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Simulation of Electromagnetic Waves in the Magnetized Cold Plasma by a DGFETD Method

Ping Li, Student Member, IEEE, and Li Jun Jiang, Senior Member, IEEE

Abstract—The properties of electromagnetic waves propagating in magnetized plasma slabs and scattering from plasma coated spheres are investigated using the discontinuous Galerkin finite-element time-domain (DGFETD) method. The magnetized plasma is a kind of gyroelectric anisotropic dispersive medium. This anisotropy must be described by an electric permittivity tensor with both nonzero diagonal and off-diagonal elements. Instead of doing the convolution directly in the time domain, an auxiliary differential equations (ADE) method is employed to represent the constitutive relation. The algorithm is validated by studying the transmission characteristics via electron-composed plasma slabs and radar cross section (RCS) of plasma-coated spheres. All numerical results demonstrate very good agreements with exact solutions and comply with the plasma physics.

Index Terms—Anisotropic magnetized plasma, auxiliary differential equation (ADE), discontinuous Galerkin finite-element time-domain (DGFETD) method, gyroelectric dispersive media.

I. INTRODUCTION

O

VER the past several decades, studies of electromagnetic wave propagation in cold plasma have continued in an increasing scale because of its importance in many practical areas [1]–[3] such as the propagation of radio waves in Earth’s ionosphere and magnetosphere [1], [12].

Many finite-difference time-domain (FDTD) models are extensively exploited to model the radio wave propagation through magnetized plasma slabs [2], [3]. Various treatments using FDTD have been developed [3], [4]. However, the spatial and time staggering of the electric/magnetic vector fields and plasma related parameters in FDTD models make the FDTD more cumbersome. An alternative to FDTD is finite-element time-domain (FETD) method that is also capable of handling anisotropic dispersive media [5]. The discontinuous Galerkin finite-element time-domain (DGFETD) method was first developed to solve the neutron transport problem in 1970s. Recently, it attracted extensive attention due to its various merits. DGFETD is a highly efficient method for solving the two first-order time-dependent Maxwell’s equations [6], [7]. It supports various types and shapes of elements and unstructured meshes with high-order accuracy. Furthermore, all the operations are local since solutions are allowed to be discontinuous at the neighboring element interfaces. In this way, the resultant mass matrix is block-diagonal, which enables a very efficient explicit time marching scheme, at the cost of added degrees of freedom. The aim of this letter is to study the propagation of electromagnetic waves in a magnetized plasma slab via DGFETD method. Recently, DGFETD has been employed to simulate the propagation of electromagnetic waves in isotropic dispersive materials [8], [9] such as unmagnetized plasma [14]. In [10], DGFETD is further extended to handle anisotropic media with only nonzero diagonal elements by reformulating the numerical flux. In our work, however, auxiliary differential equation (ADE) technique is applied while the formulation of numerical flux is the same as the isotropic case.

In this letter, a DGFETD algorithm is developed to explore the propagation of the electromagnetic wave in magnetized plasma. The method is based on the ADE formulation for the constitutive relationships. Via ADE method, an auxiliary polarization current is introduced to equivalently represent the effects of the gyroelectric medium. As far as the authors’ knowledge, it is the first time that DGFETD is employed to model such kind of gyroelectric medium.

II. ADE AND DGFETD FORMULATION

The well-known permittivity expression for the unmagnetized plasma contributed by electrons is [11]:

\[ \varepsilon(\omega) = \varepsilon_0 [1 - \omega_p^2 / \omega (\omega - j \tau_e)], \]

where \( \omega_p \) denotes the plasma frequency and \( \tau_e \) is the electron collision frequency. Notice that below \( \omega_p \), the permittivity is negative, meaning that the plasma is opaque for waves at this frequency band. When a plasma is subjected to a static magnetic field, such as in the Earth’s ionosphere, the electrons will rotate around the magnetic field vector due to the Lorentz force. The electric permittivity of the magnetized plasma (gyroelectric medium) will become a full tensor \( \tilde{\varepsilon}(\omega) \).

Supposing a plasma is subjected to an arbitrary-oriented static magnetic field, direct derivation of the ADEs is difficult. To tackle this problem, we split the biasing magnetic field \( \mathbf{B}_0 \) into \( x \)-, \( y \)-, and \( z \)-components: \( \mathbf{B}_0 = B_0^x \hat{x} + B_0^y \hat{y} + B_0^z \hat{z} \). Naturally, the total effects contributed by this biasing magnetic field are the linear superposition of \( x \)-, \( y \)-, and \( z \)-directed biasing magnetic fields. Via this decomposition, the permittivity tensor can be rewritten as \( \tilde{\varepsilon} = \tilde{\varepsilon}_x + \tilde{\varepsilon}_y + \tilde{\varepsilon}_z \), where \( \tilde{\varepsilon}_x \) corresponds to the \( x \)-directed biasing magnetic field

\[
\tilde{\varepsilon}_x = \begin{pmatrix}
\varepsilon_{xx}^x & 0 & 0 \\
0 & \varepsilon_{yy}^x & j \varepsilon_{yz}^x \\
0 & -j \varepsilon_{zy}^x & \varepsilon_{zz}^x
\end{pmatrix}
\]  

and

\[
\varepsilon_{xx}^x = \varepsilon_0 \left(1 - \frac{(\omega_p^2 / \omega)^2 [1 - (j \tau_e / \omega)]}{[1 - (j \tau_e / \omega)]^2 - (\omega_C^2 / \omega)^2}\right)
\]

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\[ \varepsilon_{yx} = \varepsilon_{xy} = -\epsilon_0 \left( \frac{(\omega_P / \omega)^2 (\omega_{P_x} / \omega)}{1 - (j \nu_x / \omega)^2} \right) \]

\[ \bar{\varepsilon}_y \text{ corresponds to the } y\text{-directed biasing magnetic field} \]

\[ \bar{\varepsilon}_x = \left( \begin{array}{ccc} \varepsilon_{xx}^x & \varepsilon_{xy}^x & 0 \\ -\varepsilon_{yx}^x & \varepsilon_{yy}^x & 0 \\ 0 & 0 & \varepsilon_{zz}^x \end{array} \right) \]

\[ \varepsilon_{yx}^x = \varepsilon_{xy}^x = \epsilon_0 \left( 1 - \frac{[\omega_{P_x} / \omega]^2 [1 - (j \nu_x / \omega)]}{[1 - (j \nu_x / \omega)]^2 - (\omega_{P_y} / \omega)^2} \right) \]

\[ \bar{\varepsilon}_x \text{ corresponds to the } x\text{-directed biasing magnetic field} \]

\[ \varepsilon_{yx}^x = \varepsilon_{xy}^x = -\epsilon_0 \left( 1 - \frac{(\nu_x / \omega)^2 [1 - (j \nu_x / \omega)]}{[1 - (j \nu_x / \omega)]^2 - (\omega_{P_y} / \omega)^2} \right) \]

In (1)–(9), \( \omega_{P_x}^i \) denotes the cyclotron frequency due to the static magnetic biasing. Based on the Fourier transform, the time-changing rate of the electric flux density can be expressed in the frequency domain as: \( \partial P_x / \partial t \), \( \varepsilon_{x}^i \cdot \mathbf{E} \rightarrow j \omega \bar{\varepsilon}(\omega) \mathbf{E}(\omega) = j \omega \epsilon_0 \mathbf{E} + \mathbf{P} \), where the auxiliary variable \( \mathbf{P} \) can be regarded as an alternating current and is expressed as \( \mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3 \), where \( \mathbf{P}_1 = j \omega \bar{\varepsilon} \mathbf{E} \rightarrow \mathbf{E}, \mathbf{P}_2 = j \omega \bar{\varepsilon} \mathbf{E} \rightarrow \mathbf{E}, \) and \( \mathbf{P}_3 = j \omega \bar{\varepsilon} \mathbf{E} \rightarrow \mathbf{E} \). To derive the auxiliary equations, we take the component \( \mathbf{P}_1 \) as an example.

From (1)–(3), we can get three equations corresponding to the \( x\text{-}, y\text{-}, \) and \( z\text{-components of } \mathbf{P}_1 \). Namely

\[ P_{1x} = -j \omega \epsilon_0 \frac{\omega_{P_x}^2}{\omega^2 - (j \nu_x / \omega)^2} E_x \]

\[ P_{1y} = -j \omega \epsilon_0 \frac{(\omega_{P_x} / \omega)^2 [1 - (j \nu_x / \omega)]}{[1 - (j \nu_x / \omega)]^2 - (\omega_{P_y} / \omega)^2} E_y \]

\[ P_{1z} = -j \omega \epsilon_0 \frac{(\omega_{P_x} / \omega)^2 [1 - (j \nu_x / \omega)]}{[1 - (j \nu_x / \omega)]^2 - (\omega_{P_z} / \omega)^2} E_z \]

By combining (11) and (12), the following two equations can be obtained:

\[ j \omega P_{1x} + v_x P_{1y} = \epsilon_0 \omega_{P_x}^2 E_y - \omega_{P_x}^2 P_{1x} \]

Next, by performing the inverse Fourier transform, (10), (13), and (14) can be reformulated as differential equations in time domain

\[ \frac{\partial P_{1x}}{\partial t} + v_x P_{1y} = \epsilon_0 \omega_{P_x}^2 E_y \]

\[ \frac{\partial P_{1y}}{\partial t} + v_y P_{1x} = \epsilon_0 \omega_{P_y}^2 E_x + \omega_{P_x}^2 P_{1x} \]

\[ \frac{\partial P_{1z}}{\partial t} + v_z P_{1x} = \epsilon_0 \omega_{P_z}^2 E_z + \omega_{P_x}^2 P_{1x} \]

Via similar operation, other two group auxiliary equations can be obtained for \( \mathbf{P}_2 \) and \( \mathbf{P}_3 \). By linear combination of these three group equations, a general expression for an arbitrary-oriented magnetic field can be formulated as

\[ \frac{\partial P_{2x}}{\partial t} + v_x P_{2y} = \epsilon_0 \omega_{P_x}^2 E_y + \omega_{P_y}^2 P_{2x} - \omega_{P_x}^2 P_{2y} \]

\[ \frac{\partial P_{2y}}{\partial t} + v_y P_{2x} = \epsilon_0 \omega_{P_y}^2 E_x - \omega_{P_x}^2 P_{2x} + \omega_{P_y}^2 P_{2y} \]

\[ \frac{\partial P_{2z}}{\partial t} + v_z P_{2x} = \epsilon_0 \omega_{P_z}^2 E_z + \omega_{P_x}^2 P_{2x} - \omega_{P_z}^2 P_{2z} \]

Through some mathematical operations, we can finally get

\[ \nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{P} \]

\[ \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}. \]

From (22) and (23), the anisotropic and dispersive behavior is fully embedded into the auxiliary field \( \mathbf{P} \). In this case, the original frequency-dependent wave impedance \( Z_0 \) becomes constant, which significantly simplifies the problem since frequency-independent wave impedance means no convolution is involved in computing the numerical flux.

To solve (21)–(23) via DGFETD, we first suppose that the interest of computational domain \( \Omega \) bounded by \( \partial \Omega \). We consider a partition \( \mathcal{I}_h \) into a number of nonoverlapping polyhedral elements \( \Omega \), with boundary \( \partial \Omega \). Inside each element, the unknown variables \( \mathbf{E}^{(i)} = \sum_{l=1}^{n} e^{(l)} \Phi_k^{(l)} \), \( \mathbf{H}^{(i)} = \sum_{l=1}^{n} h^{(l)} \Psi_l^{(l)} \) and \( \mathbf{P}^{(i)} = \sum_{l=1}^{n} p^{(l)} T_l^{(l)} \) are approximated by a linear combination of independent edge basis vector functions \( \Phi_k^{(l)}, \Psi_l^{(l)} \) and \( T_l^{(l)} \), respectively. The approximated fields are allowed to be completely discontinuous across element boundaries. By applying discontinuous Galerkin testing to (21)–(23), a linear of semi-discrete system equations can be obtained

\[ M_{k}^{(i)} \frac{\partial \mathbf{e}^{(i)}}{\partial t} = S_{k}^{(i)} \mathbf{h}^{(i)} - \mathbf{f}_{ee}^{(i)} \mathbf{e}^{(i)} + \mathbf{f}_{eh}^{(i)} \mathbf{h}^{(i)} \]

\[ M_{k}^{(i)} \frac{\partial \mathbf{h}^{(i)}}{\partial t} = -S_{k}^{(i)} \mathbf{e}^{(i)} + \mathbf{f}_{hh}^{(i)} \mathbf{h}^{(i)} - \mathbf{f}_{he}^{(i)} \mathbf{e}^{(i)} \]

\[ M_{k}^{(i)} \frac{\partial \mathbf{p}^{(i)}}{\partial t} = -v_k^{(i)} \mathbf{p}^{(i)} + \epsilon_0 \omega_{P_k}^2 \mathbf{m}^{(i)} \mathbf{e}^{(i)} + \mathbf{f}_{eh}^{(i)} \mathbf{e}^{(i)} \]

The semi-discrete equations (24)–(26) are explicitly solved by fourth-order Runge–Kutta (RK) marching scheme. To ensure stability, the time-step size is determined in terms of \( \Delta t_{DG} \leq \)
III. NUMERICAL RESULTS

The developed algorithm is employed to analyze the propagation of electromagnetic (EM) waves in a 1-D plasma slab and scattering from a 3-D plasma-coated perfectly electrical conducting (PEC) sphere.

To study the wideband propagation properties of EM waves through a plasma slab, a $z$-polarized Gaussian-derivative pulsed plane wave propagating along $x$-axis with the bandwidth more than 100 GHz is applied as the excitation. We assume that the biasing static magnetic field is also along the $x$-axis. The parameters of this magnetized plasma slab are $\omega_p = 3.14 \times 10^{11}$ rad/s, $v_c = 2.0 \times 10^{10}$ rad/s, and $\omega_{ce} = 3.0 \times 10^{11}$ rad/s [11]. To reduce the computational cost, Silver–Müller absorbing boundary condition (SM-ABC) is used to terminate the end of the plasma slab. The linear-polarized plane wave will be decomposed into the superposition of left-handed circularly polarized (LCP) and right-handed circularly polarized (RCP) waves along the propagation direction. The associated propagation properties of LCP and RCP waves are quite different based on the dispersion property in [1] and [13]. Figs. 1 and 2 show the transmission and reflection coefficients for LCP and RCP waves, respectively. For comparison, the analytical reference based on scattering matrix method [12] is provided. The coefficients for LCP and RCP are: $E_{RCP}(\omega) = E_x(\omega) + jE_y(\omega)$ and $E_{LCP}(\omega) = E_x(\omega) - jE_y(\omega)$, where $E_x(\omega)$ and $E_y(\omega)$ are calculated from the recorded time domain data using fast Fourier transform (FFT) method. For LCP wave, it is nonpropagating below $\omega_L$. For RCP wave, the nonpropagating band is from $\omega_L$ to $\omega_R$. The wave in the frequency band below $\omega_L$ is called the whistler mode and is of extreme importance in the study of ionospheric phenomena [1], [13]. The L-2 norm error of the reflection coefficient for basis functions with different orders is presented in Fig. 3. The error peaks above 1 GHz correspond to
z-direction is applied as the incident wave. To have a better absorption of the scattered wave, perfectly matched layer (PML) is employed to truncate the computational domain. The calculated radar cross section (RCS) around 1.0 GHz in the xy-plane is presented in Fig. 5. It is observed that the RCS with coated plasma is profoundly reduced since electromagnetic waves are absorbed by the plasma. This interesting phenomenon is called plasma stealth, which can be potentially exploited for stealthy aircrafts. Also, it is noted that the RCS is nonsymmetrical when the plasma is magnetized, which is due to the existence of Lorentz force.

IV. Conclusion

A DGFETD algorithm is developed to model the interaction of EM wave in magnetized plasma. ADEs are employed to tackle the constitutive relationship between the electric permittivity tensor and the electric field $E$. The DGFETD method is applied to study the EM wave propagation through a plasma slab with electrons only and scattering from a plasma-coated sphere.

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