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Reduced-order $H_{\infty}$ Filtering for Commensurate Fractional-order Systems*

Jun Shen, James Lam and Ping Li\textsuperscript{1}

Abstract—This paper is concerned with the reduced-order $H_{\infty}$ filtering problem of commensurate fractional-order systems. Our goal is to construct a reduced-order filter in such a way that the filtering error is within a prescribed $H_{\infty}$-norm error bound. Based on the bounded real lemma for commensurate fractional-order systems, a sufficient condition is established in terms of linear matrix inequalities (LMIs) under which the stability as well as the $H_{\infty}$ performance of the filtering error system can be guaranteed. Moreover, by introducing a free real matrix variable, the desired filtering matrices are decoupled with the complex matrix variable and further parameterized by the new matrix variable, which facilitates the filter synthesis. Then, an iterative LMI algorithm is proposed to compute the filtering matrices accordingly. Finally, a numerical example is presented to show the effectiveness of the proposed algorithms.

I. INTRODUCTION

Fractional-order control systems have attracted increasing attention in recent years due to the fact that fractional-order differential equations are more appropriate to describe memories and hereditary effects of various materials and physical systems which are often neglected in the classical integer-order models. As a result, fractional-order systems theory has already been applied in viscoelastic materials [1], electrical circuits and systems [2], fractional-order PID control [3] and robust control [4], [5]. A recent survey on the applications and implementations of fractional-order systems can be found in [6].

Over the past decade, considerable efforts have been devoted to stability analysis and stabilization for fractional-order systems. In [7], the stability condition of a linear time-invariant fractional-order system is firstly given based on the arguments of eigenvalues of the coefficient matrix. The LMI characterization for the stability of fractional-order systems with order $\alpha \in (1, 2)$ can be regarded as a corollary of the results on LMI regions [8]. In [9], necessary and sufficient LMI conditions for the stability of the case when $\alpha \in (0, 1)$ are derived. Moreover, in order to design stabilizing feedback controllers, the two Hermitian matrix variables are replaced by a Hermitian matrix variable and its conjugate. Then, it turns out that the feedback gain matrix is actually coupled with a real matrix variable and the classical parametrization techniques can be utilized. For fractional-order systems with parameter uncertainties, robust stability and stabilization are considered in [4], [5].

However, to the best of our knowledge, the observer and filtering problem for linear time-invariant fractional-order systems has not been well studied in the literature. Only a few results on the full-order Luenberger observer for fractional-order systems can be found in [10] and the references therein. Nevertheless, these results are no longer effective when model uncertainty and exogenous disturbance are taken into account. Although the $H_{\infty}$ filtering problem of classical integer-order systems has been widely investigated in the literature [11], [12], [13], its counterpart for fractional-order systems is still not well developed. Recently, some pioneering works on $H_{\infty}$ filtering of fractional-order systems have been presented in [14], yet the results are not valid in general.

In order to discuss the $H_{\infty}$ filtering problem, the LMI characterization for the $L_2$-gain of fractional-order systems is a prerequisite. Fortunately, the bounded real lemma for commensurate fractional-order systems was established in [15]. Although the result only provides a sufficient condition, it has also been indicated there that its applications generally lead to non-conservative results. Since the LMI characterization for $H_{\infty}$ performance of fractional-order systems involves complex matrix variables, the classical approaches such as elimination techniques [16] and cone complementarity linearization [17] are not applicable any more. Therefore, it is imperative to propose some new methods for the filter synthesis of fractional-order systems.

With the above motivation, in this paper, based on the bounded real lemma for commensurate fractional-order systems, we firstly propose a sufficient LMI condition to ensure the stability and the $H_{\infty}$ performance of the filtering error system. Then, by introducing a new free real matrix variable, some equivalent LMI conditions are derived, which naturally leads to an iterative convex optimization algorithm. It is worth mentioning that the techniques used in this paper also have their potential applications in the LMI synthesis of other control problems whenever complex matrix variables are encountered.

II. PRELIMINARIES

Throughout this paper, the Caputo’s definition is adopted for fractional derivatives. The Caputo fractional derivative of function $f(t)$ with order $\alpha$ and lower limit 0 is defined by [18]

$$\text{cD}_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \left( \frac{d}{d\tau} \right)^n f(\tau) d\tau$$

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where $\alpha$ represents the order of the derivative and $n - 1 \leq \alpha < n$. We employ a simple notation $D^\alpha f(t)$ instead, since only the Caputo fractional derivative is used in this paper.

In what follows we introduce some notations which will be used in the sequel. The symbol $\#$ denotes an entry that can be inferred by the notion of Hermitian matrices. For two Hermitian matrices $P, Q$, the notation $P > Q$ (respectively, $P \geq Q$) means that the matrix $P - Q$ is positive definite (respectively, positive semi-definite). For any square complex matrix, $\text{Her}(X) = X + X^*$ where $X^*$ denotes the conjugate transpose of $X$; $j$ denotes the imaginary unit; $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ represent the real and imaginary parts of a complex matrix, respectively; $\text{diag}(A_1, A_2, \ldots, A_n)$ denotes the block diagonal matrix with diagonal entries $A_1, A_2, \ldots, A_n$. Let $\theta = (1 - \alpha)\pi/2$ and $\varrho = e^{\theta}$ ($\varrho = e^{-\theta}$) since these two constants are frequently used in this paper. In the following, it is always assumed that the fractional-order linear systems are of commensurate order.

Some existing results on the stability of a linear time-invariant fractional-order system and its corresponding LMI characterization are presented in the following lemmas.

**Lemma 1:** [7] Let $A \in \mathbb{R}^{n \times n}$, the linear time-invariant system $D^\alpha x(t) = Ax(t)$ is asymptotically stable if and only if

$$\arg(\lambda_i(A)) > \pi \alpha/2$$

where $\lambda_i(A)$, $i = 1, 2, \ldots, n$, denote the eigenvalues of $A$ and $\arg(\cdot)$ denotes the argument of a complex number.

**Lemma 2:** [8] Let $A \in \mathbb{R}^{n \times n}$, the linear time-invariant system $D^\alpha x(t) = Ax(t)$ with $\alpha \in (1, 2)$ is asymptotically stable if and only if there exists a real symmetric matrix $P > 0$, such that

$$\begin{bmatrix} (AP + PA^T) \sin \phi & (AP - PA^T) \cos \phi \\ (AP + PA^T) \sin \phi & (AP + PA^T) \sin \phi \end{bmatrix} < 0$$

where $\phi = (1 - \alpha/2)\pi$.

By virtue of the relationship between complex and real matrix inequalities, we can restate Lemma 2 as follows.

**Lemma 3:** Let $A \in \mathbb{R}^{n \times n}$, the linear time-invariant system $D^\alpha x(t) = Ax(t)$ with $\alpha \in (1, 2)$ is asymptotically stable if and only if there exists a real symmetric matrix $P > 0$, such that $qPA^T + qAP < 0$.

As is well known, a commensurate fractional-order system admits a state-space like representation:

$$\begin{cases}
D^\alpha x(t) = Ax(t) + Bw(t), \\
y(t) = Cx(t) + Dw(t),
\end{cases}$$

where $0 < \alpha < 2$ denotes the fractional order of the system; $w(t)$ and $y(t)$ are the input and output of the system, respectively. The transfer function of system (1) can be written as $\mathcal{G}_{wy}(s) = C(s^{\alpha}I - A)^{-1}B + D$. The $L_2$-gain of system (1) can be defined through $H_\infty$-norm as $\|\mathcal{G}_{wy}\|_\infty = \sup_{\omega \in \mathbb{R}_+} \sigma(\mathcal{G}_{wy}(j\omega))$ where $\sigma$ is the maximum singular value. A sufficient LMI condition for the $L_2$-gain of system (1) is provided as follows.

**Lemma 4:** [15] The $L_2$-gain of fractional-order system (1) is bounded by $\gamma > 0$ if there exists a Hermitian matrix $P$, such that the following LMI holds:

$$\begin{bmatrix}
\text{Her}(qPA) & PB & \varrho C^T \\
\# & -\gamma I & D^T \\
\# & \# & -\gamma I
\end{bmatrix} < 0.$$

**Remark 1:** It should be noted that unlike the classical bounded real lemma for integer-order systems, Lemma 4 only provides a sufficient condition. However, as indicated in [15], the applications of the above LMI characterization often lead to non-conservative results. Another issue worth mentioning is that one cannot simply impose $P > 0$ to further characterize stability of fractional-order system (4). Therefore, in this paper, we only add one more LMI constraint to ensure stability of the system.

**III. MAIN RESULTS**

In this section, we will formulate the reduced-order filtering problem with $H_\infty$ performance for commensurate fractional-order systems. Consider the following stable fractional-order system:

$$\begin{cases}
D^\alpha x(t) = Ax(t) + Bw(t), \\
y(t) = Cx(t) + Dw(t), \\
z(t) = C_z x(t) + D_z w(t),
\end{cases}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, $w(t) \in \mathbb{R}^m$ is the exogenous input, $y(t) \in \mathbb{R}^p$ is the output that can be measured and $z(t) \in \mathbb{R}^{n_z}$ is the signal to be estimated. In order to estimate the signal $z(t)$ in system (2) using the output signal $y(t)$, we construct the following reduced-order filter:

$$\begin{cases}
D^\alpha \hat{x}(t) = \hat{A}\hat{x}(t) + \hat{B}y(t), \\
\hat{z}(t) = \hat{C}\hat{x}(t) + \hat{D}y(t),
\end{cases}$$

where $\hat{x}(t) \in \mathbb{R}^{n_f}$ ($1 \leq n_f \leq n$) is the internal state of the filter, $\hat{z}(t)$ is the estimator of $z(t)$, and $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$ are the filtering matrices to be determined.

Let $\hat{x}(t) = [x^T(t), \hat{x}^T(t)]^T$ and $e(t) = z(t) - \hat{z}(t)$, then the filtering error system can be written as

$$\begin{cases}
D^\alpha \hat{e}(t) = \hat{A}\hat{e}(t) + \hat{B}w(t), \\
e(t) = \hat{C}\hat{e}(t) + \hat{D}w(t),
\end{cases}$$

where

$$\hat{A} = \begin{bmatrix} A & 0 \\
B\hat{C} & \hat{A} \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\
\hat{B}D \end{bmatrix},
\hat{C} = \begin{bmatrix} C_z - \hat{D}\hat{C} \\
\hat{C} \end{bmatrix}, \quad \hat{D} = D_z - \hat{D}D.$$

For a given disturbance attenuation level $\gamma > 0$, we would like to design the filtering matrices $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$, such that the following two requirements are simultaneously satisfied:

1. The filtering error system (4) is asymptotically stable.
2. The closed-loop transfer function of system (4) from $w$ to $e$ given by $\mathcal{F}_{we}(s) \coloneqq C(s^{\alpha}I - \hat{A})^{-1}\hat{B} + \hat{D}$ satisfies $\|\mathcal{F}_{we}\|_\infty < \gamma$.

The following proposition gives an LMI characterization of the reduced-order $H_\infty$ filtering problem for fractional-order system (2), which directly follows from Lemmas 3 and 4.
Proposition 1. The filtering error system (4) with $\alpha \in (1,2)$ is asymptotically stable and the $H_\infty$ performance satisfies $\|T_{uv}\|_\infty < \gamma$ if there exist a real symmetric matrix $Q > 0$ and a Hermitian matrix $P$, such that

$$\text{Her}(\bar{q}A)Q < 0$$

and

$$\begin{bmatrix} \text{Her}(\bar{q}AP) & P\bar{C}^T & \bar{q}B \\ & -\gamma I & D \\ & & -\gamma I \end{bmatrix} < 0.$$  \hspace{1cm} (6)

In order to further facilitate the synthesis of the filter, we assemble all the matrices to be constructed in a matrix

$$G = \begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix},$$

then, the system matrices of the filtering error system can be written as

$$\hat{A} = \hat{A} + \hat{M} \hat{G} \hat{C}, \quad \hat{B} = \hat{B} + \hat{M} \hat{G} \hat{D},$$

$$\hat{C} = \hat{C} + \hat{N} \hat{G} \hat{C}, \quad \hat{D} = \hat{D} + \hat{N} \hat{G} \hat{D},$$

where

$$\hat{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} C_z \end{bmatrix}, \quad \hat{D} = \begin{bmatrix} 0 \\ D \end{bmatrix}.$$  

Denote $\hat{M} = \begin{bmatrix} 0 & 0 \\ \hat{A} & \hat{B} \end{bmatrix}$ and $\hat{N} = \begin{bmatrix} -\hat{C} & -\hat{D} \end{bmatrix}$. Denote $U = \hat{M} \hat{G}$ and $V = \hat{N} \hat{G}$, then to determine $G$ is equivalent to the construction of both $U$ and $V$ where

$$U \in \mathcal{U} \triangleq \left\{ U = \begin{bmatrix} 0_{n \times (n_f + p)} \\ \circ \end{bmatrix} U_{21} \in \mathbb{R}^{n_f \times (n_f + p)} \right\}.$$

Proposition 1: There exist real symmetric matrices $Q > 0, X > 0$ and a Hermitian matrix $P$, such that

$$\hat{\Theta} \triangleq \begin{bmatrix} \text{Her}(\bar{q}AP) & P\bar{C}^T & \bar{q}B \\ & -\gamma I & D \\ & & -\gamma I \end{bmatrix} < 0.$$  \hspace{1cm} (7)

Proof. First note the fact that for Hermitian matrix $\Theta < 0$ and any complex matrix $\Phi$, there exists a real symmetric matrix $X > 0$, such that $-X - \Phi \Theta^{-1} \Phi^* < 0$. By Schur complement lemma, it is obvious that $-X - \Phi \Theta^{-1} \Phi^* < 0$ and $\Theta < 0$ are equivalent to

$$\begin{bmatrix} \Theta & \Phi \\ \# & -X \end{bmatrix} < 0.$$  

Therefore, $\Theta < 0$ is equivalent to $\hat{\Theta} < 0$. Similarly, let

$$\Psi = \begin{bmatrix} 0 \\ \circ \end{bmatrix} q\Psi \bar{C}^T$$

and note that there always exists a common real symmetric matrix $X > 0$, such that $-X - \Phi \Theta^{-1} \Phi^* < 0$ and $-X - \Psi \bar{C}^T \Psi^* < 0$ simultaneously hold. By performing a congruent transformation $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1} $

$$\begin{bmatrix} X & X \\ \# & -X \end{bmatrix} T_2 = \hat{\Xi}.$$  

From the above discussion, it can be seen that $\hat{\Theta} < 0$ and $\hat{\Xi} < 0$ if and only if there exists a real symmetric matrix $X > 0$, such that $\hat{\Theta} < 0$ and $\hat{\Xi} < 0$. This completes the proof.

Theorem 1. Given system matrices $A, B, C, D, C_z, D_z, G$, fractional order $\alpha \in (1,2)$ and $\gamma > 0$, the following two statements are equivalent:

(1) there exist real symmetric matrices $Q > 0$ and a Hermitian matrix $P$, such that

$$\Theta \triangleq \text{Her}(\bar{q}A)Q < 0$$

(2) there exist real symmetric matrices $Q > 0, X > 0$ and a Hermitian matrix $P$, such that

$$\hat{\Theta} \triangleq \begin{bmatrix} \text{Her}(\bar{q}AP) & P\bar{C}^T & \bar{q}B \\ \# & -\gamma I & D \\ \# & \# & -\gamma I \end{bmatrix} < 0.$$  \hspace{1cm} (7)
and

\[
\hat{\Delta} = \begin{bmatrix}
\hat{\xi}_{11} & \hat{\xi}_{12} & \hat{\xi}_{13} \\
\hat{\xi}_{22} & \hat{\xi}_{22} & -\gamma I
\end{bmatrix} < 0, \quad (8)
\]

where

\[
\hat{\Theta}_{11} = \text{Her}(\hat{q}A\hat{Q}) - RK^T - RK^T + RX^T,
\hat{\Theta}_{12} = \text{Her}(\hat{q}A\hat{P}) - RK^T - KR^T + RX^T,
\hat{\Theta}_{13} = PC^T + \hat{q}(\hat{R}L^T - KS^T + RXS^T),
\hat{\Theta}_{22} = -\gamma I - SL^T - LS^T + SXS^T.
\]

**Proof.** (2) ⇒ (1) Note that \((K-RX)X^{-1}(K-RX)^T \geq 0\) and thus \(-KX^{-1}K^T \leq -RK^T - KR^T + RX^T\), hence \(\hat{\Theta} < 0\) implies that

\[
\text{Her}(\hat{q}A\hat{Q}) - KX^{-1}K^T \leq \text{Her}(K-RX) X^{-1} (K-RX)^T.
\]

Then select \(U = KX^{-1}\), we have \(\hat{\Theta} < 0\). Moreover, \(K \in \mathcal{K}\) implies that \(U \in \mathcal{U}\). Similarly, since

\[
\begin{bmatrix}
K - RX \\
qu(L-SX)
\end{bmatrix} X^{-1} \begin{bmatrix}
K - RX \\
qu(L-SX)
\end{bmatrix}^T \geq 0,
\]

selecting \(V = LX^{-1}\) and noting that \(U\) is already chosen as \(U = KX^{-1}\), then one can see that \(\hat{\Xi} < 0\) implies that \(\hat{\Xi} < 0\).

(1) ⇒ (2) Choose \(R = U, S = V, K = UX, L = VX\) and note that \(U \in \mathcal{U}\) implies that \(K \in \mathcal{K}\), which completes the proof. □

Summarizing Proposition 1, Theorem 1 and Theorem 2, we can obtain the following result.

**Theorem 3:** Given \(\gamma > 0\) and fractional order \(\alpha \in (1,2]\), the filtering error system (4) is asymptotically stable and the \(H_\infty\) performance satisfies \(\|\mathcal{F}_{we}\| < \gamma\) if there exist real symmetric matrices \(Q > 0, X > 0\), a Hermitian matrix \(P\) and real matrices \(W_1,W_2,W_3,V\), such that the matrix inequalities (7) and (8) hold.

According to Theorem 3, we can propose the following iterative convex optimization algorithm.

**Algorithm 1:** (Iterative Convex Optimization)

1) Set \(j = 1\). For a given attenuation level \(\gamma > 0\), solve the following LMI problems for matrix variables in \(\mathcal{Y}_1 \triangleq \{\text{real symmetric matrix } Q > 0, \text{ Hermitian matrix } P, \text{ and real matrices } W_1,W_2,W_3,V\}:

\[
\text{Her}(qQA) + \text{Her}(qW_1\tilde{C}) < 0,
\]

\[
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} \\
\Omega_{22} & \Omega_{22} & -\gamma I
\end{bmatrix} < 0,
\]

where \(\Omega_{11} = \text{Her}(qPA) + \text{Her}(q(W_2 + jW_3)\tilde{C})\) and \(\Omega_{12} = \text{Her}(qV^T)\). Then, the initial value \(R_1\) and \(S_1\) can be given by \(R_1 = Q^{-1}W_1\) and \(S_1 = V\).

2) For fixed \(R = R_j\) and \(S = S_j\), minimize \(\mu\) for matrix variables in \(\mathcal{Y}_2 \triangleq \{\text{real symmetric matrices } Q > 0, X > 0, \text{ Hermitian matrix } P, \text{ and real matrices } K \in \mathcal{K}, L\} \text{ s.t.}

\[
\begin{bmatrix}
\Theta - \text{diag}(\mu I,0) < 0, \\
\hat{\Xi} - \text{diag}(\mu I,\mu I,0) < 0,
\end{bmatrix}
\]

where \(\hat{\Theta}\) and \(\hat{\Xi}\) are defined in (7) and (8), respectively. Denote the minimized \(\mu\) as \(\mu_j\).

3) If \(\mu_j < 0\), then the desired \(U\) and \(V\) can be attained as \(U = KX_j^{-1}\) and \(V = L_jX_j^{-1}\). STOP.

4) Fix \(\mu = \mu_j\), minimize trace(\(X\)) s.t. LMI (9) for matrix variables in \(\mathcal{Y}\). Denote the obtained \(X,K\) and \(L\) as \(X_j,K_j\) and \(L_j\), respectively.

5) If \(|\mu_j - \mu_{j-1}|/\mu_{j-1} < \eta\), where \(\eta\) is a prescribed tolerance, then this algorithm fails to find a desired solution. STOP. Otherwise, update \(R_{j+1}\) and \(S_{j+1}\) as \(R_{j+1} = K_jX_j^{-1}\) and \(S_{j+1} = L_jX_j^{-1}\), respectively. Set \(j := j + 1\) and go to Step 2.

**Remark 2:** As in other iterative algorithms, one important issue is to choose a good initial value which is close to a local optimum. In this connection, one may employ an algorithm similar to Algorithm 2 in [20] to further optimize the initial value.

**IV. NUMERICAL EXAMPLE**

Consider fractional-order system (2) with system matrices

\[
A = \begin{bmatrix}
-2 & 0.9 & -1 \\
-0.2 & -0.8 & -0.5 \\
-0.4 & 0.4 & -3
\end{bmatrix}, \quad B = \begin{bmatrix}
-1 & 0 \\
0.3 & 0 \\
0.2 & 1
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
0.5 & 0.1 \\
1 & 0 & 1
\end{bmatrix}, \quad D = \begin{bmatrix}
-0.4 & 0.1 \\
0.2 & 0.3
\end{bmatrix},
\]

and fractional order \(\alpha = 1.3\). One can readily verify that the system is stable. Our purpose is to construct a reduced-order filter (3) to estimate the signal \(z(t)\). For \(\gamma = 0.1\) and \(n_f = 1\), by implementing Algorithm 1 with initial value optimization, the desired filtering matrices are given by

\[
\begin{bmatrix}
\hat{A} & \hat{B} \\
\hat{C} & \hat{D}
\end{bmatrix} = \begin{bmatrix}
-1.6921 & -0.0917 & 0.0546 \\
10.1819 & -0.2550 & 0.6175
\end{bmatrix}.
\]

Given an \(L_2\)-input \(u_1(t) = [1_4 \cos(\pi t), e^{-0.02t}]^T\) and zero initial condition, the trajectories of the signal \(z(t)\) and its estimation \(\hat{z}(t)\) are depicted in Fig. 1, respectively.

**V. CONCLUSION**

In this paper, the reduced-order \(H_\infty\) filtering problem of commensurate fractional-order systems is addressed. Based on the bounded real lemma for commensurate fractional-order systems, a sufficient condition to ensure the stability and the \(H_\infty\) performance of the filtering error system is firstly established. Furthermore, by introducing a new flexible real matrix variable, the desired filtering matrices are decoupled from the complex matrix variable and further parameterized by a new matrix variable. Then, an iterative convex optimization algorithm is proposed to solve the condition. It is worth noting that the techniques proposed in this paper...
Fig. 1. Trajectories of $z(t)$ and its estimation $\hat{z}(t)$ when $\alpha = 1.3$.

may also have its potential applications in other synthesis problems described by bilinear matrix inequalities involving complex matrix variables.

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