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A Novel Method for Optimal Life Cycle Management Scheme with Markov Model

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Abstract— Maintenance of equipment of a power system can enhance the health condition and result in the improvement of reliability of a power system. However, maintenance introduces additional cost into the total operating cost. A sophisticated maintenance strategy should be a tradeoff between maintenance costs and reliability enhancement. This paper addresses the optimal maintenance scheme for transmission equipment over their life cycles. The whole life cycle is divided into several intervals. The health states over all intervals are modeled as a Markov chain. The transformation probability between the states at interconnected intervals will be determined with different maintenance strategies. The model for optimal maintenance strategy considering minor maintenance costs, major maintenance costs, repair costs and equivalent load losses costs. Equivalent load losses are determined by Monte Carlo method based OPF algorithm associated with stochastic loads. To find solution within reasonable computing time, a space reduction approach is proposed based on the characteristics of the Markov chain. A test system is used to demonstrate the efficiency and accuracy of the method.

Index Terms— Life cycle management, maintenance, Markov chain, optimal strategy.

I. INTRODUCTION

In a power system, electric power utilities have always tried to maximize profits with acceptable reliability levels. Challenges posed by aging equipment should be solved in a scheduled manner to minimize potential crises in the future. Life cycle management (LCM) is expected to solve this problem efficiently over the whole life cycle. Some organizations, such as Department of Energy (DoE), U.S. and British Standard Institute (BSI) also highlight this issue. For instance, PAS 55 [1] [2] established by BSI has been accepted as a general standard for the life cycle management. In China, life cycle management has also been applied by State Grid Company to optimize the service equipment.

During the life cycle, the deterioration is a major factor that may increase the costs and decreases the reliability of the whole system. Usually, maintenance is used to mitigate the deterioration of components and result in the enhancement of the reliability level. However, additional costs associated with maintenance may increase the operating costs. To balance the reliability and the costs, a sophisticated maintenance schedule is critically needed.

The traditional maintenance is often pre-defined, i.e., maintenance activities are implemented at regular intervals. This method can be implemented with limited information of equipment but might be inefficient and uneconomic. The reliability-centered maintenance (RCM) [3] works as an effective method which can help to choose the most cost-effective strategy from many alternative strategies. It is an efficient tradeoff between maintenance costs and potential losses costs. However, the RCM method is usually a heuristic method. Extra judgment and experience are needed when utilization of the RCM method.

Due to the importance of maintenance strategy, many researches focus on this issue. A state diagram represented by a Markov process, is employed to demonstrate the deterioration and maintenance [4] [5]. This state diagram can illustrate transition probabilities between different deterioration states. Based on state diagrams, paper [6] discusses the properties of Markov processes. It focuses on whether these properties are realistic enough. Papers [7] and [8] analyze the optimal maintenance with the consideration of the power system operation. The genetic algorithm (GA) is used to optimize the model. However, it is a time-consuming task to find the optimal solution for large scale problems.

In this paper, the whole life cycle of a transformer is divided into a series of intervals. In each interval, the deterioration and maintenance are modeled as a Markov process. A model considering minor maintenance costs, major maintenance costs, repair costs and equivalent load losses costs, is built to optimize the strategies. The maintenance strategy is established at each interval. A concept entitled equivalent load loss is proposed. It measures the system loss associated with a given fault and stochastic loads. The optimal power flow (OPF) and the Monte Carlo method are used to assess the equivalent load loss. To deal with the large-scale multi-stages problem, a solution space reduction method is proposed. A case study is used to illustrate the efficiency of the proposed method.

The paper is organized as follows. Section II describes the equipment state model using the Markov chain. Section III...
presents the optimization model, the calculation of equivalent load losses and the approach to solve the model. A test system is performed in the Section IV. Section V presents the conclusions.

II. INTRODUCTION EQUIPMENT STATE DIAGRAM USING MARKOV CHAIN

The state of an aging component can be represented by a state diagram using the Markov chain. In this work, without loss the generality, four states are involved. They are the good state (S1), the minor deterioration state (S2), the major deterioration state (S3), and the failed state (S4). The state diagram is shown as Fig.1. The current state only determines the last state. According to the characteristic of a Markov chain, without maintenance, the state S4 will be an absorbed state.

In this paper, to simplify the model and without loss of generality, two maintenance strategies, i.e., the minor maintenance (M1) and major maintenance (M2), are considered. After the minor or major maintenance, the states S2, and S3 may be transferred to other states with different transition probabilities. For instance, with the minor maintenance M1, the equipment at state S2 can be transferred to states S1, S2, S3 or S4 with transition probabilities \( \lambda_{21} \), \( \lambda_{22} \), \( \lambda_{23} \) and \( \lambda_{24} \), respectively. The failure state, S4, can only reach the state S1 with a repair.

\[
\begin{bmatrix}
1 - \lambda_{12} - \lambda_{24} & \lambda_{12} & 0 & \lambda_{24} \\
0 & 1 - \lambda_{23} - \lambda_{24} & \lambda_{23} & \lambda_{24} \\
0 & 0 & 1 - \lambda_{34} & \lambda_{34} \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

(1)

where \( t_1 \) is the transition matrix of no maintenance, \( t_{II} \) is the transition matrix of minor maintenance, \( t_{III} \) is the transition matrix of major maintenance.

III. OPTIMAL MAINTENANCE STRATEGY

A. The Model of Optimal Maintenance

The optimal maintenance strategy is to select a proper maintenance strategy and the implementation timing to achieve the maximal profit with acceptable reliability level. To simplify the problem, the whole life cycle of the equipment is divided into \( N \) intervals. There are some assumptions in this paper.

- To avoid loss of load due to maintenance, the maintenance is implemented with the lowest load level within any intervals.
- A fault can be fixed within one interval and the equipment will be in service again in the next interval.

The model considering minor maintenance costs, major maintenance costs, repair costs and equivalent load losses costs of each state is built as follows.

\[
F = \sum_{i=1}^{N} \left[ M_{\text{minor}}(i) \cdot C_{\text{minor}} + M_{\text{major}}(i) \cdot C_{\text{major}} + P_{i} S_{4} \cdot C_{S_{4}} \right]
\]

(4)

\[
P_{i} = P_{0} \cdot T_{1} \cdot T_{2} \cdots \cdot T_{i} = P_{0} \times \prod_{j=1}^{i} T_{j}
\]

(5)

\[
T_{j} \in \{t_1, t_{II}, t_{III}\}
\]

(6)

\[
i \in \{1,2,\cdots,N\}
\]

(7)

\[
P_{0} = (P_{0}^{S_{1}}, P_{0}^{S_{2}}, P_{0}^{S_{3}}, P_{0}^{S_{4}})
\]

(8)

\[
P_{1} = (P_{1}^{S_{1}}, P_{1}^{S_{2}}, P_{1}^{S_{3}}, P_{1}^{S_{4}})
\]

(9)
\[ C_{5,4} = C_{re} + \alpha \cdot T_{re} \cdot L_{Loss} \] (10)

\[
M_{\text{minor}}(i) = \begin{cases} 0 & \text{no minor maintenance} \\ 1 & \text{minor maintenance} \end{cases}
\] (11)

\[
M_{\text{major}}(i) = \begin{cases} 0 & \text{no major maintenance} \\ 1 & \text{major maintenance} \end{cases}
\] (12)

where \( \alpha \) is the electricity price, \( L_{loss} \) is equivalent load losses, \( T_{re} \) is the mean repair time, \( C_{re} \) is the mean repair costs, \( P_0 \) is the initial probability vector of different states, \( P_i \) is the \( i^{th} \) interval probability vector of different states.

B. Equivalent Load Loss Evaluation

Equivalent load loss (ELL) is proposed to evaluate the system loss due to an equipment fault. ELL is defined as a minimal load curtailment after a fault. In this paper, OPF \[7\] is employed to optimize outputs of generators to calculate the ELL.

Because some stochastic loads affect the ELL, the Monte Carlo method is used to calculate the stochastic ELL. Loads in this paper are assumed to satisfy normal distributions. The detailed steps for calculating equivalent stochastic load curtailment are shown as follows.

- Renew the topology of the system after a fault.
- Generate \( M \) random loads according to the distribution of loads.
- Update data and run OPF to calculate the number of load curtailment \( (K) \) and the load curtailment of each simulation \( (P_{li}) \).
- The equivalent stochastic load curtailment can be calculated by:

\[ L_{loss} = \frac{\sum P_{li}}{M} \] (13)

C. Solution for the Optimization Model

According to the optimization model, the optimal strategy is a combination of the minor maintenance, the major maintenance and none maintenance. This optimization problem will become a combination explosion problem with a long time horizon. For the problem with \( N \) intervals, the number of combinations of the solutions is \( 3^N \). To model this problem accurately, the duration of each interval is often not too long. So for the equipment with 20 years’ life, the divided intervals will far more than 20.

At the \( k^{th} \) interval during the tree search, there are some strategies with approximately equal probabilities, i.e.,

\[ |P_k^m - P_k^n| < \varepsilon \] (14)

where \( \varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)^T \) is the vector of the error tolerance, \( P_k^m = P_0 \times \prod_{j=1}^{k} T_j^m, P_k^n = P_0 \times \prod_{j=1}^{k} T_j^n \), and \( T_j^m, T_j^n \in \{ t_1, t_2, t_3 \} \).

To reduce the searching space, for two states with the same (or approximately equal) state probability vectors, the state with larger cost is eliminated. The reason lies in that the vector of the state probability describes the probabilities of the equipment on different states. Due to the property of a Markov chain, two states with the same (or approximately equal) state probability vector provide the same (or approximately equal) initial states for the remaining stages. In other words, the same performance can be achieved with the same maintenance strategy for these two states and, furthermore, this performance is independent with the paths before research this state. As a result, to minimize the cost, the state with lower cost will be selected.

The process of the proposed algorithm is shown in Fig. 2.

IV. CASE STUDY

The efficiency of the proposed model and algorithm was demonstrated on the IEEE 30-bus system. The IEEE30-bus system consists of 5 generators, 37 lines and 4 transformers. This paper just analyzes the maintenance strategy of the
transformer connected between node 27 and node 28. Assume its life cycle is 20 years and each interval is one month, so the total intervals are 240.

The fault costs consist of two parts: the mean repaired cost and the load losses costs. The mean repaired cost can be calculated according to the historical data. The load losses can be calculated according to the OPF. Loads on the buses 2, 5, 7, 8, 21 and 30 are assumed to satisfy the normal distribution. 2000 simulations are implemented and the equivalent load losses are 9.682MW according to the equation (13). The simulation data are shown in TABLE I, TABLE II and TABLE III.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Value</th>
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<tbody>
<tr>
<td>λ₁₂</td>
<td>0.0175</td>
</tr>
<tr>
<td>λ₂₁</td>
<td>0.9891</td>
</tr>
<tr>
<td>λ₂₃</td>
<td>0.0100</td>
</tr>
<tr>
<td>λ₂₄</td>
<td>0.0037</td>
</tr>
<tr>
<td>λ₂₅</td>
<td>0.0056</td>
</tr>
<tr>
<td>λ₂₆</td>
<td>0.0045</td>
</tr>
<tr>
<td>λ₂₇</td>
<td>0.0057</td>
</tr>
<tr>
<td>λ₂₈</td>
<td>0.0045</td>
</tr>
<tr>
<td>λ₂₉</td>
<td>0.0057</td>
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<tr>
<td>λ₃₀</td>
<td>0.0045</td>
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<td>λ₃₁</td>
<td>0.0057</td>
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<tr>
<td>λ₃₂</td>
<td>0.0045</td>
</tr>
<tr>
<td>λ₃₃</td>
<td>0.0057</td>
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TABLE I. TRANSITION RATES OF THE EQUIPMENT

<table>
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<tbody>
<tr>
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<tr>
<td>λ³₄</td>
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<td>λ³₅</td>
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<tr>
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<tr>
<td>λ³₇</td>
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</tr>
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<td>λ³₈</td>
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TABLE II. PARAMETERS FOR CALCULATING COST

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Minor maintenance cost (10⁴RMB)</td>
<td>5</td>
</tr>
<tr>
<td>Major maintenance cost (10⁴RMB)</td>
<td>15</td>
</tr>
<tr>
<td>Repaired cost (10⁴RMB)</td>
<td>60</td>
</tr>
<tr>
<td>Repaired time(h)</td>
<td>240</td>
</tr>
<tr>
<td>Electricity cost (RMB/kWh)</td>
<td>0.6</td>
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TABLE III. PARAMETERS OF STOCHASTIC LOADS

<table>
<thead>
<tr>
<th>Bus</th>
<th>Value</th>
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<tbody>
<tr>
<td>μ (MW)</td>
<td>σ</td>
</tr>
<tr>
<td>2</td>
<td>21.7</td>
</tr>
<tr>
<td>5</td>
<td>94.2</td>
</tr>
<tr>
<td>7</td>
<td>22.8</td>
</tr>
</tbody>
</table>

To calculate the satisfactory strategy quickly while maintaining the strategy accurately, various simulations with different ε = (ε₁, ε₂, ε₃, ε₄) are implemented. In each simulation, ε₁, ε₂, ε₃, ε₄ can be set the same value. Fig. 3 shows the simulation time and objective values with different ε. According to the simulation, the simulation time of ε = (0.005, 0.005, 0.005, 0.005) is as approximate 7 times as that of ε = (0.01, 0.01, 0.01, 0.01). The error between the maximal total cost and the minimal total cost of different ε is 0.0987%. Results show that an appropriate ε can accelerate the simulation speed while sustaining a high accuracy.

To calculate the satisfactory strategy quickly while maintaining the strategy accurately, various simulations with different ε = (ε₁, ε₂, ε₃, ε₄) are implemented. In each simulation, ε₁, ε₂, ε₃, ε₄ can be set the same value. Fig. 3 shows the simulation time and objective values with different ε. According to the simulation, the simulation time of ε = (0.005, 0.005, 0.005, 0.005) is as approximate 7 times as that of ε = (0.01, 0.01, 0.01, 0.01). The error between the maximal total cost and the minimal total cost of different ε is 0.0987%. Results show that an appropriate ε can accelerate the simulation speed while sustaining a high accuracy.

Fig. 3. Simulation time and objective values with different ε

Fig. 4. Satisfactory strategies

Fig. 4 shows the satisfactory intervals in which the minor and major maintenance should be implemented. According to the results, different ε may affect maintenance timing slightly. Because of the similar total costs, the slight differences can be acceptable.

For a further analysis, this paper simulates impacts of the minor maintenance cost and the major maintenance cost on strategies. During the following simulations, ε is set (0.01, 0.01, 0.01, 0.01, 0.01) to accelerate the simulation speed. Fig. 5 shows different strategies with increasing minor maintenance costs (from 20,000RMB to 150,000RMB) and a constant major maintenance cost (150,000RMB).
According to the results, when the minor maintenance cost is 20,000RMB, no major maintenance will be implemented. With the increasing minor maintenance cost, some major maintenance will be implemented. When the minor maintenance cost reaches a certain value (65,000RMB in this paper), there are only major maintenance during the whole life time. The reason is that the major maintenance has higher probabilities to transfer a state to a better state than the minor maintenance.

V. CONCLUSION

This paper proposes a new approach to maintenance of a component. First, the Markov chain is employed to model the state transition of the component. The whole life cycle of component is divided into \( N \) intervals. Different maintenance will result in different costs which consist of minor maintenance costs, major maintenance costs, repair costs and equivalent load losses costs. OPF is used to calculate the load losses after a fault. Because some loads are stochastic, Monte Carlo method is used to calculate the equivalent load losses. According to the characteristic of the Markov process, a new method which can reduce the result space is employed to optimize the model.

REFERENCES