y_2) - s_{2\text{max}} < s_i < \theta(\pi, Y_2), s_j^* = \theta(\pi, Y_2) - s_i \\
\text{if: } & s_i \geq \theta(\pi, Y_2), s_j^* = 0
\end{align*}
\]

Stage 1: Similar to stage 2, the solution of stage 1 is (17):

\[
\begin{align*}
\text{if: } & P(\theta(\pi, Y_2) - s_{2\text{max}}, s_j^*) = 0, \\
\text{otherwise: } & P(\theta(\pi, Y_2) - s_j^*) = \frac{c_i}{c_2} P(d \geq s_{2\text{max}}, s_j^* \geq \theta(\pi, Y_2) | Y_i) = \frac{c_i}{c_2} s_j^* = 0, \\
\end{align*}
\]

By comparison, the dispatch cost in Model 3 is higher than in Model 1. The solution reveals that if the optimal output exceeds the upper limit in one stage, the rest of the optimal output must be filled up in the next stage, where the scheduling cost is much higher than current stage.

**D. Solution for Model 4**

With the guidance given by the solution of Model 2 and Model 3, we can conclude 2 theorems.

From the result of Model 2, we find that the problem can be decoupled into 2 steps. To determine the generators needed to be scheduled and to calculate the output of the generators:

**Theorem 1:** At first, to choose generators needed to be scheduled, and then to calculate the output of the generators.

From the result of Model 3, we find that the dispatch cost considering the network constraint (Model 3) is higher than it in Model 1. Thus we conclude **Theorem 2:**

**Theorem 2:** To obtain the lowest dispatch cost, the output of generators in each stage should be approximated to the result in Model 1.

In Model 4, compared with Model 3, as the number of generators to be scheduled is not only one, the approximation described in **Theorem 2** can be somehow realized by unit commitment (UC), as if the upper limitation of generator (Model 3) is broken through by UC. As the objective function is linear, this UC problem can be solved by priority list with an order formed according to the unit cost of each generator.

Specifically, in stage 3, by **Theorem 1**, at first we choose the cheapest unit cost generator, as in Model 2:

\[
c_j^* = \min(c_{i_1}, c_{i_2}, ..., c_{i_n})
\]

And then we determine the output, as in Model 3:

\[
\begin{align*}
\text{if: } & [d - s_i - s_j], s_j^* = [d - s_i - s_j] \\
\text{otherwise: } & s_j^* = s_{3\text{max}}
\end{align*}
\]

However, with **Theorem 2**, if the optimal solution exceeds the upper limitation \(s_j^* > s_{3\text{max}}\), we can realize the approximation to the optimal solution in Model 1 through UC by priority list, the optimal output is shown in (20):

\[
s_j^* = [d - s_i - s_j],
\]

and the generators to be scheduled are followed the priority list.

Stage 2 and stage 1: Similar to stage 3, we can acquire the optimal solution by choosing:

\[
\begin{align*}
c_j^* = \min(c_{i_1}, c_{i_2}, ..., c_{i_n}) \\
c_j^* = \min(c_{i_1}, c_{i_2}, ..., c_{i_n})
\end{align*}
\]

So optimal output in these stages is shown in (22), (23):

\[
s_j^* = [\theta(\pi, Y_2) - s_i],
\]

\[
\begin{align*}
\text{if: } & P(\theta(\pi, Y_2) - s_i) = 0, \\
\text{otherwise: } & P(\theta(\pi, Y_2) - s_i) = \frac{c_i}{c_2} \min(\theta(\pi, Y_2) - s_i) = 0.
\end{align*}
\]

In conclusion, when dealing with the problem of risk-limiting dispatch with network constraint, the optimal output can be obtained by choosing the lowest cost unit and using the solution in Model 1, also shown in (20), (22) and (23). After calculating the output, the generators needed to be scheduled are followed the priority list formed according to the unit cost of each generator.

**IV. ILLUSTRATED CASE**

In order to demonstrate that this solution is globally optimal, we give out a simple case study here. The network is shown in Fig. 2. Bus 4 is a wind power generator \(W\) and a load. A generator able to be scheduled and a load are connected to bus 1 and bus 2, respectively. Bus 3 is only a generator to be scheduled.

As recent studies observed that the forecast errors are distributed as a Gaussian random variable [13], we assuming that the forecast output of wind power \(W(x)\) for time \(t\) in each stage is:
Stage1: \( W(x) = \frac{X - \mu}{\sigma}, \mu = 20, \sigma^2 = 9 \)
Stage2: \( W(x) = \frac{X - \mu}{\sigma}, \mu = 25, \sigma^2 = 4 \)
Stage3: \( W(x) = \frac{X - \mu}{\sigma}, \mu = 21, \sigma^2 = 0 \)

Fig. 2 Network of the illustrated case

and the total demand \( D = 30 \). There are totally 3 generators to be scheduled, and the information of each generator is:

\[
c_1^1 = 4, c_2^1 = 6, c_3^1 = 13, s_{\text{max}}^1 = 8
\]
\[
c_1^2 = 6.5, c_2^2 = 6, c_3^2 = 12, s_{\text{max}}^2 = 6
\]
\[
c_1^3 = 7, c_2^3 = 7, c_3^3 = 10, s_{\text{max}}^3 = 10
\]

At first, we use the conventional dispatch, which means the optimal cost is calculated in each stage respectively, then:

\[
s_1^* = s_1^* = 24.85, c_1^* = c_1^1 = 4
\]
\[
s_2^* = 0, c_2^* = c_2^2 = 6
\]
\[
s_3^* = 0, c_3^* = c_3^3 = 10
\]
\[
M = c_1^* s_1^* + c_2^* s_2^* + c_3^* s_3^* = 99.4
\]

Then, we use the solution of risk-limiting dispatch with network constraint given by Model 4, and the optimal cost is shown below:

\[
s_1^* = s_1^* + s_2^* + s_3^* = 8 + 0.71 + 0 = 8.71
\]
\[
s_2^* = s_1^* + s_2^* = s_1^* = 0
\]
\[
s_3^* = s_3^* + s_3^* + s_3^* = 0 + 0 + 1.29 = 1.29
\]
\[
M = c_1^* s_1^* + c_2^* s_2^* + c_3^* s_3^* = c_1^1 s_1^1 + c_1^2 s_1^2 + c_1^3 s_1^3 = 65.515
\]

From the results of the calculation, for the conventional dispatch, it fails to solve the optimal problem globally, so the operators must schedule enough power to ensure the power balance in each stage, which leads to the large sum of power brought in in stage 1. In other words, the conventional dispatch handles the uncertainty by adding large capacity reserves.

However, the risk-limiting dispatch can realize the feasible allocation of every generator in each stage. Because slow-ramping rate units usually have lower scheduling cost than fast-ramping rate units in the early decision stage, and have higher scheduling cost than fast-ramping rate units in the stage that near the real-time, the risk limiting dispatch can utilize this characteristic and obtain the lowest dispatch cost, which can be obviously supported by the calculation results in the illustrated case.

V. CONCLUSIONS

This paper contributes to the construction of the model for risk-limiting dispatch with network constraint. In solving the risk-limiting dispatch with network constraint, through analyzing and solving 4 interrelated models, we have proved the optimization problem is convex so that the optimal solution is existent and unique. Furthermore, by proposing two theorems, the optimal output for each dispatch stage can be obtained by choosing the unit with the lowest and this total output for each particular dispatch stage is calculated by sequential optimization. Finally, the optimal output for each generator is allocated by the priority list formed according to the unit cost of each generator. The accomplishments of this paper contribute to the improvement of the risk-limiting dispatch theory, and carry the application of risk-limiting dispatch a giant step forward in smart grid.

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