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Abstract—In this brief, a new criterion for checking the global asymptotic stability of fixed-point state–space digital filters with time-varying delay and overflow arithmetics is presented. Compared with some existing results, a distinctive feature of the proposed criterion is that it is multi-bound-dependent criterion and can be less conservative than existing results. Two examples are given to show this improvement over the existing conditions.

Index Terms—Asymptotic stability, digital filters, linear matrix inequality, overflow arithmetics.

I. INTRODUCTION

DIGITAL filters are essential elements of everyday electronics such as radios, cell phones, and stereo receivers. By using them, undesirable spectral components and channel bandwidth can be removed and controlled, respectively. While implementing digital filters using fixed-point arithmetics, the finite register length of digital hardware or computer generates overflow nonlinearities [1], [9], [11], which may be saturation, zeroing, or two’s complement arithmetics. So far, there exist many criteria for two’s complement arithmetics or saturation arithmetics [3]–[8], [13], [15], [19]. Recently, a unified form of overflow nonlinearities is presented in [1], which can include two’s complement and saturation arithmetics as special cases. However, there are no generalized stability conditions for the designed digital filters using this unified form. Meanwhile, time delay is frequently encountered in many engineering applications in [2], [9], and [11], which also exists in digital filters. For example, a causal digital filter with a fixed order and cutoff frequency will delay different frequency signals. This is an undesirable yet unavoidable effect of filtering. However, the existing results for digital filters [3]–[6], [8], [13], [15], [19] are not available under the unfavorable environments with time-varying delay. Thus, it is necessary to develop a new approach to dealing with time delay and overflow arithmetic effects on the stability of the designed filters. The results will be valuable in the design of digital filters.

Motivated by the preceding discussion, in this brief, a new multi-bound-dependent stability criterion, which includes bound-dependent delay and overflow arithmetics, is proposed for fixed-point state–space digital filters using overflow arithmetics. It is shown that the presented criterion is less conservative or more general than those in [5], [7], [8], and [13]. In addition, our presented method may also extend to the design problem of digital filters under the environment of network control systems or hybrid systems [17], [23] by using some new techniques, such as the average dwell time approach given in [18].

Notation: Throughout this brief, for a real symmetric matrix $P$, notation $P > 0 \geq 0$ means that $P$ is a positive definite (positive semi-definite) matrix, $A > (\geq)B$ means $A - B > (\geq)0$, and $\text{diag}\{\cdot\}$ stands for a block-diagonal matrix. Superscripts $\top$ and $\cdot$ denote the transpose and absolute values of a vector or a matrix. The symbol $*$ stands for a term induced by symmetry in a symmetric matrix.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following digital filter:

$$
\begin{align*}
&x(k + 1) = f(y(k)) = [f_1(y_1(k)) \cdots f_n(y_n(k))]^\top \\
&y(k) = [y_1(k) \cdots y_n(k)]^\top = Ax(k) + A_dx(k - d(k)) \quad (1)
\end{align*}
$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $A, A_d \in \mathbb{R}^{n \times n}$, are the state matrices; $y(k) \in \mathbb{R}^n$ is the system output vector; $d(k)$ is the time-varying delay satisfying $h_1 \leq d(k) \leq h_2$ and $1 \leq h_1 < h_2$ with $h_1$ and $h_2$ being integers; $\phi_i(k)$, $i = 1, 2, \ldots, n$, are the initial conditions satisfying $|\phi_i(k)| \leq 1$; and $f_i(\cdot)$, $i = 1, \ldots, n$, are piecewise continuous functions.

The nonlinearities $f_i(\cdot)$, $i = 1, \ldots, n$, are given by (see [1])

$$
\begin{align*}
&-1 \leq l_i \leq f_i(y_1(k)) \leq l_i \leq 1, \quad y_i(k) > 1 \quad (2a) \\
&f_i(y_1(k)) = y_i(k), \quad -1 \leq y_i(k) \leq 1 \quad (2b) \\
&-1 \leq -l_2i \leq f_i(y_1(k)) \leq -l_i \leq 1, \quad y_i(k) < -1 \quad (2c)
\end{align*}
$$

With an appropriate choice of $l_i$, $l_1i$, and $l_2i$, (2) represents the various overflow arithmetics, namely, saturation ($l_i = l_1i = l_2i = 1$), zeroing ($l_i = l_1i = l_2i = 0$), and two’s complement
(l_i = -1, l_{ij} = l_{ji} = 1), for instance. For the nonlinear functions satisfying (2), we have the following lemmas.

**Lemma 1:** For given nonlinear functions satisfying (2), the following inequalities hold:

\[ [y_i(k) - f_i(y_i(k))] f_i(y_i(k)) - l_i y_i(k) \geq 0 \quad (3) \]

where \( \hat{l}_i = \min\{l_i, 0\} \).

**Proof:** When \( l_i < 0 \), it means \( \hat{l}_i = l_i \). There are three possible cases: 1) \( y_i(k) > 1 \); 2) \( 1 \leq y_i(k) \leq 1 \); and 3) \( y_i(k) < -1 \). For case 1, we can obtain \( [y_i(k) - f_i(y_i(k))] > 0 \) from (2) and

\[ f_i(y_i(k)) - l_i y_i(k) = [f_i(y_i(k)) - l_i] + [l_i - l_i y_i(k)] 
\]

It is obvious that (3) holds for this case. For case 2, because \( y_i(k) = f_i(y_i(k)) \), the inequalities \([y_i(k) - f_i(y_i(k))]|f_i(y_i(k)) - l_i y_i(k)| = 0 \) are true. For case 3, we have \( [y_i(k) - f_i(y_i(k))] < 0 \) and

\[ f_i(y_i(k)) - l_i y_i(k) = [f_i(y_i(k)) - l_i] - [l_i - l_i y_i(k)] \]

It is clear that (3) holds.

When \( l_i \geq 0 \), it means \( \hat{l}_i = 0 \). We only need to show that \([y_i(k) - f_i(y_i(k))] f_i(y_i(k)) > 0 \) holds. When \( y_i(k) > 1 \), by using (2), we have the terms \([y_i(k) - f_i(y_i(k))] f_i(y_i(k)) \) to have the same positive or negative sign, which can guarantee \([y_i(k) - f_i(y_i(k))] f_i(y_i(k)) > 0 \). When \( y_i(k) \leq 1 \), it is easy to see that \([y_i(k) - f_i(y_i(k))] f_i(y_i(k)) = 0 \). Hence, we always have (3) holds with \( \hat{l}_i = \min\{l_i, 0\} \). This completes the proof of Lemma 1.

**Remark 1:** Recently, Kar and Singh in [1] had considered a special case of the nonlinearities in (2) with \( l_1 = l_2 = \cdots = l_n = L, \ l_{11} = l_{12} = \cdots = l_{nn} = L_1, \ l_{21} = l_{22} = \cdots = l_{nn} = L_2 \). In [1], they divided the condition in (2) into two parts: \(-1 \leq L < 0 \) and \( 0 \leq L \leq 1 \). When \(-1 \leq L < 0 \), \([y_i(k) - f_i(y_i(k))]|f_i(y_i(k)) - L_i y_i(k)| > 0 \) are obtained in [1], which is similar to (3) in Lemma 1. When \( 0 \leq L \leq 1 \), by introducing some new parameters \( \lambda_{ij} \) with constraint \( \sum_{j=1}^{n} \lambda_{ij} = 1 \), \([y_i(k) - f_i(y_i(k))]|f_i(y_i(k)) - L_i y_i(k)| > 0 \) are derived in [1]. Instead of separating the bound of the nonlinearities in (2) and introducing the new variables from [1], a unified result only including \( \hat{l}_i \) is obtained in Lemma 1, which is more general than that used in [1].

**Lemma 2 ([15]):** For a given system (1) satisfying (2), if there exist a diagonal matrix \( S = \text{diag}\{s_1, s_2, \ldots, s_n\} \in \mathbb{R}^{n \times n} \geq 0 \) and matrices \( M = [m_{ij}], N = [n_{ij}] \in \mathbb{R}^{n \times n} \) satisfying \( \sum_{i=1}^{n} m_{ij} + \sum_{i=1}^{n} n_{ij}, \ i, j = 1, \ldots, n \), then the following inequality holds:

\[ [y^T(k) S + x^T(k) M^T + f^T(y(k)) N^T] [y(k) - f(y(k))] \geq 0. \]

The purpose of this brief is to develop multi-bound-dependent stability criterion for system (1) satisfying the condition in (2) and to compare it with the existing results in [5], [7], [8], [13].

### III. MAIN RESULTS

First, we present a new stability criterion for system (1).

**Theorem 1:** Given system (1) satisfying condition (2), if there exist matrices \( P > 0; \ Q_1 > 0; \ Q_2 > 0; \ Q_3 > 0; \ Z_1 > 0; \ Z_2 > 0; D = \text{diag}\{d_1, d_2, \ldots, d_n\} \geq 0, S = \text{diag}\{s_1, s_2, \ldots, s_n\} \geq 0, D, S \in \mathbb{R}^{n \times n} \); and \( M = [m_{ij}], N = [n_{ij}] \in \mathbb{R}^{n \times n} \), such that

\[
\Omega = \begin{bmatrix}
\bar{\Omega} & Y \\
Y^T & \Delta
\end{bmatrix} < 0 \tag{5a}
\]

\[
s_j \geq \sum_{i=1}^{n} m_{ij} + \sum_{i=1}^{n} n_{ij}, \ j = 1, \ldots, n
\]

\[
\bar{\Omega} = \begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} \\
\Omega_{12}^T & \Omega_{22} & \Omega_{23} \\
\Omega_{13}^T & \Omega_{23}^T & \Omega_{33}
\end{bmatrix}, \ Y = \begin{bmatrix}
0 & 0 \\
\frac{1}{h_1} Z_1 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
\Delta = \begin{bmatrix}
-Q_1 - \frac{1}{h_1} Z_1 & -\frac{1}{h} Z_2 & \frac{1}{h} Z_2 \\
-\frac{1}{h} Z_2 & -Q_2 - \frac{1}{h} Z_2
\end{bmatrix} \tag{5b}
\]

where

\[
\Omega_{11} = P + h_1 Z_1 + \hat{h} Z_2 - 2 D - N - N^T
\]

\[
\Omega_{12} = -h_1 Z_1 - \hat{h} Z_2 + (D \hat{L} + D - S) A - M + N^T A
\]

\[
\Omega_{13} = (D \hat{L} + D - S + N^T) A_d
\]

\[
\Omega_{22} = -P + Q_1 + Q_2 + (1 + \hat{h}) Q_3 + \left(\frac{h_1}{h_1} - \frac{1}{h_1}\right) Z_1 + \hat{h} Z_2 - 2 A^T (D \hat{L} - S) A + M^T A + A^T M
\]

\[
\Omega_{23} = -2 A^T (D \hat{L} - S) A_d + M^T A_d
\]

\[
\Omega_{33} = -Q_3 - 2 A_d^T (D \hat{L} - S) A_d
\]

\[
\hat{h} = h_2 - h_1
\]

\[
\hat{L} = \text{diag}\{\hat{l}_1, \hat{l}_2, \ldots, \hat{l}_n\}
\]

then the zero solution of system (1) is globally asymptotically stable.

**Proof:** Define a Lyapunov function as

\[
V(x(k)) = x^T(k) P x(k) + \sum_{j=1}^{2} \sum_{i=k-h}^{k-1} x^T(i) Q_j x(i)
\]

\[
+ \sum_{i=k-d}^{k-1} x^T(i) Q_3 x(i) + \sum_{j=h_2+1}^{k-1} \sum_{i=k+j}^{k-1} x^T(i) Q_3 x(i)
\]

\[
+ \sum_{i=-h_1}^{k-1} \sum_{i=k+j}^{k-1} \eta^T(i) Z_1 \eta(i) + \sum_{j=-h_2+1}^{k-1} \sum_{i=k+j}^{k-1} \eta^T(i) Z_2 \eta(i)
\]

(6)

where \( P > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, Z_1 > 0, Z_2 > 0 \), and \( \eta(i) = x(i+1) - x(i) \). Along the trajectories of system (1), one has

\[
\Delta V(x(k)) = x^T(k+1) P x(k+1) - x^T(k) P x(k)
\]

\[
+ x^T(k) Q_1 + Q_2 + Q_3 x(k)
\]

\[
- x^T(k - h_1) Q_1 x(k - h_1)
\]

\[
- x^T(k - h_2) Q_2 x(k - h_2)
\]
where $\alpha = 2[y^T(k) - f^T(y(k))]^\top D[f(y(k)) - \hat{L} y(k)] + D = \text{diag}(d_1, \ldots, d_n) \geq 0$, $\hat{L} = \text{diag}\{\hat{\ell}_1, \ldots, \hat{\ell}_n\}$, and $\beta = 2[y^T(k) S + x^T(k) M^T + f^T(y(k)) M^T] y(k) - f(y(k))$. From Lemmas 1 and 2, $\alpha \geq 0$ and $\beta \geq 0$ hold. Use the method in [14] to obtain

$$- \sum_{i = k - h_1}^{k - 1} \eta^i(i) Z_1 i(i) \leq - \frac{1}{h_1} [x(k) - x(k - h_1)]^\top \times Z_1 [x(k) - x(k - h_1)]$$

(8)

$$- \sum_{i = k - h_2}^{k - h_1 - 1} \eta^i(i) Z_2 i(i) \leq - \frac{1}{h} [x(k - h_1) - x(k - h_2)]^\top \times Z_2 [x(k - h_1) - x(k - h_2)].$$

(9)

Note that

$$- \sum_{i = k + 1 - d(k) + 1}^{k - 1} \eta^i(i) Q_3 x(i) - \sum_{i = k + 1 - d(k) + 1}^{k - 1} \eta^i(i) Q_3 x(i) \leq \sum_{i = k + 1 - d(k) + 1}^{k - 1} \eta^i(i) Q_3 x(i).$$

(10)

From (7)–(10), we can get that

$$\Delta V(x(k)) \leq \zeta^T(k) \Omega \zeta(k) - \alpha - \beta$$

(11)

where $\zeta(k) = [f^T(y(k)) x^T(k) x^T(k - d(k)) x^T(k - h_1)]^\top$, and $\Omega$ is defined in (5). Therefore, if the condition in (5) is satisfied, $\Delta V(x(k)) < 0$ is satisfied. Thus, the system described in (1) is asymptotically stable. This completes the proof. □

In this brief, we extend the idea in [14] to analyze the stability problem of fixed-point state–space digital filters with time-varying delay and overflow arithmetics. However, we do not introduce some free-weighting matrices in solving a difference process such as Proposition 1 [14]. These matrices may lead to larger computational burden. The other significant contribution is that the characteristic of overflow arithmetics is introduced in (7). Thus, Theorem 1 depends not only on delay parameters $h_1$ and $h_2$ but also on the overflow arithmetics bounds $\hat{\ell}_i$. A multi-bound-dependent stability condition for system (1) is first proposed. On the other hand, recently, there are some discussions about complex networks [20]–[22]. It is noted that complex networks usually can be described as discrete-time systems with nonlinear transmission and time-varying delays [22]. A similar description method is also used in digital filters, as shown in system (1). Thus, our method may be extended to some relevant issues associated with complex networks.

Remark 2: Theorem 1 is a unified stability criterion for overflow arithmetics, which can include saturation case, zeroing case, and two’s complement case, etc. In practice, a digital filter usually only experiences one of the above overflow arithmetic cases within a certain period. Thus, Theorem 1 can be further simplified to handle one specific overflow case, such as subsequent Corollaries 1 (the two’s complement case) and 2 (the saturation case). The conditions given in Corollaries 1 and 2 are much simpler. On the other hand, Theorem 1 is described in linear matrix inequality (LMI) form without any extra slack variables. Thus, it is easy to verify by using the Matlab LMI toolbox with lesser computational burden.

In order to compare with the results in [5], [7], [8], and [13], we can obtain the following two corollaries from Theorem 1 when there is no time delay.

**Corollary 1:** Given system (1) satisfying condition (2) with $\hat{L} = -I_n$, if there exist a matrix $P > 0$, $P \in \mathbb{R}^{n \times n}$; diagonal matrices $D = \text{diag}\{d_1, d_2, \ldots, d_n\} \geq 0$, $S = \text{diag}\{s_1, s_2, \ldots, s_n\} \geq 0$, $D, S \in \mathbb{R}^{n \times n}$; and matrices $M = [m_{ij}]$, $N = [n_{ij}] \in \mathbb{R}^{n \times n}$ such that (5b) holds and

$$\Pi = \begin{bmatrix} \Pi_{11} & \ast \\ -P + 2A^\top(D + S)A + M^\top A + A^\top M & \ast \end{bmatrix} < 0$$

(12)

where $\Pi_{11} = P - N - N^\top - 2D$ then the zero solution of system (1) is globally asymptotically stable.

For the $\hat{L} = -I_n$ case, a similar result can be found in Theorem 4 [1]. However, it is noted that the result in [1] can be covered by setting $N = M = S = 0$ in Corollary 1.

**Corollary 2:** Given system (1) satisfying condition (2) with $\hat{L} = 0_n$, if there exist a matrix $P > 0$, $P \in \mathbb{R}^{n \times n}$; diagonal matrices $D = \text{diag}\{d_1, d_2, \ldots, d_n\} \geq 0$, $S = \text{diag}\{s_1, s_2, \ldots, s_n\} \geq 0$, $D, S \in \mathbb{R}^{n \times n}$; and matrices $M = [m_{ij}]$, $N = [n_{ij}] \in \mathbb{R}^{n \times n}$ such that (5b) holds and

$$\bar{\Pi} = \begin{bmatrix} \Pi_{11} & \ast \\ -P + 2A^\top(SA + M^\top A + A^\top M) & \ast \end{bmatrix} < 0$$

(13)

where $\Pi_{11}$ is defined in (12), then the zero solution of system (1) is globally asymptotically stable.

Corollaries 1 and 2 are global asymptotic stability conditions for the system (1) using two’s complement arithmetics and saturation arithmetics, respectively. It is obvious that two corollaries are different for considering the bounds of overflow arithmetics. However, since the methods in [6], [13], and [15] neglect the bounding information of overflow arithmetics, the stability criteria are the same for two’s complement arithmetics and saturation arithmetics.
IV. COMPARISON ON SATURATION ARITHMETICS

In this brief, we will mainly compare the saturation case in [7] and [13], where the following assumptions are made:

\begin{align}
  k_i > 1, & \quad i = 1, \ldots, m \\
  k_i \leq 1, & \quad i = m + 1, \ldots, n 
\end{align}

(14a)\hspace{1cm}(14b)

where

\[ k_i = \sum_{j=1}^{n} |a_{ij}|, \quad i = 1, \ldots, n \]

\[ A = [a_{ij}], \quad m \text{ is an integer with } 1 \leq m \leq n. \]

As noted in [15], the criteria in [7] and [13] are equivalent to Mills–Mullis–Roberts criterion [5] when \( m = n \). Meanwhile, it has been shown in [19] that the Mills–Mullis–Roberts criterion [5] is only a special case of the Singh criterion in [8] when \( m = n \). Hence, the main purpose of this part is to compare Corollary 2 with the result in [8]. First, we provide the result in [8] as follows.

**Lemma 3 ([8]):** The null solution of the system described in (1) and (2) with \( l_1 = 1, l_2 = 1, i = 1, \ldots, n \), is asymptotically stable if there exist a matrix \( G > 0 \) and a diagonal matrix \( T > 0 \) such that

\[ \begin{bmatrix} G & -A^T \\bar{T} \\ -TA & 2T - G \end{bmatrix} > 0. \]

(15)

\[ \theta \]

**Remark 3:** It is noted that Lemma 3 can be recovered by letting \( M = N = S = 0 \), \( G = P \), and \( D = T \) in (13). This means Corollary 2 is less conservative than the existing results in [5], [7], [8], and [13] when \( m = n \). Moreover, the assumption in (14) is not used in our method. Thus, our result is applicable for the arbitrary system matrix \( A \), regardless of the values of \( k_i \).

**Remark 4:** Shen et al. [16] presented a new stability condition for system (1) with saturation nonlinearities. The result in [16] may be less conservative than Corollary 2 by introducing general matrices \( M \) and \( D \) defined in [16] and our Corollary 2, respectively. However, this will lead to requiring more computational burden than our Corollary 2.

**Remark 5:** Recently, a new stability criterion in [19] is derived for system (1) without time delay by assuming a decomposition on the system matrix \( A \), as in (14). Thus, it requires less computational effort than our corollaries when \( m < n \). However, it is also noted that constraint \( m \geq 1 \) in (14) established in [19] will not be applicable to the case \( |k_i| \leq 1 \), \( i = 1, \ldots, n \). Our results do not have such a restriction.

V. NUMERICAL EXAMPLES

In this section, we use two examples to illustrate the usefulness of our results.

**Example 1:** Consider the second-order system in (1) without time delay, and

\[ A = \begin{bmatrix} \theta_1 & -0.4 \\ 0.6 & \theta_2 \end{bmatrix}, \quad l_1 = l_2 = 1. \]

(16)

When we choose \( (\theta_1, \theta_2) = (1, 0.7) \) and \( (0.8, 0.1) \), it leads to case \( m = n \) and case \( m < n \) from (14), respectively. Moreover, for the two cases, it can be verified that the criteria on saturation arithmetics given in [5], [7], [8], and [13] fail to conclude whether this system is asymptotically stable or not. On the other hand, applying Corollary 2 to this example, it can deduce the stability of the system, and the feasible solution sets are given as follows. When \( (\theta_1, \theta_2) = (1, 0.7) \), we have

\[ P = \begin{bmatrix} 9.26 & -2.08 \\ -2.08 & 6.16 \end{bmatrix}, \quad M = \begin{bmatrix} -0.66 & -0.58 \\ -1.33 & -0.27 \end{bmatrix}, \quad N = \begin{bmatrix} 0.10 & -0.71 \\ -1.08 & 0.01 \end{bmatrix}, \quad D = \text{diag}(5.24, 3.68), \quad S = \text{diag}(4.00, 3.65). \]

When \( (\theta_1, \theta_2) = (0.8, 0.1) \), we have

\[ P = \begin{bmatrix} 2.41 & -0.38 \\ -0.38 & 1.32 \end{bmatrix}, \quad M = \begin{bmatrix} -0.06 & 0.01 \\ -0.09 & 0.03 \end{bmatrix}, \quad N = \begin{bmatrix} 0.02 & -0.05 \\ -0.10 & 0.01 \end{bmatrix}, \quad D = \text{diag}(1.45, 1.17), \quad S = \text{diag}(1.05, 1.06). \]

such that (13) holds. Therefore, our presented criterion can ensure this system stability. The state \( x(k) \) of the system (16) is illustrated in Figs. 1 and 2 when the initial condition is \([0.8, -0.8]\), which further show that the system (16) is globally stable.

It is noted that, in Example 1, the results in [7] and [13] are about the two’s complement arithmetics case. In fact, saturation arithmetics is only a special case of two’s complement arithmetics from inequality (2). Then, it is clear that, if the stability criteria for two’s complement arithmetics in [7] and [13] are not satisfied, it means that the methods in [7] and [13] certainly fail to conclude whether this system is asymptotically stable or not for saturation arithmetics case.

**Example 2:** In this example, the time delay is considered. Given the system in (1) with \( l_1 = -1, l_2 = 0, h_1 = 1 \), and

\[ A = \begin{bmatrix} 0.3 & -0.4 \\ 0.5 & 0.7 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \]
the maximum value of $b_2$ such that (5) holds is equal to 8, and a feasible solution set is given by

\[
P = \begin{bmatrix} 895.22 & 167.04 \\ 167.04 & 661.37 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 4.06 & 1.18 \\ 1.18 & 3.57 \end{bmatrix}
\]

\[
Q_2 = \begin{bmatrix} 4.09 & 0.32 \\ 0.32 & 1.41 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 50.79 & -3.88 \\ -3.88 & 22.48 \end{bmatrix}
\]

\[
Z_1 = \begin{bmatrix} 4.24 & 10.67 \\ 10.67 & 33.25 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 0.45 & 1.12 \\ 1.12 & 3.45 \end{bmatrix}
\]

\[
M = \begin{bmatrix} -14.29 & 315.70 \\ -10.90 & -39.99 \end{bmatrix}, \quad N = \begin{bmatrix} 74.26 & 0.64 \\ 0.64 & 177.62 \end{bmatrix}
\]

\[
D = \text{diag}(404.18, 388.71), \quad S = \text{diag}(405.84, 310.33).
\]

Therefore, our criterion given by Theorem 1 can ensure the stability of this system. It is noted that the delay-independent result in [24] is not feasible.

VI. CONCLUSION

In this brief, a multi-bound-dependent stability criterion for fixed-point state–space digital filters employing overflow arithmetic has been established. The obtained stability criterion has been shown to be less restrictive than some existing results. Further reduction of conservatism is expected when using some new technique such as the delay-partitioning approach to improve the stability criterion of fixed-point state–space digital filters with time delay.

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