Dominant Bidding Strategy in Mobile App Advertising Auction

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***Abstract*—The widespread use of intelligent mobile phone has promoted prosperity of mobile App advertising in recent years. Based on existing bidding status, this paper presents the dominant bidding strategy for mobile advertising auction. Firstly, our study characterizes multiple Nash Equilibria resulting from different bidding strategies in wGSP (weighted Generalized Second-Price) auction. Further more, we prove that advertiser's rank and utility will not decrease by using the dominant bidding strategy. We also consider the situation where the reserve price is set by the mobile advertising platform. It turns out that that advertiser's payment will be no less than reserve price. Finally, a practical implementation for a virtual market simulates the dynamic bidding process in real world environments.**

Keywords : Mobile App Advertising Auction, wGSP mechanism, Nash Equilibrium, Dominant Bidding Strategy.

I. Introduction

The increasing of mobile communication equipments has spurred the growth of all kinds of intelligent systems. We can compare the price of goods, download games and browse commodities whenever and wherever. With popular game Plants vs. Zombies creating astonishing record, more and more advertisers promote their brand on mobile App. App has become key driving force of the current advertising market and advertisers' total spending for advertising on mobile App will surpass 2.9 billion.

Compared with traditional internet advertising auction(i.e. sponsored search auction), mobile advertising platform is the intermediary between the App developer and the advertiser. When a user use the App, the mobile App advertising platform would provide suitable advertisements with certain auction mechanism, such as GSP(Generalized-Second-Price) and GFP(Generalized-First-Price). The position of advertisement on the page decides how many clicks the advertisement receives. The advertiser will pay limited price for user's click. This payment will be shared by platform and App developer.

Our research is based on the sponsored search auction. However, being different from the related literature [7] which studied the behavior of advertisers and search engine in sponsored search auction, mobile advertising platform is between advertisers and App developer in mobile App advertising auction. Platform can be accurate to follow advertisement's geographical location, time, type, price, brand and previous average click through rates. Thus, similar to Aggarwal G et al. [8], we introduce the notion of weight which indicates the effect of those factors. Our mobile advertising auction mechanism generally uses the wGSP (weighted Generalized-Second-Price) mechanism.

Our contribution is to present the dominant bidding strategy for mobile advertising auction. But our work is different from Bu T M et al. [4] whose forward looking response function do not consider the weight. To be closer to the actual situation, advertisers, in our paper, are ranked by the product of his/her bid and weight. Another line of work closely related to ours is [3] in which the author studied multiple Nash Equilibria. We propose that multiple Nash Equilibria also exist in mobile advertising auction. Moreover, our work take the reserve price into account. The payment will be the maximum of the previous payment and the reserve price. This change may affect the dominant bidding strategy.

The paper is structured as follows: next we discuss the related literatures. Section III introduces the basic definitions, properties and formally describes the mobile advertising auction model. In Section IV, we firstly analyse the existence of multiple Nash Equilibria that satisfy the wGSP mechanism. Then, we propose the dominant bidding strategy with or without reserve price. Additionally, a practical implementation for a virtual market is presented to confirm the dominant bidding strategy. Section V concludes the paper.

II. Related Work

Because of the growing popularity of GSP (Generalized Second-Price) mechanism, this problem has attracted intensive studies in recent years. Renato G et al. [10] develop a Bayes-Nash analysis of the generalized second-price (GSP) auction, their results characterizes the efficient Bayes-Nash equilibrium of the GSP and provides a necessary and sufficient condition that guarantees existence of such an equilibrium. Baichun Xiao et al. [9] proposed sponsored search advertising model, which was widely used by most web search engines such as Google, Yahoo. [5], [6] pointed out "locally-free equilibrium", when the GSP auction achieved Nash Equilibrium. This notion could be extended to "global envy free equilibrium". Namely, none of the auction participants is willing to suffer a deficit. Thus, paper [1] put forward that GSP mechanism must satisfy individual rationality, which means that the price can't exceed

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value of the advertisements.

The related literature on bidding strategy has also been growing. Y. Kamijo. [2] explores the bidding behavior of advertisers in a sealed-bid environment, where each bidder does not know the current bids of others. It is shown that the SBT(secure bidding with a trial) bid adjustment process converges to some equilibrium point in a one-shot game irrespective of the initial bid profile. [11] presented an intelligent advertiser for bidding on CPC(Cost Per Click) sponsored search auctions. The advertiser developed a future look-ahead bidding plan that enabled it to hold back cash for more desirable times of the day. Cary M et al. [12] considered best-response bidding strategies for a repeated auction. Zhou Y et al. [13] showed that vindictive bidding was prevalent in sponsored search auctions, and it led to instability of most traditional Nash Equilibrium.

III. The Model

A. Basic settings

In this section, we will explain the basic definition and the mobile advertising auction model. Our payment rule is CPC (Cost Per Click). It essentially means that advertisers will pay when a user clicks on their advertisements. In the mobile App advertising auction environment, assume n risk-neutral advertisers compete for k ($n \geq k$) slots inside the mobile App. Formally, a bid submitted by advertiser i is denoted by b_i. w_i represents advertiser i's weight which is assigned by mobile Ad platform. Advertisers are ranked by the product of his/her bid and weight w_i (i.e.$w_i b_i$). If advertiser i got j-th highest position among k slots, his payment per click will be $p_i(k, j)$, which is defined as

$$p_i(k, j) \quad = \quad \frac{w_{(j+1)} b_{(j+1)}}{w_i} \tag{1}$$

where (j) denotes the bidder who gets slot j. For any two slots $(m < l)$, $w_m b_{(m)} \geq w_l b_{(l)}$ holds.

Advertisers' bids constitute the bid vector $\mathbf{b} = (b_1, b_2, ..., b_n)$. $x_{i,j}(\mathbf{b})$ represents the probability of advertiser i who gets slot j. Each advertiser can be allocated to one slot at most and each slot can be allocated to one advertiser at most, too. It can be represented as:

$$\sum_{i=1}^{n} x_{i,j}(\mathbf{b}) \quad \leq \quad 1 \tag{2}$$

$$\sum_{j=1}^{k} x_{i,j}(\mathbf{b}) \quad \leq \quad 1 \tag{3}$$

Moreover, each advertiser i has a privately known information v_i which represents the expected return of per-click to advertiser i. For simplicity, we assume that all the bidders' private values would be always different. q_i is a factor related to the quality of advertiser i. Let $\alpha_{i,j}$ denote click-through rate of advertiser i who got j-th slot among k slots. And it is defined as follows:

$$\alpha_{i,j} \quad = \quad e_j \, q_i \tag{4}$$

where e_j denotes the impact of slot j. There is $\alpha_{i,1} > \alpha_{i,2} > ...\alpha_{i,k}$ for the same advertiser.

We introduce the following five properties for mobile advertising auction:

Risk neutral: Each advertiser's target in the auction is to maximize their expected utility;

Private value: Only advertiser himself knows true value which is private information;

Independent: The advertiser's value $v_1, ..., v_n$ is independent random variables;

Symmetry: True value is the same probability distribution from a continuous random variables i.e. $F(v)$;

Individual rationality: Payment of each advertiser can't exceed value, otherwise advertiser will quit.

Assume $v_1 > v_2 >, ..., > v_n$. Then, the necessary conditions for Nash Equilibrium is

- if $n \leq k$,
  $w_{(1)} b_{(1)} > w_{(2)} b_{(2)} >, ..., w_{(n)} b_{(n)}$
- if $n > k$,
  $w_{(1)} b_{(1)} > w_{(2)} b_{(2)} >, ..., w_{(k)} b_{(k)} > w_{(k+1)} b_{(k+1)}$

Particularly, for $\forall i < j$ and $w_{(i)} b_{(i)} = w_{(j)} b_{(j)}$, advertiser i will be assigned above advertiser j.

B. Utility of Advertiser

Given bid vector $\mathbf{b}$, the expected click through rate of advertiser i is defined as

$$Q_i(\mathbf{b}) \quad = \quad \sum_{j=1}^{k} x_{i,j}(\mathbf{b}) \alpha_{i,j} \tag{5}$$

The expected payment of advertiser i is represented as follows:

$$P_i(\mathbf{b}) \quad = \quad \sum_{j=1}^{k} x_{i,j}(\mathbf{b}) \alpha_{i,j} p_i(k, j) \tag{6}$$

Thus, the utility of advertiser i can be represented as:

$$\begin{aligned} u_i(\mathbf{b}) \quad &= \quad \sum_{j=1}^{k} x_{i,j}(\mathbf{b}) \alpha_{i,j} (v_i - p_i(k, j)) \\ &= \quad v_i Q_i(\mathbf{b}) - P_i(\mathbf{b}) \end{aligned} \tag{7}$$

IV. Analysis

A. Multiple Equilibria in Mobile App Advertising Auction

1) Envy-free Equilibrium: If there is $1 \leq s < a < t \leq k$ and (8),(9), we say that this equilibrium is envy-free.

$$\begin{aligned} &\alpha_{i,a}(v_i - \frac{w_{(a+1)} b_{(a+1)}}{w_i}) \\ \geq \quad &\alpha_{i,s}(v_i - \frac{w_{(s+1)} b_{(s+1)}}{w_i}) \end{aligned} \tag{8}$$

$$\begin{aligned} & \alpha_{i,a}(v_i - \frac{w_{(a+1)}b_{(a+1)}}{w_i}) \\ \geq \quad & \alpha_{i,t}(v_i - \frac{w_{(t+1)}b_{(t+1)}}{w_i}) \end{aligned} \tag{9}$$

2) Nash Equilibria Algorithm: While some advertisers adopt different bidding strategies, they still get the same allocation in equilibrium. We call these Nash Equilibria "E-quivalence Class". For the purpose of calculating the number of Equivalence Classes, we design the Algorithm 1.

Algorithm 1 Algorithm of Nash equilibria

Input: Different kinds of bidding vector
Output: The number of Nash Equilibria

1: Set $m = 0$;
2: Select bidding vector, determine the allocation;
3: Determine whether it is the different equivalence class;
4: **if** It is the same **then**
5:     go to step (2);
6: **end if**;
7: **if** It is the different **then**
8:     go on;
9: **end if**;
10: **if** The bid vector satisfy Nash Equilibrium (all advertisers satisfy global envy free equilibrium) **then**
11:     go on;
12: **end if**;
13: **if** The bid vector satisfy Nash Equilibrium (all advertisers satisfy global envy free equilibrium) **then**
14:     go to step (2);
15: **end if**;
16: go on;
17: $m \leftarrow m + 1$, go to step(2).

According to permutation and combination, we can get the number of equivalence classes. If $n \leq k$ holds, the number will be $n!$ at most. Otherwise, the number will be $k! + (n-k)(k-1)!$ at most.

B. Dominant Bidding Strategy for Advertiser

1) Without Considering the Reserve Price: Mobile advertising auction allows advertisers to change their bids anytime. Once some bids are changed, platform will refresh the rank automatically and instantaneously. All the bidders' rank and utility will also be recalculated. Other advertisers can then have incentive to change their bids to increase their utility. While other advertisers respond to his/her bid, he/she must change bid once again to get the higher possible rank, ensuring his/her utility does not reduce.

Lemma 1: For any t: $t < s$, the necessary and sufficient condition for $u_i(k,s) \leq u_i(k,t)$ is

$$b_i \quad \leq \quad \frac{e_s}{e_t}(v_i - \frac{w_{(s+1)}b_{(s+1)}}{w_i}) \tag{10}$$

Proof: If advertiser i gets slot s, the expected utility will be

$$u_i(k,s) = (v_i - \frac{w_{(s+1)}b_{(s+1)}}{w_i})\alpha_{i,s}$$

If advertiser i gets slot t, the expected utility will be

$$u_i(k,s) = (v_i - \frac{w_{(t+1)}b_{(t+1)}}{w_i})\alpha_{i,t}$$

When advertiser i gets slot t, $w_i b_i \geq w_{(t+1)}b_{(t+1)}$ is satisfied. Then, the utility of advertiser i at least is:

$$\begin{aligned} u_i(k,t) &= (v_i - \frac{w_i}{w_i}b_i)\alpha_{i,t} \\ &= (v_i - b_i)\alpha_{i,t} \end{aligned}$$

$$\begin{aligned} b_i &\leq v_i - \frac{\alpha_{i,s}}{\alpha_{i,t}}(\alpha_{i,t})(v_i - \frac{w_{(s+1)}b_{(s+1)}}{w_i}) \\ &= v_i - \frac{e_s}{e_t}(v_i - \frac{w_{(s+1)}b_{(s+1)}}{w_i}) \end{aligned}$$

■

Owing to $t < s$, we can get $\alpha_{i,s-1} \leq \alpha_{i,t}$,

$$\begin{aligned} b'_i &= v_i - \frac{e_s}{e_{s-1}}(v_i - \frac{w_{(s+1)}b_{(s+1)}}{w_i}) \\ &\leq v_i - \frac{e_s}{e_t}(v_i - \frac{w_{(s+1)}b_{(s+1)}}{w_i}) \end{aligned} \tag{11}$$

If the advertiser i's bid is b'_i and he/she gets slot t', it will be obviously that $t' \leq s$ and $u_i(k,s) \leq u_i(k,t')$.

According to the above reasoning, we infer the following theorem.

Theorem 1: Let $\mathbf{b}_{-i} = (b_1, ..., b_{i-1}, b_{i+1}, ..., b_n)$ denote the bids of all other bidders except i. If the rank of advertiser i is s, the dominant bidding strategy for advertiser i is

$$F_i(\mathbf{b}_{-i}) = \begin{cases} v_i & s = 1 \\ v_i & s > k \\ v_i - \frac{e_s}{e_{s-1}}(v_i - \frac{w_{(s+1)}}{w_i}b_{(s+1)}) & 2 \leq s \leq k \end{cases} \tag{12}$$

Proof: First, let's prove that the dominant bidding strategy for advertiser i is an Nash Equilibrium strategy . According to (12), we will have

$$q_i e_l b_{(s+1)} \quad = \quad q_i \sum_{j=s}^{k}(e_j - e_{j+1})\frac{w_{(j+1)}v_{(j+1)}}{w_i} \tag{13}$$

Advertiser i earns

$$\begin{aligned} u_i(\mathbf{b}) &= \sum_{j=1}^{k} x_{i,j}(\mathbf{b})\alpha_{i,j}(v_i - p_i(k,j)) \\ &= \alpha_{i,s}(v_i - b_{(s+1)}) \\ &= q_i\sum_{j=s}^{k}(e_j - e_{j+1})v_i - q_i\sum_{j=s}^{k}(e_j - e_{j+1})\frac{w_{(j+1)}v_{(j+1)}}{w_i} \\ &= q_i\sum_{j=s}^{k}(e_j - e_{j+1})(v_i - \frac{w_{(j+1)}v_{(j+1)}}{w_i}) \end{aligned} \tag{14}$$

But, if advertiser i changes the bid to get slot ϕ ($\phi \neq s$), he/she will earns

$$\begin{aligned} u_i(\mathbf{b})' &= q_i e_\phi (v_i - b_{(\phi+1)}) \\ &= q_i \sum_{j=\phi}^{k} (e_j - e_{j+1})(v_i - \frac{w_{(j+1)} v_{(j+1)}}{w_i}) \end{aligned} \quad (15)$$

For any $\phi > s$, the net utility from this deviation is equal to

$$\triangle u_i(\mathbf{b}) = -q_i \sum_{j=s}^{\phi-1} (e_j - e_{j+1})(v_i - \frac{w_{(j+1)} v_{(j+1)}}{w_i}) \quad (16)$$

Since $w_i b_i > w_{(s+1) b_{(s+1)}} >, ..., > w_{(\phi-1)} b_{(\phi-1)}$, we know that $\triangle u_i(\mathbf{b}) < 0$.

For $\phi < s$, the net utility is equal to

$$\triangle u_i(\mathbf{b})' = q_i \sum_{j=\phi}^{s-1} (e_j - e_{j+1})(v_i - \frac{w_{(j+1)} v_{(j+1)}}{w_i}) \quad (17)$$

Since $w_i b_i < w_{(s-1) b_{(s-1)}} <, ..., < w_{(\phi)} b_{(\phi)}$, it is obviously that $\triangle u_i(\mathbf{b})' < 0$. Hence, the deviation is not profitable and strategy $F_i(\mathbf{b}_{-i})$ is a Nash Equilibrium strategy.

Then, in this new created Nash Equilibrium, advertiser i's rank and utility will not decrease according to Lemma1. Therefore, $F_i(\mathbf{b}_{-i})$ is advertiser i's dominant bidding strategy. ■

2) Considering the Reserve Price: In order to guarantee the utility of the App developer, mobile advertising platform may set reserve price for each slot. Then, some advertisers whose true value is lower than the reserve price may quit the auction. If advertiser i gets slot j, the payment will be

$$p_i(k, j) = \max(\frac{w_{(j+1)} b_{(j+1)}}{w_i}, r_j) \quad (18)$$

subject to $\quad v_i \geq r_j$

where r_j denotes the reserve price of the slot j.

If advertiser i gets slot t ($v_i \geq r_t$), the utility is

$$u_i(k, t) = (v_i - p_i(k, t)) \alpha_{i,t} \quad (19)$$

where

$$p_i(k, t) = \max(\frac{w_{(t+1)} b_{(t+1)}}{w_i}, r_t)$$

According to $w_i b_i \geq w_{(s+1)} b_{(s+1)}$, if there exists slot s ($v_i \geq r_s$), the utility of advertiser i would be at least:

$$u_i(k, s) = (v_i - \frac{w_i}{w_i} b_i) \alpha_{i,s} = (v_i - b_i) \alpha_{i,s}$$

In order to ensure

$$u_i(k, s) \geq u_i(k, t)$$

We obtain the following inequality:

$$\begin{aligned} (v_i - p_i(k, t)) \alpha_{i,t} &\leq (\frac{w_i}{w_i} b_i) \alpha_{i,s} \\ &= (v_i - b_i) \alpha_{i,s} \end{aligned}$$

$$b_i \leq v_i - \frac{e_t}{e_s}(v_i - \max(\frac{w_{(t+1)} b_{(t+1)}}{w_i}, r_t) \quad (20)$$

Then, we get the following corollary:

Corollary 1: If the rank of advertiser i is t and the reserve price of slot t is r_t, the dominant bidding strategy for advertiser i is as follows:

$$G_i(\mathbf{b}_{-i}) = \begin{cases} v_i & t = 1 \\ v_i & t > k \\ v_i - \frac{e_t}{e_{t-1}}(v_i - max(\frac{w_{(t+1)}}{w_i} b_{(t+1)}, r_t) & 2 \leq t \leq k \end{cases} \quad (21)$$

Proof: If $p_i(k, t) = \frac{w_{(t+1)}}{w_i} b_{(t+1)} \geq r_t$, the reserve price would have no effect on the bidding strategies . Therefore, bidding strategy for advertiser i is the same with (12). Otherwise, advertiser i should compare the payment with reserve price. ■

C. Practical implementation for a virtual market

We implemented the dominant bidding strategy for a virtual market. This subsection demonstrates the result of this implementation.

The initial data is as TABLE I:

TABLE I
INITIAL PARAMETERS

$n = 3$	$k = 2$	
$v_1 = 5$	$v_2 = 4.5$	$v_3 = 1$
$w_1 = 3$	$w_2 = 3$	$w_3 = 1.5$
$\alpha_{1,1} = 0.2$	$\alpha_{1,2} = 0.1$	
$\alpha_{2,1} = 0.2$	$\alpha_{2,2} = 0.1$	
$b_1 = 2$	$b_2 = 3$	$b_3 = 1$

Because all advertisers have no incentive to change the bid in this state, Nash Equilibrium exists for the above bidding strategies. If advertiser 1 changes the bid to b'_1 based on the dominant bidding strategy. i.e.

$$\begin{aligned} b'_1 &= v_1 - \frac{\alpha_{1,2}}{\alpha_{1,1}}(v_1 - \frac{w_3 b_3}{w_1}) \\ &= 5 - \frac{0.1}{0.2}(5 - \frac{1.5}{3}) \\ &= 2.75 \end{aligned}$$

Then,

$$\begin{aligned} u'_2(2, 1) &= (v_2 - \frac{w_1 b'_1}{w_2}) \times \alpha_{2,1} \\ &= (4 - \frac{2.75 \times 3}{3}) \times 0.2 \\ &= 0.25 \end{aligned}$$

For $u'_2(2, 1) < u_2(2, 1)$, advertiser 2 must get slot 2 by reducing bid. Then,

$$\begin{aligned} u_1''(2,1) &= (v_1 - \frac{w_2 b_2'}{w_1}) \times \alpha_{1,1} \\ &= (5 - \frac{6}{3}) \times 0.2 \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} u_2''(2,2) &= (v_2 - \frac{w_3 b_3}{w_3}) \times \alpha_{2,2} \\ &= (4 - \frac{1.5}{3}) \times 0.2 \\ &= 0.35 \end{aligned}$$

If advertiser 3 get slot 1 or 2, advertiser 1 or 2 will increase bid. So advertiser 3 can not get advertising slot and his utility is 0.

TABLE II shows that bidder 2 has to decrease the bid to get the optimal utility when bidder 2 changes the bid. After simulating this bidding process for the virtual market, a new created Nash Equilibrium exists.

As we expected, both advertiser 1's utility and the slot increase, comparing with the initial state.

TABLE II
VARIETY OF PARAMETERS

	Previous bidding	After adjusted bidding
b_1	2	2.75
b_2	3	1.5
b_3	1	1
u_1	0.45	0.6
u_2	0.4	0.35
u_3	0	0
Rank of advertiser 1	2	1
Rank of advertiser 2	1	2
Rank of advertiser 3	3	3

The results of our implementation for the virtual market presented in this subsection shows that the dominant bidding strategy will work in real world environments.

V. Conclusion

This paper discusses two major problems in mobile advertising auction − multiple Nash Equilibria and dominant bidding strategy. This paper firstly designs a new mobile advertising auction model, in which mobile advertising platform allocates slots to advertisers considering weight. After providing existence of multiple Nash Equilibria, an algorithm for calculating the number of "Equivalence Class" is presented. Then, we put forward dominant bidding strategy. The objective of this bidding strategy is to increase advertiser's rank and the utility, comparing with the previous status. At last, the results of implementation for the virtual market presented shows that the dominant bidding strategy will work in real world environments.

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