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Solving a static repositioning problem in bike-sharing systems using iterated tabu search

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Highlights

- We study the static bike repositioning problem
- We modify the problem to improve its realism and reduce the solution space
- We solve the problem by iterated tabu search with specific operators
- We obtain high quality solutions efficiently
Solving a static repositioning problem in bike-sharing systems using iterated tabu search

Abstract

In this paper, we study the static bike repositioning problem where the problem consists of selecting a subset of stations to visit, sequencing them, and determining the pick-up/drop-off quantities (associated with each of the visited stations) under the various operational constraints. The objective is to minimize the total penalties incurred at all the stations. We present an iterated tabu search heuristic to solve the described problem. Experimental results show that this simple heuristic can generate high quality solutions using small computing times.

Keywords: Bike-sharing; Static repositioning; Tabu search
1 Introduction

Bike sharing is very popular in many places nowadays. In April 2013, there were 535 schemes worldwide, in 49 countries (Larsen, 2013). It is therefore important to research bike sharing. Currently, bike sharing research topics include bike network design (dell’Olio et al., 2011; Lin and Yang, 2011; Martinez et al., 2012; Romero et al., 2012; Sayarshad et al., 2012; Lin et al., 2013; Lu, 2013), bike demand analysis (Froehlich et al., 2008; Borgnat et al., 2009; Kaltenbrunner et al., 2010; Borgnat et al., 2011; Vogel et al., 2011; Lathia et al., 2012), bike network flow analysis (Shaheen et al., 2011; Kitthamkesorn et al., 2013; Shu et al., 2013), bike service level analysis (Nair et al., 2013; Raviv and Kolka, 2013), and many others. As we can see, the bike sharing research has only received attention recently.

One fundamental problem of bike-sharing is that the numbers of bikes required at some stations are not enough to satisfy the bike user demand. Hence, in practice, trucks are deployed to transport bikes from surplus stations or depots to deficit stations to meet the demand. This problem is called a bike repositioning problem, which determines the optimal vehicle (truck) routes and the number of bikes loaded and unloaded at each station to meet the objective, such as satisfying bike user demand, subject to various constraints including routing, vehicle, station, and operational constraints. Very often, it is not necessary for the trucks to visit all bike stations. This means that the problem involves selecting and determining the sequence of the visited stations. This problem is more complicated than the classical vehicle routing problem (VRP) (Ho and Gendreau, 2006; Szeto et al., 2011) and the classical traveling salesman problem (TSP) (Gendreau et al., 1992) because the repositioning problem further requires determining the pick-up and drop-off quantities. These extra variables imply that the solution space is larger than that of the classical VRP and TSP. The repositioning problem was also referred to as a bike sharing rebalancing problem (BRP) (Dell’Amico et al., 2014) and belongs to the class of many-to-many pickup and delivery problems (Parragh et al., 2008). When the BRP only considers only one vehicle, the problem becomes one-commodity pickup-and-delivery TSP (1-PDTSP) introduced by Hernández-Pérez and Salazar-González (2004a). When the BRP considers multiple vehicles and a maximum duration constraint for each route, the problem belongs to one-commodity pickup-and-delivery VRP introduced by Shi et al. (2009).

Although bike sharing is very popular and the repositioning problem is a practical one, the literature on the problem is rather sparse (around 15 papers) compared with those of the classical VRP and TSP. In the literature, various bike repositioning problems with different constraints and objective functions have been studied. The problems can be broadly classified into two types, namely static and dynamic repositioning problems. The static repositioning problem considers the night time repositioning operation in which the numbers of bikes required by and presence in each station are fixed and known before repositioning takes place. The dynamic repositioning problem considers the case in which the number of bikes required by and presence in each station are changing over time. This problem is always found in the daytime repositioning operation or the 24-hour bike-sharing operation. To the best of our knowledge, very few (Nair and Miller-Hooks, 2011; Caggiani and Ottomanelli, 2012; Contardo et al., 2012) studied the dynamic problem and most of the existing studies are related to the static bike repositioning problem.

In the repositioning problem, the objectives considered include minimizing the total unmet demand or total user dissatisfaction (Contardo et al., 2012), minimizing the sum of relocation and lost user costs (Caggiani and Ottomanelli, 2012), minimizing the total travel cost (Benchimol et al., 2011; Chemla et al., 2013), minimizing the weighted sum of the deviation from the targeted number of bikes in each station, the numbers of loading and unloading quantities, and the total travel time on all routes (Papazek et al., 2013; Di Gaspero et al., 2013), minimizing the
maximum tour length (Schuijbroek et al., 2013), and minimizing the sum of travel and handling costs (Erdoğan et al., 2013). The choice of objectives should be determined by the application of bike-sharing operations. Very often, the concern of the operator governs the choice of the objective. If the bike sharing system is operated by the government, the societal benefit measure, such as user satisfaction, the maximum deviation, and the penalty cost, should be included in the objective function. However, if the bike sharing system is operated by a private operator, the travel distance or travel time is the key concern and the cost minimizing objective is considered when determining the truck route. Nevertheless, we can observe that some objectives are more general than the others. For example, minimizing the total penalty cost is more general than minimizing the total user dissatisfaction and minimizing the sum of the deviation from the targeted number of bikes in each station, because we can choose a penalty function that assigns a value of zero to the level equal to or greater than the demand level and a very large number to other levels to replicate the effect of minimizing the total user dissatisfaction, and we can select a penalty function that assigns a value to a level equal to the absolute difference between that level and the target level to replicate the effect of minimizing the sum of deviations.

Various operational constraints are also considered in the repositioning problem. For example, despite the maximum time for each repositioning activity, Benchimol et al. (2011) proposed a routing constraint that requires each node being visited exactly once by the vehicle as in the traveling salesman problem. In terms of demand-related constraints, Benchimol et al. (2011) and Chemla et al. (2013) set the perfect balance requirement as a hard constraint but relaxed the time constraint. Lin and Chou (2012) considered road conditions, traffic regulations, and geographical factors in the constraint. Nair and Miller-Hooks (2011) introduced a probabilistic level-of-service constraint such that the repositioning activity must satisfy a certain proportion of the most nearest future demand but ignored the routing constraint. The operational constraints included in the problem should be related to the application. They also determine the complexity of the problem.

Most of the preceding bike sharing studies do not predefine a station to be 1) a pick-up one, 2) a drop-off one, or 3) a neither pick-up nor drop-off station. In fact, a station can be easily classified into one of these three types. We only need to handle the demand at drop-off stations. It is not necessary for a truck to visit all pick-up stations to satisfy the demand of each drop-off station or minimize the total penalty cost. Hence, the problem can be refined so that a more efficient solution method can be proposed. Based on this approach, Ting and Liao (2013) modeled the bike repositioning problem as the selective pick-up and drop-off problem. The input of the bike repositioning problem includes the sets of pick-up and drop-off stations. The truck must visit all the drop-off stations to satisfy the demand but it is unnecessary for the truck to visit all the pick-up stations. However, they considered the total travel time of the vehicle in the objective function and the number of bikes required by each drop-off station is explicitly given. In reality, not all the demand can be satisfied because the supply of bikes from depots and pick-up stations may be insufficient or the operational period is too short to reposition bikes to some of the drop-off stations. In these two cases, it is unwise to ensure the truck to visit all drop-off stations and satisfy all the demand. Moreover, the penalty cost of not satisfying one unit of demand at a station may also vary from one to another. In an area with high (low) bike station density or many (few) transport mode alternatives, the penalty cost may be low (high) because the users can (cannot) easily walk to other nearby bike stations or take other transport modes. It is more reasonable to consider such penalty cost than just user demand for the bike service provided by the government.

To address this issue, we propose a refined problem to consider such penalty cost. We use the concept of the penalty function proposed by Raviv et al. (2013) and consider a fixed planning horizon. The objective is to minimize the total penalty cost. The constraints in Raviv et al.
(2013) are modified to consider the sets of pick-up and drop-off stations individually and to reduce the solution space. As pointed out by Ting and Liao (2013), the original problem is already $\mathcal{NP}$-hard. The refined problem with a more complicated objective function is also $\mathcal{NP}$-hard. It is intractable to use exact methods such as those shown in Table 1 to solve large, realistic repositioning problems. Moreover, because the existing heuristics or approximation methods shown in Table 1 are tailored for solving their bike repositioning problems that are different from ours (in terms of objective function or constraints), their methods cannot be directly applied to solve our problem. Hence, we develop a heuristic that makes use of the problem’s properties to solve our problem.

Table 1: A summary on the existing solution methods for bike repositioning problems

<table>
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<tr>
<th>Algorithm</th>
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<tr>
<td><em>Exact algorithms:</em></td>
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<tr>
<td>Dantzig-Wolfe and Benders decomposition</td>
<td>Contardo et al. (2012)</td>
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<tr>
<td>Benders decomposition</td>
<td>Erdoğan et al. (2013)</td>
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<td><em>Approximation method:</em></td>
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<tr>
<td>9.5-approximation algorithm</td>
<td>Benchimol et al. (2011)</td>
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<tr>
<td><em>Heuristics:</em></td>
<td></td>
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<tr>
<td>Variable neighborhood search</td>
<td>Papazek et al. (2013)</td>
</tr>
<tr>
<td>Cluster-first route-second</td>
<td>Rainer-Harbach et al. (2013)</td>
</tr>
<tr>
<td>Ant colony and constraint programming</td>
<td>Schuijbroek et al. (2013)</td>
</tr>
<tr>
<td>PILOT/GRASP</td>
<td>Di Gaspero et al. (2013)</td>
</tr>
<tr>
<td><em>Hybrid algorithm:</em></td>
<td></td>
</tr>
<tr>
<td>Branch-and-cut method with tabu search</td>
<td>Chemla et al. (2013)</td>
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As the proposed problem involves a routing problem, we considered to enhance an existing heuristic for routing problems to solve our proposed problem. Because tabu search is well-known to be very efficient to solve routing problems compared with other non-hybrid methods and tabu search has rarely been used to solve the static repositioning problem as shown in Table 1, we choose tabu search as the backbone of our solution method. However, the repositioning problem involves pick-up and drop-off quantity variables in addition to routing variables. Hence, we could not directly apply tabu search for routing problems to solve our proposed problem and some new operators must be added to tabu search to handle the extra variables. For this purpose, we develop specific operators to ensure solution feasibility. In order to accelerate the search when evaluating a neighbor solution, several ideas are incorporated into the heuristic in order to avoid re-computing all the pick-up/drop-off quantities. To improve the solution quality, the tabu search procedure is embedded into an iterative framework, where several intensification/diversification mechanisms are applied to the solution obtained from tabu search. To show the efficiency and accuracy of our method, we set up different test scenarios and compare the results obtained from IBM Ilog CPLEX.

The remainder of the paper is organized as follows. Section 2 presents the formulation. Section 3 depicts the solution method. Section 4 describes the test cases and discusses the results. Section 5 gives the conclusion.
2 Mathematical formulation

In this section, we develop the formulation based on Raviv et al. (2013), but modify their formulation, consider single vehicle only, and explicitly define pick-up and drop-off stations similar to the studies by Caggiani and Ottomanelli (2012) and Ting and Liao (2013). However, unlike their studies, pick-up and drop-off stations are deduced from their penalty functions. A station is defined as a drop-off station if the initial number of bikes at the station is smaller than the optimal number of bikes at that station (i.e., the level at which the penalty cost is minimum). On the other hand, if the initial number of bikes at a station is larger than the optimal number of bikes at that station, then the station is defined as a pick-up station.

The formulation of Raviv et al. (2013) was modified by performing the following sequentially: 1.) Split each constraint into a pair of constraints, one for pick-up stations and one for drop-off stations. 2.) Simplify each resultant constraint according to the definition of the associated station. 3.) Refine the meanings of loading and unloading variables according to the definition of stations. 4.) Simplify the whole formulation for the single vehicle case by eliminating redundant notations such as the notations’ subscript for vehicles and the notation for the set of vehicles, and 5.) Simplify the objective function to only consider the sum of the penalty cost of each station and define a specific form of the penalty function for the objective function. Note that if we removed step 4 in the preceding procedure, we could actually have a multi-vehicle formulation for the static bike repositioning problem.

The modified formulation assumes that the depot is both a pick-up and drop-off node and has sufficient bikes and capacity. The formulation also assumes that each station can only be visited by the vehicle at most once as in the literature (e.g., Raviv et al., 2013) because this assumption can make the solution space smaller, leading to the development of an efficient heuristic for solving large-scale bike repositioning problems much easier. Moreover, the formulation allows the vehicle leaving the depot with some bikes loaded on board and returning to the depot non-empty. Furthermore, the formulation allows the vehicle visiting the depot multiple times if necessary.

To ease the description of the mathematical formulation of the static bike repositioning problem, we start by introducing the notations.

\[ \mathcal{N} \] Set of stations, indexed by \( i = 1, \ldots, |\mathcal{N}| \)

\[ \mathcal{N}_0 \] Set of nodes, including the stations and the depot (denoted by \( i = 0 \))

\( c_i \) Number of lockers installed at station \( i \in \mathcal{N}_0 \), referred to as the station’s capacity

\( f_i(s_i) \) A convex penalty function for station \( i \in \mathcal{N} \); the function is defined over the integers \( s_i = 0, \ldots, c_i \)

\( s_{0i}^i \) Number of bikes at node \( i \) before the repositioning operation starts

\( s_{1i}^i \) arg min \( s_i \), i.e., optimal number of bikes at node \( i \)

\( \mathcal{P} \) Set of pick-up stations, i.e., \( \{ i \in \mathcal{N} | s_{0i}^i > s_{1i}^i \} \)

\( \mathcal{D} \) Set of drop-off stations, i.e., \( \{ i \in \mathcal{N} | s_{0i}^i < s_{1i}^i \} \)

\( k \) Vehicle capacity

\( t_{ij} \) Traveling time from station \( i \) to station \( j \)

\( T \) Repositioning time, i.e., time allotted to the repositioning operation

\( L \) Time required to remove a bike from a station and load it onto the vehicle

\( U \) Time required to unload a bike from the vehicle and hook it to a locker in a station

\( a_i, b_i \) Parameters associated with the penalty function for station \( i \)

\( M \) A very large number

Decision variables:

\[ x_{ij} = \begin{cases} 
1, & \text{if the vehicle travels directly from node } i \text{ to node } j; \\
0, & \text{otherwise.} 
\end{cases} \]
\( y_{ij} \) Number of bikes carried on the vehicle when it travels directly from node \( i \) to node \( j \).

\( y_{ij} \) is zero if \( x_{ij} = 0 \).

\( s_i \) Inventory level at station \( i \) at the end of the repositioning operation

\( y_i^P \) Number of bikes dropped off at node \( i \in D \cup \{0\} \) by the vehicle

\( y_i^D \) Number of bikes picked up at node \( i \in P \cup \{0\} \) by the vehicle

\( q_i \) Auxiliary variable associated with node \( i \) used for the sub-tour elimination constraints

The objective function can be stated mathematically as:

\[
\min \sum_{i \in N} f_i(s_i) = \sum_{i \in N} (a_i(s_i - s_i^0)^2 + b_i)
\]  
(1)

s. t. \( s_i = s_i^0 - y_i^P \) \( \forall i \in P \)  
(2)

\( s_i = s_i^0 + y_i^D \) \( \forall i \in D \)  
(3)

\( s_0 = s_0^0 - y_0^P + y_0^D \)  
(4)

\( y_i^P = \sum_{j \in N_0 \cup \{0\}} y_{ij} - \sum_{j \in N_0 \cup \{0\}} y_{ji} \) \( \forall i \in P \)  
(5)

\( y_i^D = \sum_{j \in N_0 \cup \{0\}} y_{ji} - \sum_{j \in N_0 \cup \{0\}} y_{ij} \) \( \forall i \in D \)  
(6)

\( y_0^P = \sum_{j \in N} y_{0j} \)  
(7)

\( y_0^D = \sum_{j \in N} y_{j0} \)  
(8)

\( y_{ij} \leq k x_{ij} \) \( \forall i, j \in N_0, i \neq j \)  
(9)

\( \sum_{j \in N} x_{0j} \geq 1 \)  
(10)

\( \sum_{j \in N_0 \cup \{0\}} x_{ji} - \sum_{j \in N_0 \cup \{0\}} x_{ij} = 0 \) \( \forall i \in N_0 \)  
(11)

\( \sum_{j \in N_0 \cup \{0\}} x_{ij} \leq 1 \) \( \forall i \in N \)  
(12)

\( y_i^P \leq s_i^0 \) \( \forall i \in P \)  
(13)

\( y_i^D \leq c_i - s_i^0 \) \( \forall i \in D \)  
(14)

\( \sum_{i \in P \cup \{0\}} y_i^P - \sum_{i \in D \cup \{0\}} y_i^D = 0 \)  
(15)

\( \sum_{i \in P \cup \{0\}} Ly_i^P + \sum_{i \in D \cup \{0\}} U y_i^D + \sum_{i,j \in N_0, i \neq j} t_{ij} x_{ij} \leq T \)  
(16)

\( q_j \geq q_1 + 1 - M (1 - x_{ij}) \) \( \forall i \in N_0, j \in N, i \neq j \)  
(17)

\( x_{ij} \in \{0,1\} \) \( \forall i \in N_0, j \in N, i \neq j \)  
(18)

\( y_i^P \geq 0, \text{integer} \) \( \forall i \in P \cup \{0\} \)  
(19)

\( y_i^D \geq 0, \text{integer} \) \( \forall i \in D \cup \{0\} \)  
(20)

\( y_{ij} \geq 0 \) \( \forall i, j \in N_0, i \neq j \)  
(21)

\( s_i \geq 0 \) \( \forall i \in N_0 \)  
(22)

\( q_i \geq 0 \) \( \forall i \in N_0 \)  
(23)

The objective function (1) is defined as the sum of the penalty cost incurred at each station, which differs from that of Raviv et al. (2013) that used the weighted sum of the total penalty and operational costs. The penalty cost at each station is calculated by the associated penalty
function. Each penalty function is defined over the integers from zero to the maximum station capacity because these integers are the possible number of bikes at that station. Because static repositioning occurs during the night, the numbers of bikes at each station before and after the repositioning operation are usually different. The penalty cost of a station is defined by the number of bikes at that station after the operation. Raviv et al. (2013) suggested that the penalty function or cost represents the expected number of shortages for bicycles or lockers during the next working day. They argued that the penalty function has been captured the effect of stochastic bicycle demand and can be estimated using the method proposed by Raviv and Kolka (2013).

Constraints (2)-(4) define the final inventory level at each node at the end of repositioning. The inventory level of the pick-up station at the end of repositioning equals the initial level minus the pick-up quantity whereas the inventory level of the drop-off station at the end of repositioning equals the initial level plus the delivery quantity. Constraints (5) state that the number of bikes obtained from a pick-up station equals the difference in the number of bikes on the vehicle between after and before visiting the station. Constraints (6) state that the number of bikes delivered to a drop-off station equals the difference in the number of bikes on the vehicle between before and after visiting the station. Constraints (7) and (8) define the pick-up and drop-off quantities at each pick-up and drop-off station, respectively. Constraints (9) are the vehicle capacity constraints, requiring the number of bikes on the vehicle not greater than its capacity. Constraint (10) ensures that the vehicle must leave the depot at least once. Constraints (11) make sure that if the vehicle visits a station, it must leave that station. Constraints (12) ensure that the vehicle can visit a station at most once. Constraints (13) and (14) require that the pick-up and drop-off quantities at each pick-up and drop-off station are not larger than the number of bikes available at the pick-up station and the remaining capacity of the drop-off station, respectively. Constraints (15) make sure that all the picked-up bikes are delivered eventually. Constraints (16) limit the total operating time, including loading, unloading, and travel times, is not greater than the total time available for repositioning. Constraints (17) are the sub-tour elimination constraints (see Miller et al. (1960)). Finally, constraints (18) define $x_{ij}$ to be a binary variable. Constraints (19) and (20) restrict pick-up and drop-off quantities to be nonnegative integers. Constraints (21)-(23) ensure that the number of bikes on the vehicle, the inventory level at each station, and the auxiliary variables are non-negative.

As (1) is not linear, it needs to be linearized in order to be solved by CPLEX. Raviv et al. (2013) show how (1) can be linearized. They also show how their mathematical formulation can be strengthened. Based on their idea, we add the following constraints:

$$ y_i^P \leq \min\{s_i^0, k\} \sum_{j \in N_0, i \neq j} x_{ij} \quad \forall i \in P \quad (24) $$

$$ y_i^D \leq \min\{c_i - s_i^0, k\} \sum_{j \in N_0, i \neq j} x_{ij} \quad \forall i \in D \quad (25) $$

$$ y_i^P \geq \sum_{j \in N_0, i \neq j} x_{ij} \quad \forall i \in P \quad (26) $$

$$ y_i^D \geq \sum_{j \in N_0, i \neq j} x_{ij} \quad \forall i \in D \quad (27) $$

Constraints (24)-(25) further tighten the solution space for loading and unloading quantities at each of the pick-up and drop-off stations, respectively, by including the vehicle’s capacity, and conditioning the quantities only for cases in which the vehicle visits the corresponding station.
Constraints (26)-(27) tighten the solution space by ensuring that a vehicle enters a station must have a loading and an unloading bike activity, respectively. These constraints are valid when the distance matrix satisfies the triangle inequality because then it is always possible to skip a station without loading and unloading activities.

3 Solution methodology

In our proposed solution, a solution $x$ consists of two parts: 1) a routing sequence $(i_0, i_1, i_2, \ldots, i_n)$ where $i_0 = i_n = 0$ (i.e., the depot), $i_h \in \mathcal{N}$, $h = 1, 2, \ldots, n - 1$, the subscript $h$ is used to define the order of stations being visited, and 2) the decision variable $y_{ih}^P$ or $y_{ih}^D$ that describes how many bikes to be picked up or dropped off at each station $i_h$, respectively. Except for the depot, only one of the decision variables is defined for each $i_h$, as the station has been classified into either a pick-up or drop-off station before constructing an initial solution. The evaluation function is defined as $z(x) = \sum_{i \in \mathcal{N}} f_i(s_i)$, where $s_i$ is defined by equations (2) and (3). Every neighbor solution $x$ is evaluated based on the evaluation function $z(x)$.

The proposed method consists of the following: the initial solution construction procedure, new solution generation method (based on the insertion, removal and exchange moves) incorporated into a tabu search procedure, and intensification and diversification procedures to further improve and diversify the search.

3.1 Initial solution construction

The initial solution to the repositioning problem is obtained by a simple construction heuristic depicted below:

**Step 0** Sort the pick-up and the drop-off stations individually in a descending order based on $|f_i(s_0^i) - f_i(s_I^i)|$, $\forall i \in \mathcal{N}$. Set $l = 1$. Initialize the route as $(i_0, i_n)$.

**Step 1** Choose the $l$-th pick-up station from the list of ordered pick-up stations. Let $g$ be the station number of the station placed in the $l$th place, and $\tau$ be the remaining time of the repositioning operation if station $g$ is appended at the end of the route (i.e., before $i_n$). If $\tau > 0$, then set $y_{g}^P = \min\{k - y_{n-1}^n, s_0^g - s_I^g, \lfloor \tau/(U + L) \rfloor\}$. If $y_{g}^P > 0$, then station $g$ is added to the end of the route. Remove station $g$ from the ordered list. Set $l = l + 1$. Step 1 is repeated until no more pick-up stations can be added to the route without violating the constraints. Set $l = 1$.

**Step 2** Repeat Step 1 for drop-off stations, replace $y_{g}^P$ with $y_{g}^D$, and the min function with $\min\{y_{n-1}^n, s_I^g - s_0^g\}$.

**Step 3** Repeat Steps 1 and 2 until no more stations can be appended at the end of the route without violating the constraints.

3.2 Tabu search

Tabu search is a well known metaheuristic proposed by Fred Glover in 1986 (Glover, 1986). Many difficult combinatorial optimization problems (e.g., vehicle routing problems) have been efficiently solved by tabu search. Tabu search is based on the local search principle: At each
iteration, the best solution in the neighborhood is chosen. To avoid getting stuck in a local
optimum and to prevent cycling of solutions, solutions possessing some attributes of previously-
visited-solutions are banned from being chosen for a certain number of iterations (i.e., those
attributes are recorded in a tabu list). Sometimes tabu lists may be too powerful as they
restrain excellent solutions to be chosen. Aspiration criteria are very useful to overcome this
obstacle.

The tabu search heuristic runs $\gamma$ iterations. At each iteration, a feasible solution that minimizes
$z(\bar{x})$ is chosen from the three neighborhoods: $\mathcal{R}(x)$, $\mathcal{I}(x)$ and $\mathcal{E}(x)$. The selected solution is
either a non-tabu solution or a solution that satisfies the aspiration criterion. If the objective
function value evaluated at the selected solution is less than that evaluated at the current best
solution, we update the current best solution $z^*_1$. The reverse move is set tabu for a certain number
of iterations. Two tabu lists are maintained: $\text{TABU}_1(i)$ denotes when station $i$ can be removed
and $\text{TABU}_2(i)$ states when station $i$ can be inserted into the solution again.

The selected solution is set to be the current solution. The algorithmic description of the
heuristic is given in Algorithm 1.

```
Algorithm 1 TabuSearch($x_0$)

Require: Initial solution $x_0$
1: Set $\bar{x}^* = x_0$ and $z^*_1 = z(x_0)$.
2: Set $\text{TABU}_1(i) = -1$ and $\text{TABU}_2(i) = -1$ $\forall i \in \mathcal{N}$.
3: Set $x = x_0$.
4: for $c = 1, \ldots, \gamma$ do
5:   Select a solution $\bar{x} \in \mathcal{R}(x) \cup \mathcal{I}(x) \cup \mathcal{E}(x)$ that minimizes $z(\bar{x})$, and the solution is either a
6:   non-tabu solution or a solution that satisfies the aspiration criterion (i.e., $z(\bar{x}) < z^*_1$).
7:   if $\bar{x} \neq \emptyset$ then
8:     if $z(\bar{x}) < z^*_1$, set $\bar{x}^* = \bar{x}$ and $z^*_1 = z(\bar{x})$.
9:     Set the move from $\bar{x}$ to $x$ tabu for $\theta_1$ and/or $\theta_2$ iterations.
10:    Set $x = \bar{x}$.
11:   end if
12: end for
13: return $\bar{x}^*$
```

### 3.3 The removal move

The neighborhood $\mathcal{R}(x)$ is a set of feasible neighbor solutions obtained by applying the removal
moves. Each of the removal moves removes station $i_h \in W(x) \setminus \{0\}$, where $W(x)$ is the set
of stations used to define the route sequence in the solution $x$. Sometimes when a station is
removed from the solution, the resultant solution may no longer remain feasible as the values
of the variables $y^P_d$ and $y^D_d$ where $d > h$ may no longer satisfy the vehicle capacity constraint
and/or the non-negativity constraint. Instead of recomputing the pick-up or drop-off quantity
at each station for the resultant solution, we only modify the quantity $y^{P}_{i_0}$, $y^{P}_{i_{h-1}}$, $y^{D}_{i_{h-1}}$, $y^{P}_{i_{h+1}}$
or $y^{D}_{i_{h+1}}$ in the resultant solution to generate a feasible neighbor solution efficiently. With each
$i_h$ that gets removed, a maximum of three feasible neighbor solutions can be generated. These
three potential neighbor solutions have the same routing sequence. The only difference between
the first potential neighbor solution and solution $x$ is that they have different values of $y^{P}_{i_0}$.
The difference between the second (third) potential neighbor solution and $x$ is that they have
different values of $y^{P}_{i_{h-1}}$ or $y^{D}_{i_{h-1}}$ ($y^{P}_{i_{h+1}}$ or $y^{D}_{i_{h+1}}$). The determination of the values of $y^{P}_{i_0}$, $y^{P}_{i_{h-1}}$
$y^{D}_{i_{h-1}}$, $y^{P}_{i_{h+1}}$ or $y^{D}_{i_{h+1}}$ is described in Sections 3.3.1 and 3.3.2.
3.3.1 Station $i_h$ is a pick-up station

When $i_h$ is a pick-up station, we consider three possible ways in adjusting the pick-up/drop-off quantities in the resultant solution:

Adjusting $y^P_{i_h}$. Denote $r$ as the minimum residual capacity of the vehicle before visiting station $i_h$. Then, $r$ is obtained by \( \min_{i=0,1,\ldots,h-1} \{ k - y_{i(i+1)} \} \). It is possible to adjust the quantity $y^P_{i_h}$ in the resultant solution to obtain a feasible neighbor solution if the following three conditions are satisfied by the current solution simultaneously: $r \geq y^P_{i_h}$, $y^0_{i_h} + y^P_{i_h} \leq k$, and $s^0_{i_h} - y^0_{i_h} - y^P_{i_h} \geq 0$. The three conditions ensure that $y^P_{i_h}$ extra bikes can be loaded to the vehicle at the depot while satisfying the vehicle capacity constraint. If the three conditions are satisfied, then the pick-up quantity at the depot in the neighbor solution is set to be $y^P_{i_h} + y^P_{i_h}$, and the quantities associated with other stations remain unchanged.

Adjusting $y^P_{i_{h-1}}$ or $y^P_{i_{h+1}}$ If $i_{h-1}$ is a pick-up station, then it is possible to adjust $y^P_{i_{h-1}}$ in the resultant solution to form a feasible neighbor solution if the current solution satisfies $s^0_{i_{h-1}} - y^P_{i_{h-1}} - y^P_{i_{h}} \geq 0$. This condition ensures that node $i_{h-1}$ can provide $y^P_{i_{h-1}}$ extra bikes for pick-up. The pick-up quantity at that node in the resultant solution is set to be $y^P_{i_{h-1}} + y^P_{i_{h}}$ to obtain a feasible neighbor solution. If $i_{h-1}$ is a drop-off station, then the neighbor solution is feasible if $s^0_{i_{h-1}} + y^P_{i_{h-1}} - y^P_{i_{h}} \leq c_{i_{h-1}}$ (i.e., drop off fewer bikes at $i_{h-1}$) and $y^D_{i_{h-1}} > y^D_{i_{h}}$. If it is feasible, then set $y^D_{i_{h-1}} = y^D_{i_{h-1}} - y^P_{i_{h}}$. If in the latter case, $y^D_{i_{h-1}} = y^D_{i_{h}}$, then $i_{h-1}$ is also eliminated from the route and that is not ideal.

Adjusting $y^P_{i_{h+1}}$ or $y^P_{i_{h+1}}$ Same as above, except replace $h - 1$ with $h + 1$.

3.3.2 Station $i_h$ is a drop-off station

We also consider three possible ways in adjusting the pick-up/drop-off quantities in the resultant solution when $i_h$ is a drop-off station. They are depicted below.

Adjusting $y^P_{i_h}$. Let $u$ be the minimum vehicle load before visiting station $i_h$. Then, $u$ is computed as $\min_{i=0,1,\ldots,h-1} \{ y_{i(i+1)} \}$. Then, we may adjust $y^P_{i_h}$ in the resultant solution to obtain a feasible neighbor solution if the current solution $x$ satisfies the following: $u - y^D_{i_h} \geq 0$ and $y^0_{i_h} - y^D_{i_h} \geq 0$. It is because the vehicle can be loaded with fewer bikes at the depot while satisfying the constraints if the two conditions are satisfied. In this case, $y^P_{i_h}$ is set to be $y^P_{i_h} - y^D_{i_h}$, and the other quantities remain unchanged.

Adjusting $y^P_{i_{h-1}}$ or $y^P_{i_{h+1}}$. If $i_{h-1}$ is a drop-off station, then the resultant solution is feasible if the current solution satisfies $s^0_{i_{h-1}} + y^P_{i_{h-1}} + y^D_{i_h} \leq c_{i_{h-1}}$. In this case, more bikes will be dropped off at $i_{h-1}$ and $y^D_{i_{h-1}}$ is set to be $y^P_{i_{h-1}} + y^D_{i_h}$ If $i_{h-1}$ is a pick-up station, then the resultant solution is feasible if $s^0_{i_{h-1}} - y^P_{i_{h-1}} + y^D_{i_h} \geq 0$ and $y^P_{i_{h-1}} > y^D_{i_h}$. If it is feasible, then set $y^P_{i_{h-1}} = y^P_{i_{h-1}} - y^D_{i_h}$. We do not allow $y^P_{i_{h-1}}$ equal to $y^D_{i_h}$ to avoid $i_{h-1}$ being eliminated from the route.

Adjusting $y^P_{i_{h+1}}$ or $y^P_{i_{h+1}}$ Same as above, except replace $h - 1$ with $h + 1$. 

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3.4 The insertion move

The neighborhood \( \mathcal{I}(x) \) consists of all feasible neighbor solutions obtained by applying the insertion moves. Each of the insertion moves involves two steps. The first step is to insert a station \( i \in (\mathcal{P} \cup \mathcal{D}) \setminus \mathcal{W}(x) \) in the routing sequence in the current solution \( x \). The station can be added between every pair of nodes \( i_{h-1} \) and \( i_h \) in the sequence. The station can also be added after the depot but the next visiting node must also be the depot. The second step is to set the quantity at each station in the resultant solution, including the quantity at the inserted station. The insertion move is feasible if the travel time on the resultant route does not violate constraint (16) and the new quantities computed satisfy constraints (5)-(8), (13)-(15), (19)-(20). To save computation time, instead of recomputing the quantities at each station in the resultant solution, we set some of the quantities equal to the corresponding quantities in the current solution, compute \( y^P_i \) or \( y^D_i \), and only adjust the quantity \( y^P_0, y^P_{i_{h-1}}, y^D_{i_{h-1}}, y^P_i \) or \( y^D_i \) to obtain a feasible neighbor solution. The rules used for modifying the quantities are described in Sections 3.4.1 and 3.4.2.

3.4.1 Station \( i \) is a pick-up station

We consider four possible ways in adjusting the quantities in the resultant solution after inserting pick-up station \( i \): 

No need to adjust other stations’ quantities Let \( \tau \) be the remaining time left if station \( i \) is inserted between \( i_{h-1} \) and \( i_h \), and \( \kappa \) be the maximum vehicle load after visiting station \( i_{h-1} \). Then, \( \kappa = \max_{i=h-1,h,...,n-1}\{y_{ni_i+1}\} \) and \( y^P_i = \min\{\lfloor \tau/(U + L) \rfloor, s^0_i, \kappa - \kappa \} \), which depends on the remaining time left, the number of bikes available at the station, and the spare vehicle capacity. If \( y^P_i > 0 \), then the insertion of \( i \) between \( i_{h-1} \) and \( i_h \) is feasible without altering any of the other stations’ pick-up/drop-off quantities.

Adjusting \( y^P_0 \) This case is possible if the original pick-up quantity \( y^P_0 > 0 \) as we aim to reduce the pick-up quantity at the depot and the reduced number is compensated by the pick-up quantity at station \( i \), i.e., \( y^P_i = \min\{s^0_i, u\} \), where \( u = \min_{i=0,...,n-1}\{y_{ni_i+1}\} \). Including \( u \) in the formula is because we cannot reduce too many bikes from \( i_0 \); otherwise, the load on the vehicle traveling between some stations would be negative. If \( y^P_i > 0 \), then station \( i \) can be inserted between \( i_{h-1} \) and \( i_h \) with depot’s quantity reduced by \( y^P_i \) (i.e., \( y^P_0 = y^P_0 - y^P_i \)).

Adjusting \( y^P_{i_{h-1}} \) or \( y^D_{i_{h-1}} \) There are two mutually exclusive cases:

1. If \( i_{h-1} \) is a pick-up station: It is possible to pick-up fewer bikes at \( i_{h-1} \) and let the vehicle pick-up the rest from \( i \). The quantity \( y^P_i \) in the resultant solution is determined from 

\[
\arg\min_{v=0,1,...,\min\{s^0_i,y^P_{i_{h-1}}\}} \frac{f_{i_{h-1}}(s^0_{i_{h-1}} - y^P_{i_{h-1}} + v) + f_i(s^0_i - v)}{\nu_{i_{h-1}}}
\]

The insertion is feasible if \( y^P_i > 0 \) and \( y^P_{i_{h-1}} > y^P_i \) as this move aims to pick-up \( y^P_i \) fewer bikes from \( i_{h-1} \). In this case, \( y^P_{i_{h-1}} = y^P_{i_{h-1}} - y^P_i \).

2. If \( i_{h-1} \) is a drop-off station: For every additional bike the vehicle drops off at station \( i_{h-1} \), a bike is picked up from station \( i \). The quantity \( y^P_i \) equals \( \min\{y_{i_{h-1}i_k} - s^0_{i_{h-1}} - y^D_{i_{h-1}}, \lfloor \tau/(U + L) \rfloor \} \). The insertion is feasible if \( y^P_i > 0 \) because this move can drop off \( y^D_i \) more bikes at \( i_{h-1} \). In this case, \( y^D_{i_{h-1}} = y^D_{i_{h-1}} + y^P_i \).
Adjusting $y_{ih}^P$ or $y_{ih}^D$. There are two mutually exclusive cases:

1. If $i_h$ is a pick-up station: Same as above, except replace $h - 1$ with $h$.

2. If $i_h$ is a drop-off station: For every bike the vehicle picks up from station $i$, a bike is dropped off at station $i_h$. The quantity $y_{ih}^P$ is $\min\{k - y_{ih-1}^P, c_i - s_i^0 - y_{ih}^D, \frac{\tau}{(U + L)}\}$. The insertion is feasible if $y_i^P > 0$ so that the vehicle can drop off $y_i^P$ more bikes at $i_h$. In this case, $y_{ih}^D = y_{ih}^P + y_i^P$.

3.3.2 Station $i$ is a drop-off station

We consider five possible ways in adjusting the quantities in the resultant solution after inserting drop-off station $i$:

No need to adjust other stations’ quantities Denote $\nu$ as the minimum vehicle load after visiting station $i_{h-1}$. Then, $\nu = \min_{i=h-1,h,...,n-1}{\{y_{iti+1}\}}$ and $y_{i}^D = \min\{c_i - s_i^0, \nu\}$. If $y_{i}^D > 0$, then the insertion of $i$ between $i_{h-1}$ and $i_h$ is feasible without altering any of the other stations’ pick-up/drop-off quantities.

Adjusting $y_{i0}^P$. This case is possible if $y_{i0}^P < k$ as it is only possible to drop off at $i$ if we increase the pick-up quantity at the depot. The quantity $y_i^D = \min\{\frac{\tau}{(U + L)}], c_i - s_i^0, r, s_{i_0} - y_{i0}^P\}$, where $r$ is defined in Section 3.3.1. If $y_i^D > 0$, then it is feasible to insert $i$ between $i_{h-1}$ and $i_h$, where $y_i^D$ more bikes are picked up from the depot (i.e., $y_{i0}^P = y_{i0}^P + y_i^P$).

Adjusting $y_{ih-1}^D$ or $y_{ih-1}^D$. There are two mutually exclusive cases:

1. If $i_{h-1}$ is a drop-off station: It is possible to drop off fewer at $i_{h-1}$ and drop off the rest at $i$. The quantity $y_i^D$ is determined from

$$\arg\min_{u=0,1,...,\min(c_i - s_i^0, y_{ih-1}^D)} f_{ih-1}(s_{ih-1}^0 + y_{ih-1}^D - u) + f_i(s_i^0 + u).$$

The insertion is feasible if $y_i^D > 0$ and $y_{ih-1}^D > y_i^D$ as the vehicle can drop off $y_i^D$ fewer bikes at $i_{h-1}$ (i.e., $y_{ih-1}^D = y_{ih-1}^D - y_i^D$).

2. If $i_{h-1}$ is a pick-up station: For every additional bike the vehicle picks up from station $i_{h-1}$, a bike is dropped off at station $i$. The quantity $y_i^D$ is $\min\{k - y_{ih-1}^P, c_i - s_i^0 - y_{ih}^P, \frac{\tau}{(U + L)}\}$. The insertion is feasible if $y_i^D > 0$ because the vehicle can pick up $y_i^D$ more bikes at $i_{h-1}$. In this case, $y_{ih}^P = y_{ih}^P + y_i^P$.

Adjusting $y_{ih}^D$ or $y_{ih}^P$. There are two mutually exclusive cases:

1. If $i_h$ is a drop-off station: Same as above, except replace $h - 1$ with $h$.

2. If $i_h$ is a pick-up station: For every bike the vehicle drops off at station $i_h$, a bike is picked up at station $i_h$. The quantity $y_i^D$ is $\min\{y_{ih-1}^D, c_i - s_i^0, s_{ih-1} - y_{ih}^P, \frac{\tau}{(U + L)}\}$. The insertion is feasible if $y_i^D > 0$ so that the vehicle can pick up $y_i^D$ more bikes at $i_h$. In this case, $y_{ih}^D = y_{ih}^D + y_i^D$.

Adjusting pick-up quantities at the depot due to creating a new trip When a new trip is created by inserting both $i$ and the depot after the depot in the route sequence (i.e., 0 - $i$ - 0), the quantity, $y_i^P$, is set to be $\min\{c_i - s_i^0, k, s_{i_0} - y_{i0}^P, \frac{\tau}{(U + L)}\}$. This is also the pick-up quantity from the depot.
Note that the term "trip" of a vehicle means the vehicle leaves the depot, visits one or more stations, and then return to the depot. The vehicle may make multiple trips on a single route and hence the vehicle can visit the depot multiple times if necessary.

3.5 The exchange move

The neighborhood $\mathcal{E}(x)$ consists of all feasible neighbor solutions obtained by applying the exchange moves. Each of the exchange moves works as follows: It exchanges station $i_h \in W(x) \cap \mathcal{P}$ with station $i \in \mathcal{P} \setminus W(x)$ in position $h$ in the routing sequence in the current solution $x$ and set the pick-up quantity at $i$ to be that at $i_h$. This exchange is feasible if $s_i^0 - y_{ih}^P \geq 0$ and the resultant traveling time does not violate constraint (16). Similarly, this move also exchanges station $i_h \in W(x) \cap \mathcal{D}$ with station $i \in \mathcal{D} \setminus W(x)$ and set the drop-off quantity at $i$ equal to that at $i_h$. The exchange is feasible if $s_i^0 + y_{ih}^D \leq c_i$ and constraint (16) is not violated.

3.6 Tabu tenures and aspiration criterion

Tabu tenures are set according to the planning horizon $T$ and the number of stations $|\mathcal{N}|$. We utilize two tabu lists; one for the removal of stations, $TABU_1(\cdot)$, and one for the insertion of stations, $TABU_2(\cdot)$. The first tabu tenure $\theta_1$ is set to equal $\lceil (T/3600) \log_{10} |\mathcal{N}| \rceil$, where $T$ is given in seconds and $\lceil \cdot \rceil$ is the nearest integer function. The second tabu tenure $\theta_2$ equals $2\theta_1$ for short planning horizon instances, and $\theta_2 = \lceil 1.5\theta_1 \rceil$ for the instances of long planning horizon.

Insertion moves are set tabu as follows: $TABU_1(i) = c + \theta_1$ if the solution $\bar{x}$ is selected and obtained by inserting station $i$ into the solution $x$ at iteration $c$ in Algorithm 1. Hence, the reverse move is declared forbidden for $\theta_1$ iterations. Removal moves are set tabu in a similar way: $TABU_2(i_h) = c + \theta_2$ if $\bar{x}$ is selected and obtained by removing station $i_h$ from $x$ at iteration $c$. Thus, inserting $i_h$ back to the solution is declared forbidden for $\theta_2$ iterations. Tabu exchange moves are set as follows: $TABU_1(i) = c + \theta_1$ and $TABU_2(i_h) = c + \theta_2$ if $\bar{x}$ is selected and obtained by removing station $i_h$ from $x$ and inserting $i$ into $x$ at iteration $c$.

When evaluating the removal neighborhood $\mathcal{R}(x)$ and the insertion neighborhood $\mathcal{I}(x)$, a neighbor solution is considered tabu if $TABU_1(i_h) \geq c$ and $TABU_2(i) \geq c$, respectively. For the exchange neighborhood $\mathcal{E}(x)$, a neighbor solution is considered tabu if both $TABU_1(i_h) \geq c$ and $TABU_2(i) \geq c$.

Tabu lists can sometimes be too powerful as excellent solutions are not allowed to be chosen. A remedy is to apply an aspiration criterion in order to revoke the tabu status. The aspiration criterion used in the heuristic is the most widely used criterion. If $z(x) < z(x^*)$, where $x^*$ is the best solution encountered so far, then the solution $x$ can be chosen even though the solution is obtained by a forbidden move as it is obvious that $x$ has never been encountered before.

3.7 Intensification and diversification procedures

After tabu search, the following intensification and diversification procedures are applied to the best solution from tabu search.

**Adjusting the quantities** Sometimes a solution can be improved by re-adjusting the assigned quantities among the stations (in a heuristically way). In this procedure, we have four
Intra-relocation

1. If stations $i_m$ and $i_h$ are pick-up stations and $m < h$: The number of bikes picked up at these two stations can be modified by either the forward or backward adjustment method. The forward (backward) adjustment procedure requires that some of the bikes, which are originally obtained from $i_m$ ($i_h$), can be picked up at $i_h$ ($i_m$) instead. For the forward adjustment method, we compute the adjustment $\rho = \arg\min_{\nu=0,1,\ldots,\min\{y^P_{i_m},y^D_{i_h}\}} f_{i_m}(s^0_{i_m} - y^P_{i_m} + \nu) + f_{i_h}(s^0_{i_h} - y^P_{i_h} - \nu)$ subject to $y_{i_l+i+1} - \nu > 0$, $l = m + 1, \ldots, h - 1$. Hence, $y^P_{i_m} = y^P_{i_m} - \rho$ and $y^P_{i_h} = y^P_{i_h} + \rho$. The backward adjustment is computed as $\rho = \arg\min_{\nu=0,1,\ldots,\min\{y^P_{i_m},y^D_{i_h}\}} f_{i_m}(s^0_{i_m} - y^P_{i_m} - \nu) + f_{i_h}(s^0_{i_h} - y^P_{i_h} + \nu)$ subject to $y_{i_l+i+1} + \nu \leq k$, $l = m + 1, \ldots, h - 1$. Hence, $y^P_{i_m} = y^P_{i_m} + \rho$ and $y^P_{i_h} = y^P_{i_h} - \rho$.

2. If stations $i_m$ and $i_h$ are drop-off stations and $m < h$: The forward adjustment is computed as $\rho = \arg\min_{\nu=0,1,\ldots,\min\{y^D_{i_m} - s^0_{i_h},y^D_{i_h} - s^0_{i_h}\}} f_{i_m}(s^0_{i_m} + y^D_{i_m} - \nu) + f_{i_h}(s^0_{i_h} + y^D_{i_h} + \nu)$ subject to $y_{i_l+i+1} + \nu \leq k$, $l = m + 1, \ldots, h - 1$. Hence, $y^D_{i_m} = y^D_{i_m} - \rho$ and $y^D_{i_h} = y^D_{i_h} + \rho$. For the backward adjustment, we compute $\rho = \arg\min_{\nu=0,1,\ldots,\min\{s^D_{i_m},s^D_{i_h}-y^D_{i_m}\}} f_{i_m}(s^0_{i_m} + y^D_{i_m} + \nu) + f_{i_h}(s^0_{i_h} + y^D_{i_h} - \nu)$ subject to $y_{i_l+i+1} - \nu \geq 0$, $l = m + 1, \ldots, h - 1$. Hence, $y^D_{i_m} = y^D_{i_m} + \rho$ and $y^D_{i_h} = y^D_{i_h} - \rho$.

3. If $i_h$ is a drop-off station, $i_{h+1} = 0$, and $y_{i_h+h+1} > 0$: We compute $\rho = \arg\min_{\nu=0,1,\ldots,\min\{s^D_{i_h}-y^D_{i_h},y^D_{i_h},s^D_{i_{h+1}}\}} f_{i_h}(s^0_{i_h} + y^D_{i_h} + \nu)$.

If $\rho > 0$, then the drop-off quantity at $i_h$ is increased by $\rho$ (i.e., $y^D_{i_h} = y^D_{i_h} + \rho$).

4. If $i_m$ is a pick-up station, $i_{m-1} = 0$, and $y_{i_{m-1}i_m} > 0$: We calculate the adjustment $\rho = \arg\min_{\nu=0,1,\ldots,\min\{y^P_{i_{m-1}} - s^0_{i_m},y^P_{i_{m-1}} - s^0_{i_m}\}} f_{i_m}(s^0_{i_m} - y^P_{i_m} - \nu)$. If $\rho > 0$, then $y^P_{i_{m-1}} = y^P_{i_{m-1}} + \rho$.

If $\rho > 0$, then $y^P_{i_m} = y^P_{i_m} + \rho$ and $y^P_{i_{m-1}} = y^P_{i_{m-1}} - \rho$. Otherwise, $\rho = \arg\min_{\nu=0,1,\ldots,\min\{k-y^P_{i_{m-1}},y^P_{i_m}\}} f_{i_m}(s^0_{i_m} - y^P_{i_m} + \nu)$.

If $\rho > 0$, then $y^P_{i_m} = y^P_{i_m} - \rho$ and $y^P_{i_{m-1}} = y^P_{i_{m-1}} + \rho$.

Note that by adjusting the pick-up/drop-off quantities, one or several quantities may be reduced to zero. In case they equal zero, the corresponding stations are removed from the routing sequence.

2-opt This local search procedure removes two edges from the solution and adds two new edges back so it remains a tour (Lin, 1965). The objective is about reducing the tour-length. The procedure disregards the pick-up or drop-quantities and employs the best-improvement (BI) strategy.

Intra-relocation This local search procedure is also about reducing the tour-length (and disregarding the pick-up/drop-off quantities) by relocating a station from its position in the route sequence and reinserting it into some other position until no improvements of the tour-length can be achieved.
Removing a station  This procedure removes station \(i\) from the solution \(x\). It chooses the station with the least reduction of the value of objective function (1). The selection criterion is: \(\arg\min_{i \in W(x)} \{f_i(s_i^0) - f_i(s_i^0 - r) : r = \begin{cases} y_i^P, & \text{if } i \in \mathcal{P}; \\ -y_i^D, & \text{if } i \in \mathcal{D}. \end{cases} \} \).

Inserting a station  This is the same as described in Section 3.4 where the best solution is selected from the neighborhood, except that tabu list is not used in this procedure.

Algorithm 2 Perturbation(\(\hat{x}, z^*\))

Require: Tabu search solution \(\hat{x}\) and the best overall solution value \(z^*\)

1: \(\hat{z} = z(\hat{x})\)
2: if \(z(\hat{x}) < z^*\) then
3: \(\hat{x} = \text{AdjustQuantities}(\hat{x})\)
4: \(\hat{x} = \text{2-opt}(\hat{x})\)
5: \(\hat{x} = \text{IntraRelocation}(\hat{x})\)
6: \(\hat{x} = \text{Repairing}(\hat{x})\)
7: \(\hat{x} = \text{AdjustQuantities}(\hat{x})\)
8: for \(a = 1\) to \(5\) do
9: \(\hat{x} = \text{InsertStation}(\hat{x})\)
10: \(\hat{x} = \text{AdjustQuantities}(\hat{x})\)
11: end for
12: \(\hat{x} = \text{RemoveStation}(\hat{x})\)
13: \(\hat{x} = \text{Repairing}(\hat{x})\)
14: \(\hat{x} = \text{AdjustQuantities}(\hat{x})\)
15: else
16: \(\hat{x} = \text{AdjustQuantities}(\hat{x})\)
17: for \(a = 1\) to \(\phi\) do
18: \(\hat{x} = \text{RemoveStation}(\hat{x})\)
19: end for
20: \(\hat{x} = \text{2-opt}(\hat{x})\)
21: \(\hat{x} = \text{IntraRelocation}(\hat{x})\)
22: \(\hat{x} = \text{Repairing}(\hat{x})\)
23: \(\hat{x} = \text{AdjustQuantities}(\hat{x})\)
24: for \(a = 1\) to \(5\) do
25: \(\hat{x} = \text{InsertStation}(\hat{x})\)
26: \(\hat{x} = \text{AdjustQuantities}(\hat{x})\)
27: end for
28: end if
29: \textbf{return} the best solution \(\hat{x}^*\) and the final solution \(\hat{x}\)

After applying the 2-opt, the intra-relocation, and the removal-of-station procedures, the solution is usually infeasible in terms of the pick-up/drop-off quantities. The solution needs to be feasible before the AdjustQuantities procedure can be applied. Hence, the solution will be repaired by initializing all the pick-up/drop-off quantities at stations with zero and then assigning the pick-up/drop-off quantity at each station with a minimal number of bikes so that the solution becomes feasible. Then, one pick-up station and one drop-off station are selected and their pick-up and drop-off quantities are increased by at most two to maximize the reduction of the objective value.

The problem of determining this pair of stations \((p, d)\) and the increase in the pick-up/drop-off
quantity $r$ is formulated as follows.

$$
(p, d) = \arg \max_{i \in W(x) \cap P, j \in W(x) \cap D} \{f_i(s_i^0 - y_i^p) - f_i(s_i^0 - y_i^p - r) + f_j(s_j^0 + y_j^p) - f_j(s_j^0 + y_j^p + r) : \\
\quad r = \min\{s_i^0 - y_i^p, c_j - s_j^0 - y_j^D, \lfloor \tau/(U + L) \rfloor, 2 \}\}
$$

subject to the capacity and loading constraints. Afterwards, the pick-up and drop-off quantities are updated ($y_p^p = y_p^p + r$ and $y_d^D = y_d^D + r$) and the procedure is repeated until $\tau$ does not permit to further increase the pick-up/drop-off quantity of any stations or the objective value could not be reduced.

The perturbation procedure is given in Algorithm 2 and is divided into two parts; the first part is devoted to when an overall best solution has been attained in the preceding tabu search, and the other part is for when an overall best solution has not been obtained. There are some differences between the two parts. First, the above mentioned procedures are applied in a slightly different order. In the first part, the removal of a station is executed at the end of Algorithm 2, while in the second part it is executed in the beginning. This is because in the second part tabu search has not been able to generate a better solution than the current best solution $x^*$. Hence, the algorithm removes $\phi$ stations in the beginning in order to diversify the search in the hope of moving into some other more promising regions of the solution space. Second, the strategy used in 2-opt of the first part is the BI strategy. In the second part, the BI strategy is also used but with some exceptions. If the BI strategy results in relocating station $i_1 \in P$ somewhere else in the route and station $i_2 \in D$, then this change is not implemented. Rather, the next best choice is implemented instead. This is because the objective value is usually deteriorated if the first visited pick-up station is relocated, where most likely its pick-up quantity is picked up at the depot instead (which does not contribute anything to the objective function value).

After the route length is shortened by 2-opt and IntraRelocation, there is usually room for inserting one or more stations into the route. Our preliminary experiments found that inserting a maximum of five stations into the route is a good choice.

If any of the solutions improves $\tilde{x}$, then the solution is saved as $\tilde{x}^*$ and $\tilde{z}$ is updated. The best solution $\tilde{x}^*$ and the final solution $\tilde{x}$ are returned from the Perturbation procedure.

### 3.8 The overall algorithm

To further improve the solutions, the tabu search procedure is embedded into an iterative framework (see Algorithm 3), where several intensification/diversification mechanisms (see Section 3.7) are applied to the solution obtained from tabu search. Algorithm 3 is run until no improvement can be achieved or it has run for at least $\beta$ iterations. Preliminary computational experiments have shown that when $K = 1$ and $w < \beta$ (with $\beta \geq 5$) is left out, the algorithm may sometimes terminate after only two or three iterations which results in inferior solutions. When the algorithm runs for at least five iterations, the solutions are significantly better.

The algorithm continues with the final solution $\tilde{x}$ from the Perturbation procedure rather than from the best solution $x'$. Computational experiments have also shown that restarting the search from $\tilde{x}$ is more beneficial from a diversification point of view. Otherwise, it may be difficult to leave valleys in the search space (especially the deep ones) as only small changes are made in each iteration of the heuristic.
Algorithm 3  Iterated Tabu Search

1: $x_0 = \text{InitialSolution}()$
2: Set $x^* = x_0$, $z^* = z(x^*)$, $iterWI = 0$ and $w = 0$.
3: while $iterWI < K$ or $w < \beta$ do
4: \hspace{1em} $\hat{x} = \text{TabuSearch}(x_0)$
5: \hspace{1em} $(x', \hat{x}) = \text{Perturbation}(\hat{x}, z^*)$
6: \hspace{1em} if $z(x') < z^*$ then
7: \hspace{2em} Set $x^* = x'$, $z^* = z(x^*)$ and $iterWI = 0$.
8: \hspace{1em} else
9: \hspace{2em} Set $iterWI = iterWI + 1$.
10: \hspace{1em} end if
11: Set $w = w + 1$.
12: Set $x_0 = \hat{x}$.
13: end while
14: return $x^*$

4 Computational experiments

The heuristic was coded in C++ and all computational experiments were carried out on a Dell notebook with an Intel Core i5-2520M CPU @ 2.5 GHz. Three sets of instances were used to conduct the experiments in this study. The sets include:

Set 1 It contains the Paris instances (up to 100 stations) used in the study by Raviv et al. (2013). They showed the results for 30 and 60 stations, and up to two vehicles. However, in this paper, instances of varying numbers of stations (30 to 100 stations) were used. The vehicle capacity is fixed to 20. There is a total of 12 instances in this set.

Set 2 This set contains instances of 100-400 stations, with a step size of 25. The smaller instances are subsets of the larger ones. The locations of the different stations were taken from the benchmark instances for the 1-PDTSP studied by Hernández-Pérez and Salazar-González (2004b). As in Raviv et al. (2013), the following setting was used. a) Manhattan distances (see the definition in http://en.wiktionary.org/wiki/Manhattan_distance) were used as they are more realistic than Euclidean distances. b) Two repositioning times (in seconds) were considered: $T = 9000$ and $T = 18000$. c) The time for picking up or dropping off one bike is 60 seconds. d) Other input data (i.e., $s^0_i$, $c_i$, and $f_i(\cdot)$) were taken from their instances. As their data do not cover all 400 stations, the original data were replicated when necessary. The vehicle capacity in each instance varied from 10 to 40, with a step size of 10. This set contains 104 instances.

Set 3 This set contains instances ranging from 200 to 400 stations. The locations of the stations were also taken from the benchmark instances for the 1-PDTSP. The vehicle capacity is 20. Repositioning times and loading/unloading times are the same as above. Other input data were randomly generated. This set contains a total of 30 instances.

The first set of instances was included in the experiments because our problem is closely related to the problem studied by Raviv et al. (2013). Because of the same reason, their solution method (i.e. CPLEX) was used for comparison. The instances used by Raviv et al. (2013) contain 30-100 stations and we believe that the sizes of the instances do not justify the usage of heuristics. Hence, the experiments were also conducted on larger instances, i.e., the second and third sets of instances with 100-400 stations.
The results from the heuristic were obtained by setting $K = 1$, $\beta = 5$, and $\gamma = 200$ after some preliminary experiments. The parameter $\phi$ was set to $[0.5\eta]$ for the instances of short planning horizon (i.e., $T = 9000$), and $[0.3\eta]$ for the instances of long planning horizon (i.e., $T = 18000$), where $\eta$ is the number of stations visited by the vehicle.

The results obtained from the heuristic were compared to those obtained from CPLEX 12.4 with the default settings and a maximum running time of 2 hours. For each instance, CPLEX returned a lower bound, an upper bound ($z^{UB}$), and a solution $x^*$ (if feasibility was achieved within those 2 hours). In principle, if CPLEX terminates before the 2-hour limit, it implies that $x^*$ is the optimal solution and $z(x^*) = z^{UB}$. However, this may not always be true regarding the experiments conducted in this study as it has been verified that for some instances, this holds: $z(x^*) < z^{UB}$. The discrepancy between $z(x^*)$ and $z^{UB}$ may be due to some internal rounding errors in CPLEX. Hence, $z(x^*)$ instead of $z^{UB}$ is reported for all instances in which optimality has been proven by CPLEX.

Tables 2 and 3 present the results from the experiments conducted on the first set of instances in which optimality has been proven by CPLEX for most of them. Opt denotes the true optimal objective value of the solution $x^*$, i.e., $z(x^*)$, and Heuristic denotes the result from the heuristic. Gap denotes the deviation (in %) of the results from Opt. CPU is the running time of CPLEX or the heuristic (in seconds). $\psi = \sum_{i\in N} y^{D}_i$ and $\varphi = \sum_{i\in N} y^{P}_i$ denote the total number of bikes dropped off and picked up at all of the visited stations, respectively. Done is the percentage of job done, and is the percentage reduction in the expected shortages. This is computed using the formula stated in Raviv et al. (2013). Finally, $\varsigma = \sum_{i,j\in N_0,i\neq j} h_{ij} x_{ij}$ is the total travel time in seconds.

**Table 2: Results of Raviv’s instances, $\beta = 5$**

| $|N_0|$ | $T$ | Opt | Gap  | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\varsigma$ |
|-----|-----|-----|------|-----|-----------|-----|--------|--------|----------|-------|------------|
| 30  | 9000| 214.16 | 0.570 | 22.75 | 215.38 | 0.225 | 8   | 51    | 42    | 40.26 | 2854 |
| 18000| 167.07 | 0.431 | 523.75 | 167.79 | 0.245 | 18   | 95    | 75    | 66.63 | 6488 |
| 45  | 9000 | 335.05 | 0.546 | 790.75 | 336.88 | 0.305 | 10  | 45    | 45    | 30.60 | 3552 |
| 18000| 273.09 | 1.058 | 757.04 | 275.98 | 0.375 | 18   | 100   | 80    | 55.49 | 5992 |
| 60  | 9000 | 463.45 | -0.047 | 7200.00 | 463.23 | 0.230 | 9   | 46    | 45    | 22.50 | 3468 |
| 18000| 382.52 | 1.406 | 638.10 | 387.90 | 0.450 | 20   | 100   | 81    | 45.27 | 5950 |
| 75  | 9000 | 568.57 | 0.026 | 1437.20 | 586.72 | 0.465 | 8   | 47    | 47    | 21.01 | 3270 |
| 18000| 486.15 | 0.084 | 1275.20 | 486.56 | 0.455 | 19   | 97    | 96    | 42.54 | 6350 |
| 90  | 9000 | 692.00 | 0.062 | 323.09 | 692.43 | 0.745 | 11  | 49    | 49    | 18.86 | 3042 |
| 18000| 609.48 | 0.000 | 1999.15 | 609.57 | 0.375 | 20   | 97    | 97    | 37.40 | 6334 |
| 100 | 9000 | 782.69 | 0.055 | 760.00 | 782.12 | 0.450 | 11  | 49    | 49    | 17.07 | 3042 |
| 18000| 700.17 | 0.734 | 2654.00 | 705.31 | 0.785 | 20   | 93    | 93    | 32.80 | 6814 |
| Avg |     | 0.410 | 4.531.748 |     | 0.488 | 0.815 | 0.570 | 22.75 | 215.38 | 0.225 | 8   |

From Tables 2 and 3, we observe that CPLEX found the optimal solution to almost all of the instances from Set 1 within the 2-hour limit. The results obtained by our heuristic are 0.41% from the optimal values on average and mean computing time is 0.488 second. Our heuristic found the optimal solution for one instance (the optimal objective value is bolded in Tables 2 and 3), and improved the bound on another instance (which gives a negative gap value in Tables 2 and 3). When letting the heuristic run a little longer (e.g., $\beta = 30$), the average deviation is then reduced to 0.187% with a slightly increase in computing time (on average 2 seconds more). Compared with the average running time of CPLEX (i.e., 1531.748 seconds), our heuristic could...
get a very good upper bound efficiently.

Tables 2 and 3 also show that when the operation duration is longer, the number of stations visited is larger. Moreover, we observe that the number of stations visited is much less than the number of stations in the network, because the vehicle has a limited capacity and the repositioning time is not long enough to allow all stations to be visited.

Because our problem is different from Raviv et al. (2013) in the sense that we explicitly define pick-up and drop-off stations and do not consider the operational cost in the objective function, their results may differ from ours shown in Tables 2 and 3. In principle, without rounding errors due to CPLEX and the weight of the total operational cost relative to the total penalty cost equal to zero, the optimal shortage of their problem obtained could be better than that of ours, because our solution set is smaller as a result of predetermining pick-up and drop-off stations. Meanwhile, as in their results, the total number of bikes dropped off at all of the visited stations can be larger than that picked up at these stations. This implies that some bikes that are dropped off at some visited stations are actually picked up from the depot.

Tables 4-11 present the results from the computational experiments conducted on the second set of instances. LB and UB stand for the lower and upper bounds obtained from CPLEX, respectively. CPLEX could not find the optimal solution to any of the 104 instances from the second set. Feasibility was achieved for less than half of the instances within the 2-hour limit. The UBs are usually not too good compared to the results obtained from our heuristic which produces solutions with better quality in more than half of the cases (where feasibility was achieved by CPLEX) using much less computing time. The overall improvement of the results obtained by our heuristic over the UBs from CPLEX (for the 45 instances where feasibility was achieved) is 0.456%.

As can be seen in these tables, the heuristic is very fast and produces solutions with high quality. For the short planning horizon instances, the average gap (from the LBs) is 0.687%, and for the longer horizon instances, the average gap is 1.261%. The overall average deviation is 0.974%.

According to the summary table 12, it seems that the heuristic is better in planning vehicle routes and determining the pick-up/drop-off quantities for the instances of short planning horizon. The results can be further improved by running the heuristic longer (e.g., \( \beta = 30 \), Table 13) where

### Table 3: Results of Raviv’s instances, \( \beta = 30 \)

| \( |N_0| \) |  \( T \)  |  \( \text{Opt} \)  |  \( \text{Gap} \)  |  \( \text{CPU} \)  |  \( \text{Heuristic} \)  |  \( \text{CPU} \)  |  \( \eta \)  |  \( \psi \)  |  \( \varphi \)  |  \( \text{Done} \)  |  \( \varsigma \)  |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 30 | 9000 | 214.16 | 0.570 | 22.75 | 215.38 | 0.857 | 8 | 51 | 42 | 40.26 | 2854 |
| 18000 | 167.07 | 0.305 | 523.75 | 167.58 | 1.279 | 17 | 96 | 76 | 66.75 | 6390 |
| 45 | 9000 | 335.05 | 0.546 | 790.75 | 336.88 | 1.403 | 10 | 45 | 45 | 30.60 | 3552 |
| 18000 | 273.09 | 0.549 | 757.04 | 274.59 | 1.653 | 19 | 100 | 80 | 56.06 | 5986 |
| 60 | 9000 | 463.45 | -0.222 | 7200.00 | 462.42 | 1.450 | 9 | 47 | 47 | 22.75 | 3296 |
| 18000 | 382.52 | 0.251 | 638.10 | 383.48 | 2.542 | 17 | 103 | 85 | 46.61 | 5564 |
| 75 | 9000 | 568.57 | 0.026 | 1437.20 | 586.72 | 1.840 | 8 | 47 | 47 | 21.01 | 3270 |
| 18000 | 486.15 | 0.084 | 1275.20 | 486.56 | 3.213 | 19 | 97 | 96 | 42.54 | 6350 |
| 90 | 9000 | 692.00 | 0.062 | 323.09 | 692.43 | 3.432 | 11 | 49 | 49 | 18.86 | 3042 |
| 18000 | 609.48 | 0.000 | 1999.15 | **609.48** | 4.180 | 20 | 97 | 97 | 37.40 | 6334 |
| 100 | 9000 | 782.69 | 0.055 | 760.00 | 783.12 | 3.962 | 11 | 49 | 49 | 17.07 | 3042 |
| 18000 | 700.17 | 0.013 | 2654.00 | 700.26 | 4.586 | 20 | 97 | 97 | 33.82 | 6334 |
| Avg | | | | | **0.187** | **1531.748** | **2.533** | | | | |
Table 4: Results of the second set of instances, $T = 9000$ and $k = 10$

| $|N_0|$ | LB Gap | UB Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\varsigma$ |
|------|--------|--------|-----|-----------|-----|-------|-------|-------|-------|-------|
| 100  | 765.689| 0.843  | 772.91| -0.092    | 7200| 772.20| 0.759 | 14     | 53    | 53    | 19.29 | 2610 |
| 125  | 1021.255| 0.643  | 1034.20| -0.613    | 7200| 1027.86| 1.444 | 15     | 55    | 55    | 15.15 | 2400 |
| 150  | 1247.729| 0.545  | 1257.66| -0.246    | 7200| 1254.57| 1.438 | 18     | 55    | 55    | 12.66 | 2374 |
| 175  | 1406.761| 0.711  | 1430.66| -0.967    | 7200| 1416.83| 1.986 | 16     | 58    | 58    | 11.65 | 2016 |
| 200  | 1629.402| 0.697  | -     | -        | 7200| 1640.84| 2.334 | 17     | 57    | 57    | 10.51 | 2136 |

| 225  | 1885.456| 0.624  | 1948.84| -2.645    | 7200| 1897.30| 2.989 | 19     | 56    | 56    | 9.16  | 2276 |
| 250  | 2106.665| 0.817  | -     | -        | 7200| 2124.02| 3.281 | 17     | 57    | 57    | 8.11  | 2150 |
| 275  | 2270.393| 0.685  | -     | -        | 7200| 2286.06| 3.764 | 16     | 57    | 57    | 7.84  | 2144 |
| 300  | 2493.339| 0.785  | -     | -        | 7200| 2513.06| 3.703 | 18     | 55    | 55    | 7.20  | 2382 |
| 325  | 2752.142| 0.920  | -     | -        | 7200| 2777.69| 4.195 | 16     | 52    | 52    | 6.15  | 2750 |
| 350  | 2974.340| 0.732  | -     | -        | 7200| 2996.27| 3.577 | 16     | 55    | 55    | 5.97  | 2394 |
| 375  | 3138.892| 0.705  | -     | -        | 7200| 3161.19| 6.195 | 18     | 56    | 56    | 5.74  | 2264 |
| 400  | 3375.275| 0.659  | -     | -        | 7200| 3397.68| 8.186 | 15     | 55    | 55    | 5.38  | 2390 |

| Avg  | 0.721  |        |       | 3.373    |     |       |       |       |       |       |       |

Table 5: Results of the second set of instances, $T = 18000$ and $k = 10$

| $|N_0|$ | LB Gap | UB Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\varsigma$ |
|------|--------|--------|-----|-----------|-----|-------|-------|-------|-------|-------|
| 100  | 677.236| 1.582  | 688.07| 0.007     | 7200| 688.12| 1.250 | 30     | 106   | 106   | 36.28 | 5268 |
| 125  | 922.087| 1.974  | 932.67| 0.857     | 7200| 940.66| 1.675 | 32     | 108   | 107   | 28.27 | 5014 |
| 150  | 1142.561|1.113   |1158.77|0.289     | 7200| 1155.42|2.213 | 33     | 111   | 111   | 25.03 | 4662 |
| 175  | 1300.503|1.108   | -     | -        | 7200| 1315.08|3.330 | 31     | 112   | 112   | 23.26 | 4558 |
| 200  | 1520.752|1.006   |1542.33|0.397     | 7200| 1536.21|2.737 | 36     | 113   | 113   | 20.99 | 4422 |
| 225  | 1773.529|1.241   |1833.10|2.034     | 7200| 1795.81|3.917 | 32     | 112   | 112   | 17.91 | 4538 |
| 275  | 2153.474|1.129   | -     | -        | 7200| 2178.06|5.947 | 35     | 116   | 116   | 15.71 | 4076 |
| 300  | 2376.437|1.408   | -     | -        | 7200| 2410.37|5.899 | 31     | 108   | 108   | 14.08 | 5034 |
| 325  | 2631.309|1.210   | -     | -        | 7200| 2663.54|9.975 | 37     | 111   | 111   | 13.02 | 4652 |
| 350  | 2852.320|1.334   | -     | -        | 7200| 2890.89|6.619 | 31     | 109   | 109   | 11.83 | 4884 |
| 375  | 3014.210|1.397   | -     | -        | 7200| 3056.93|7.876 | 32     | 109   | 109   | 11.31 | 4910 |
| 400  | 3249.340|1.156   | -     | -        | 7200| 3287.34|10.052| 32     | 111   | 111   | 10.91 | 4672 |

| Avg  | 1.286  |        |       | 5.268    |     |       |       |       |       |       |

the overall deviation is reduced to 0.87%.

As expected, the runtime of the heuristic increases with the size of the network and repositioning time because the solution space is larger. However, the runtime does not decrease with vehicle capacity as shown in Table 12. The runtime decreases with vehicle capacity but increases with it at the end. It may be because an increase in vehicle capacity reduces the chance of having infeasible solutions and this is beneficial only if vehicle capacity is small. However, when vehicle capacity is larger, the feasible solution space is also larger. Hence, a further increase in vehicle capacity leads to a longer computation time. It may be further explained by the fact that the heuristic does not terminate after a fixed number of iterations, but rather it terminates after $K$ consecutive iterations without improvement. Hence, the number of iterations needed by the
Table 6: Results of the second set of instances, $T = 9000$ and $k = 20$

| $|N_0|$ | LB | Gap | UB | Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\varsigma$ |
|------|----|-----|----|-----|-----|-----------|-----|--------|--------|--------|-------|--------|
| 100  | 757.675 | 0.845 | 762.19 | 0.255 | 7200 | 764.13 | 0.476 | 10 | 56 | 56 | 20.91 | 2268 |
| 125  | 1015.481 | 0.708 | 1020.08 | 0.259 | 7200 | 1022.72 | 0.968 | 15 | 58 | 58 | 15.93 | 2036 |
| 150  | 1241.588 | 0.581 | 1251.86 | -0.240 | 7200 | 1248.85 | 1.249 | 12 | 56 | 56 | 13.37 | 2262 |
| 175  | 1398.796 | 0.780 | 1415.29 | -0.389 | 7200 | 1409.79 | 1.557 | 12 | 58 | 58 | 12.45 | 1994 |
| 200  | 1620.869 | 0.853 | 1651.72 | -1.024 | 7200 | 1634.81 | 1.791 | 9 | 57 | 57 | 11.11 | 2150 |
| 225  | 1878.321 | 0.748 | 1908.03 | -0.816 | 7200 | 1892.47 | 2.865 | 11 | 58 | 58 | 9.58 | 2032 |
| 250  | 2103.136 | 0.587 | - | - | 7200 | 2115.51 | 2.708 | 14 | 59 | 59 | 8.77 | 1912 |
| 275  | 2266.508 | 0.630 | - | - | 7200 | 2280.87 | 2.652 | 14 | 59 | 59 | 8.22 | 1902 |
| 300  | 2488.187 | 0.710 | - | - | 7200 | 2505.99 | 3.948 | 15 | 60 | 60 | 7.67 | 1772 |
| 325  | 2745.796 | 0.636 | - | - | 7200 | 2763.36 | 3.315 | 14 | 60 | 60 | 7.01 | 1800 |
| 350  | 2970.251 | 0.744 | - | - | 7200 | 2992.52 | 2.871 | 12 | 56 | 56 | 6.18 | 2274 |
| 375  | 3134.805 | 0.481 | - | - | 7200 | 3149.95 | 4.851 | 11 | 61 | 61 | 6.34 | 1678 |
| 400  | 3370.077 | 0.687 | - | - | 7200 | 3393.38 | 3.642 | 11 | 56 | 56 | 5.59 | 2264 |
| Avg  | 0.691 | &nbsp;  | &nbsp; | &nbsp; | 7200 | 1.173 | 2.530 | &nbsp; | &nbsp; | &nbsp; | &nbsp; | &nbsp; |

Table 7: Results of the second set of instances, the second set of instances, $T = 18000$ and $k = 20$

| $|N_0|$ | LB | Gap | UB | Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\varsigma$ |
|------|----|-----|----|-----|-----|-----------|-----|--------|--------|--------|-------|--------|
| 100  | 657.489 | 1.501 | 670.73 | -0.480 | 7200 | 667.51 | 0.961 | 23 | 114 | 114 | 40.44 | 4300 |
| 125  | 905.537 | 2.006 | 919.31 | 0.518 | 7200 | 924.07 | 1.273 | 24 | 111 | 111 | 30.76 | 4680 |
| 150  | 1128.336 | 1.286 | 1144.23 | -0.104 | 7200 | 1143.04 | 2.797 | 29 | 114 | 114 | 26.58 | 4294 |
| 175  | 1285.336 | 1.198 | 1307.70 | -0.518 | 7200 | 1300.92 | 1.570 | 27 | 115 | 115 | 24.88 | 4198 |
| 200  | 1505.106 | 1.187 | 1527.85 | -0.306 | 7200 | 1523.18 | 2.877 | 22 | 119 | 119 | 22.30 | 3682 |
| 225  | 1759.620 | 1.102 | 1845.61 | -3.597 | 7200 | 1779.22 | 2.443 | 25 | 117 | 117 | 19.34 | 3952 |
| 250  | 1979.037 | 1.047 | - | - | 7200 | 1999.98 | 3.872 | 24 | 121 | 121 | 17.67 | 3474 |
| 275  | 2143.107 | 1.140 | - | - | 7200 | 2167.81 | 4.134 | 24 | 118 | 118 | 16.46 | 3840 |
| 300  | 2363.469 | 1.251 | - | - | 7200 | 2393.41 | 6.962 | 27 | 116 | 116 | 15.22 | 4078 |
| 325  | 2618.046 | 1.003 | - | - | 7200 | 2644.56 | 6.417 | 28 | 121 | 121 | 14.17 | 3468 |
| 350  | 2840.673 | 0.991 | - | - | 7200 | 2869.10 | 4.612 | 28 | 120 | 120 | 13.05 | 3598 |
| 375  | 3003.741 | 0.807 | - | - | 7200 | 3028.17 | 6.594 | 24 | 124 | 124 | 12.85 | 3120 |
| 400  | 3273.559 | 0.734 | - | - | 7200 | 3261.30 | 5.554 | 26 | 126 | 126 | 12.21 | 2852 |
| Avg  | 1.173 | &nbsp;  | &nbsp; | &nbsp; | 7200 | 3.851 | &nbsp; | &nbsp; | &nbsp; | &nbsp; | &nbsp; | &nbsp; |

heuristic may vary from instance to instance which results in different computing times.

Table 14 shows how imbalanced each instance of the second set is. Ideal denotes the objective value to an instance at its balanced state, while Initial denotes the objective value to an instance before repositioning takes place. Every instance is highly imbalanced initially, and according to Tables 4-11, only a fraction of the imbalance can be covered. In particular, the larger the network is, the smaller fraction of the imbalance can be covered. This is because under the restrictions in vehicle capacity and operating hours, the vehicle can only visit very limited stations to reduce their shortages. Hence, even if the network is larger, the number of stations visited and the total number of bikes picked up are about the same. Hence, the fraction of the imbalance to...
Table 8: Results of the second set of instances, $T = 9000$ and $k = 30$

| $|N_0|$ | LB | Gap | UB | Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\zeta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 755.698 | 0.384 | 757.55 | 0.140 | 7200 | 758.61 | 0.671 | 13 | 60 | 60 | 22.02 | 1788 |
| 125 | 1012.685 | 0.641 | 1017.36 | 0.183 | 7200 | 1019.22 | 0.919 | 11 | 60 | 60 | 16.45 | 1790 |
| 150 | 1239.010 | 0.493 | 1244.57 | 0.047 | 7200 | 1245.15 | 1.167 | 13 | 59 | 59 | 13.83 | 1904 |
| 175 | 1397.390 | 0.854 | 1409.70 | -0.020 | 7200 | 1409.42 | 1.275 | 12 | 59 | 59 | 12.49 | 1894 |
| 200 | 1621.292 | 0.930 | 1636.25 | 0.016 | 7200 | 1636.51 | 1.925 | 12 | 57 | 57 | 10.94 | 2156 |

Avg | 0.651 | 2.256 |

Table 9: Results of the second set of instances, $T = 18000$ and $k = 30$

| $|N_0|$ | LB | Gap | UB | Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\zeta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 650.735 | 1.964 | 662.06 | 0.258 | 7200 | 663.77 | 0.767 | 23 | 113 | 113 | 41.20 | 4418 |
| 125 | 901.479 | 1.824 | 916.74 | 0.163 | 7200 | 918.23 | 1.544 | 20 | 116 | 116 | 31.64 | 4072 |
| 150 | 1126.147 | 1.286 | 1137.96 | 0.251 | 7200 | 1140.82 | 1.683 | 24 | 118 | 118 | 26.85 | 3836 |
| 175 | 1281.509 | 1.420 | 1305.91 | -0.455 | 7200 | 1299.97 | 1.922 | 24 | 114 | 114 | 24.98 | 4316 |
| 200 | 1499.720 | 1.266 | - | - | 7200 | 1518.95 | 1.875 | 24 | 120 | 120 | 22.72 | 3600 |
| 225 | 1754.574 | 1.366 | 1787.79 | -0.498 | 7200 | 1778.88 | 2.978 | 25 | 116 | 116 | 19.37 | 4078 |
| 250 | 1978.997 | 0.900 | - | - | 7200 | 1996.96 | 5.150 | 27 | 121 | 121 | 17.91 | 3478 |
| 275 | 2137.996 | 1.233 | - | - | 7200 | 2164.69 | 4.580 | 28 | 119 | 119 | 16.69 | 3694 |
| 300 | 2358.116 | 0.986 | - | - | 7200 | 2381.61 | 4.837 | 25 | 124 | 124 | 16.01 | 3118 |
| 325 | 2613.583 | 0.881 | - | - | 7200 | 2636.80 | 5.280 | 27 | 125 | 125 | 14.63 | 2996 |
| 350 | 2835.486 | 0.955 | - | - | 7200 | 2862.84 | 6.257 | 27 | 124 | 124 | 13.39 | 3110 |
| 375 | 2998.819 | 0.869 | - | - | 7200 | 3025.11 | 6.015 | 28 | 125 | 125 | 13.01 | 2988 |
| 400 | 3368.221 | 0.449 | - | - | 7200 | 3383.40 | 4.334 | 13 | 62 | 62 | 6.09 | 1506 |

Avg | 1.208 | 3.667 |

be covered is smaller for larger networks. Tables 4-11 also show that when the operating hours (or the vehicle capacity) is reduced, the fraction of the imbalance to be covered is decreased because the number of stations visited (or the number of bikes picked up or dropped off at visited stations) is reduced in general.

Regarding the third set of instances, CPLEX could not find the optimal solution to any of the 30 instances. Feasibility was achieved for four of the instances within the 2-hour limit of CPU time (see Tables 15-16). Our heuristic managed to find better bounds than those provided by CPLEX. The average gaps (from LBs) are 2.927% and 3.878% for short planning horizon instances and long planning horizon instances, respectively. These gaps are further reduced to 2.668% and 3.43% when the heuristic runs a little longer (e.g., $\beta = 30$).
Table 10: Results of the second set of instances, $T = 9000$ and $k = 40$

| $|N_0|$ | LB | Gap | UB | Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\phi$ | Done | $\varsigma$ |
|------|-----|-----|----|-----|-----|----------|-----|--------|--------|--------|-------|--------|
| 100  | 755.229 | 0.668 | 755.88 | 0.586 | 7200 | 760.31 | 0.713 | 11 | 57 | 57 | 21.68 | 2148 |
| 125  | 1011.117 | 0.944 | 1019.15 | 0.157 | 7200 | 1020.75 | 0.918 | 13 | 60 | 60 | 16.22 | 1796 |
| 150  | 1237.474 | 0.619 | 1244.79 | 0.031 | 7200 | 1245.18 | 1.654 | 12 | 60 | 60 | 13.83 | 1798 |
| 175  | 1394.505 | 0.772 | 1423.98 | -1.308 | 7200 | 1405.35 | 2.015 | 14 | 60 | 60 | 12.96 | 1794 |
| 200  | 1619.833 | 0.722 | - | - | 7200 | 1631.62 | 2.000 | 10 | 59 | 59 | 11.43 | 1898 |
| 225  | 1876.497 | 0.621 | - | - | 7200 | 1888.23 | 2.583 | 10 | 60 | 60 | 9.94 | 1786 |
| 250  | 2099.115 | 1.099 | - | - | 7200 | 2122.44 | 3.114 | 17 | 55 | 55 | 8.24 | 2370 |
| 275  | 2263.230 | 0.602 | - | - | 7200 | 2276.93 | 2.357 | 12 | 60 | 60 | 8.51 | 1794 |
| 300  | 2486.304 | 0.627 | - | - | 7200 | 2502.00 | 3.136 | 11 | 61 | 61 | 7.94 | 1662 |
| 325  | 2743.351 | 0.689 | - | - | 7200 | 2762.38 | 2.603 | 12 | 62 | 62 | 7.07 | 1542 |
| 350  | 2968.040 | 0.594 | - | - | 7200 | 2985.77 | 5.541 | 11 | 60 | 60 | 6.55 | 1798 |
| 375  | 3131.912 | 0.595 | - | - | 7200 | 3150.66 | 4.780 | 13 | 60 | 60 | 6.30 | 1792 |
| 400  | 3368.217 | 0.377 | - | - | 7200 | 3380.95 | 7.955 | 13 | 64 | 64 | 6.22 | 1302 |
| Avg  | 117.378 | 0.258 | 121.275 | 0.062 | 7200 | 121.87 | 0.308 | 11 | 59 | 59 | 11.43 | 1898 |

Table 11: Results of the second set of instances, $T = 18000$ and $k = 40$

| $|N_0|$ | LB | Gap | UB | Gap | CPU | Heuristic | CPU | $\eta$ | $\psi$ | $\phi$ | Done | $\varsigma$ |
|------|-----|-----|----|-----|-----|----------|-----|--------|--------|--------|-------|--------|
| 100  | 648.455 | 2.237 | 661.68 | 0.243 | 7200 | 663.29 | 0.987 | 24 | 115 | 115 | 41.30 | 4188 |
| 125  | 899.603 | 2.602 | 913.45 | 1.116 | 7200 | 923.64 | 1.267 | 25 | 114 | 114 | 30.83 | 4320 |
| 150  | 1124.078 | 1.434 | 1138.24 | 0.192 | 7200 | 1140.43 | 1.776 | 27 | 115 | 115 | 26.90 | 4172 |
| 175  | 1279.517 | 1.533 | 1296.87 | 0.198 | 7200 | 1299.44 | 2.003 | 27 | 118 | 118 | 25.05 | 3816 |
| 200  | 1499.273 | 1.241 | 1521.71 | -0.237 | 7200 | 1518.11 | 1.923 | 29 | 125 | 125 | 22.81 | 2998 |
| 225  | 1753.076 | 1.279 | 1792.08 | -0.909 | 7200 | 1775.79 | 3.521 | 25 | 119 | 119 | 19.64 | 3698 |
| 275  | 2136.965 | 1.348 | - | - | 7200 | 2166.17 | 4.098 | 27 | 116 | 116 | 16.58 | 4074 |
| 300  | 2358.169 | 1.023 | - | - | 7200 | 2382.55 | 5.316 | 25 | 125 | 125 | 15.94 | 2994 |
| 325  | 2612.708 | 0.885 | - | - | 7200 | 2636.03 | 7.554 | 26 | 126 | 126 | 14.68 | 2864 |
| 350  | 2835.190 | 1.150 | - | - | 7200 | 2868.17 | 7.901 | 23 | 123 | 123 | 13.10 | 3238 |
| 375  | 2998.582 | 0.996 | - | - | 7200 | 3028.76 | 5.721 | 27 | 123 | 123 | 12.82 | 3224 |
| 400  | 3233.586 | 0.772 | - | - | 7200 | 3258.75 | 7.149 | 24 | 125 | 125 | 12.34 | 2980 |
| Avg  | 1.378 | 0.057 | 1.552 | 0.202 | 7200 | 2.038 | 0.430 | 24 | 123 | 123 | 11.90 | 3038 |

5 Conclusion

In this paper, we present an iterated tabu search heuristic for the static bike repositioning problem. The implemented heuristic makes use of specialized neighborhood structures and perturbation to obtain diversification in the search. Experiments were conducted on 156 instances. Computational results show that this simple iterated tabu search heuristic produces high quality solutions using very short computing times.

The contributions of this paper to the literature include the following. 1) This paper proposes a modification to the existing bike reposition problem to improve its realism and to reduce the
solution space. 2) This paper develops an efficient and novel heuristic to obtain high quality solutions to solve the proposed problem. 3) The specific operators developed can also be incorporated in other heuristics to solve the same repositioning problem or other repositioning problems with different objective functions.

In the future, we extend our method to solve the static repositioning problem with multiple
Table 15: Results of the third set of instances, $T = 9000$ and $k = 20$

| $|\mathcal{N}|$ | LB    | Gap | UB    | Gap | CPU | Heuristic CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\varsigma$ |
|-------|-------|-----|-------|-----|-----|---------------|-------|-------|---------|-------|----------|
| 200   | 4705.242 | 2.590 | 5027.60 | -3.924 | 7200 | 4830.33 | 1.996 | 19 | 46 | 46 | 24.18 | 3476 |
| 200   | 4627.208 | 5.379 | - | - | 7200 | 4890.28 | 1.861 | 15 | 45 | 45 | 24.17 | 3570 |
| 200   | 4569.509 | 1.599 | - | - | 7200 | 4643.78 | 2.152 | 18 | 52 | 52 | 25.70 | 2758 |
| 200   | 4060.484 | 3.006 | - | - | 7200 | 4186.33 | 2.075 | 22 | 49 | 49 | 26.78 | 3082 |
| 200   | 4644.023 | 3.182 | - | - | 7200 | 4796.63 | 2.028 | 17 | 48 | 48 | 25.15 | 3202 |
| 300   | 6708.367 | 3.167 | - | - | 7200 | 6927.74 | 2.520 | 21 | 47 | 47 | 17.58 | 3338 |
| 300   | 7689.757 | 2.926 | - | - | 7200 | 7921.54 | 3.820 | 19 | 51 | 51 | 17.63 | 2864 |
| 300   | 7059.826 | 3.455 | - | - | 7200 | 7312.46 | 3.731 | 19 | 53 | 53 | 19.65 | 2636 |
| 300   | 7896.429 | 2.899 | - | - | 7200 | 8132.14 | 3.626 | 16 | 47 | 47 | 17.89 | 3348 |
| 300   | 7810.430 | 3.312 | - | - | 7200 | 8077.99 | 3.548 | 18 | 46 | 46 | 17.68 | 3464 |
| 400   | 9867.358 | 2.469 | - | - | 7200 | 10117.10 | 5.263 | 16 | 51 | 51 | 16.36 | 2870 |
| 400   | 9464.528 | 2.616 | - | - | 7200 | 9718.80 | 4.450 | 17 | 54 | 54 | 17.01 | 2504 |
| 400   | 10450.917 | 2.491 | - | - | 7200 | 10717.90 | 5.785 | 24 | 48 | 48 | 15.50 | 3228 |
| 400   | 11064.190 | 1.938 | - | - | 7200 | 11282.90 | 6.584 | 22 | 50 | 50 | 14.45 | 2992 |
| 400   | 9294.085 | 2.876 | - | - | 7200 | 9569.30 | 5.460 | 15 | 49 | 49 | 16.88 | 3114 |
| Avg   |        | 2.927 |     |     |     |            |       |     |        |       | 3.660 |

Table 16: Results of the third set of instances, $T = 18000$ and $k = 20$

| $|\mathcal{N}|$ | LB    | Gap | UB    | Gap | CPU | Heuristic CPU | $\eta$ | $\psi$ | $\varphi$ | Done | $\varsigma$ |
|-------|-------|-----|-------|-----|-----|---------------|-------|-------|---------|-------|----------|
| 200   | 3878.686 | 2.793 | 4501.27 | -11.355 | 7200 | 3990.14 | 2.418 | 40 | 101 | 101 | 43.65 | 5830 |
| 200   | 3755.538 | 4.299 | - | - | 7200 | 3924.25 | 2.992 | 36 | 100 | 100 | 46.02 | 5994 |
| 200   | 3806.262 | 3.289 | 3943.67 | -0.202 | 7200 | 3935.71 | 2.964 | 41 | 108 | 108 | 42.74 | 5034 |
| 200   | 3346.358 | 4.498 | - | - | 7200 | 3503.97 | 3.166 | 47 | 98 | 98 | 45.79 | 6192 |
| 200   | 3784.330 | 4.204 | 4401.57 | -10.250 | 7200 | 3950.41 | 4.617 | 37 | 104 | 104 | 44.70 | 5516 |
| 300   | 5789.674 | 4.088 | - | - | 7200 | 6036.42 | 5.532 | 45 | 103 | 103 | 33.57 | 6123 |
| 300   | 6641.535 | 4.118 | - | - | 7200 | 6926.79 | 5.327 | 43 | 109 | 109 | 32.30 | 4920 |
| 300   | 6001.242 | 4.527 | - | - | 7200 | 6285.83 | 5.092 | 39 | 104 | 104 | 36.22 | 5510 |
| 300   | 6754.097 | 3.647 | - | - | 7200 | 7009.76 | 3.342 | 42 | 110 | 110 | 33.78 | 4792 |
| 300   | 6657.507 | 4.434 | - | - | 7200 | 6966.41 | 5.121 | 39 | 102 | 102 | 33.61 | 5740 |
| 400   | 8773.665 | 3.187 | - | - | 7200 | 9062.52 | 7.726 | 43 | 104 | 104 | 28.92 | 5504 |
| 400   | 8336.122 | 4.224 | - | - | 7200 | 8703.79 | 11.255 | 42 | 104 | 104 | 30.11 | 5520 |
| 400   | 9237.014 | 3.412 | - | - | 7200 | 9563.27 | 8.909 | 40 | 101 | 101 | 28.70 | 5862 |
| 400   | 9806.238 | 4.129 | - | - | 7200 | 10228.60 | 8.175 | 53 | 98 | 98 | 25.64 | 6228 |
| 400   | 8144.318 | 3.324 | - | - | 7200 | 8424.37 | 5.840 | 39 | 109 | 109 | 31.75 | 4918 |
| Avg   |        | 3.878 |     |     |     |            |       |     |        |       | 5.498 |

vehicles. We will also develop an efficient heuristic that allows a station to be visited more than once.
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