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Efficient Low-Frequency Integral Equation Solver for Wireless Power Transfer Modeling

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Abstract — In this paper, the wireless power transfer system based on magnetic-coupled resonators is modeled and optimized by using low-frequency integral equation solver. For the low-frequency transfer system, the mesh size after discretization is usually much smaller than the wavelength. Hence, the low-frequency solvers are proposed to model this kind of structures with tiny meshes. After the spiral resonators are determined at specific frequency, we only need to optimize the distance between resonators and two loops. The numerical results show that we are no need to re-mesh the whole transfer system during the distance searching procedure, and the optimized distance can be easily obtained.

Index Terms — Low-frequency solvers, integral equation, wireless power transfer, magnetic-coupled resonators.

I. INTRODUCTION

Surface integral equation (SIE) based methods have been widely used to solve the circuit and scattering problems, because the unknowns are only along the surface of the problem. In comparison with other differential equation based methods, it is much faster in simulation time and more suitable for large scale problems [1], [2].

It is well known that the traditional electric field integral equation (EFIE) solver suffers from the low-frequency breakdown problem due to unbalance the vector potential part and the scalar potential part. At low frequency, the current can be decomposed as a divergence-free part and a curl-free part, which are equally important in capturing the inductive and capacitive physics. The popular remedy of low-frequency breakdown problem is to separate the vector and the scalar potential contributions by using loop-tree/-star basis functions [3]-[5]. Alternatively, by introducing the charges as additional unknowns, the augmented electric field integral equation (A-EFIE) was proposed to balance the two potentials down to DC [6], [7]. Recently, based on the Calderón identities, the multiplicative preconditioner was presented to convert the system matrix into a second kind integral equation system [8]. Very recently, this Calderón based preconditioner is further applied on A-EFIE system, where the spectrum of the low-frequency system can be largely improved [9].

On the other hand, the design of wireless power transfer (WPT) system [10], [11] has attracted a lot of attentions. As detailed in [12], this kind of links based magnetic-coupled resonators can actually be considered as a matching network, which has been well developed from the filter design point of view. However, the layout optimization is still a time-consuming and challenging task in full-wave simulations, including the completely re-meshing time for each dimension. Because of the tiny discretization along the narrow spiral metallic strips, the edge length would be much smaller than the wavelength, and the number of unknowns will also be very large when multiple resonators are involved as shown in Fig. 1. Therefore, an efficient and fast integral equation based low-frequency solvers is extremely important for the design of wireless power transfer system.

In this paper, the A-EFIE based low-frequency solvers are employed to simulate the wireless power transfer system using magnetic-coupled resonators. The numerical examples show that the solver converges fast over all the concern frequencies, and the optimal system with maximum transfer efficiency can be obtained efficiently without re-meshing the whole system.

II. DESIGN AND MODELING OF WIRELESS POWER TRANSFER SYSTEM

Fig. 2 shows the configuration of wireless power transfer system based on two magnetic-coupled resonators. It consists of two spiral resonators, transmit and receiving loops. The width of the spiral strip is 2 mm, and the spacing is 0.2 mm. The outer radius is 34 mm, which is equivalent to 22.75 turns. In order to achieve maximum transfer efficiency, the distance

Fig. 1. Configuration of wireless power transfer link with multiple magnetic-coupled resonators and two feeding loops.
between loops and resonators, and the distance between resonators have to be optimized. Here, the transfer efficiency ($\eta$) is defined in terms of $S$-parameter as

$$\eta = \left| S_{21} \right|^2 \tag{1}$$

where $|S_{21}|$ is the amplitude of the transfer coefficient of the system. Therefore, maximizing $|S_{21}|$ is equivalently achieving the best of the transfer efficiency. Fig. 3 shows the simulated $S$-magnitudes of the wireless power transfer links with different distance between spiral resonators. As the distance increases from 3, 18 to 28 mm, the two resonant peaks are closed to each other and merged together. Since only the distances are needed to be optimized during the design, once the loops and resonators are determined based on the filter theory in [12], only 564s are needed for 100 frequency points when the number of unknowns is 2000.

III. NUMERICAL STABILITY

For the magnetic-coupled resonator based wireless power transfer system, the minimum mesh size is usually very smaller (<0.001$\lambda$). It implies the EFIE-based full-wave solver will break down at these low frequencies. In other words, the system matrix becomes ill-conditioned because of the small eigenvalues of the dense discretization. Hence, the low-frequency solvers have to be employed, such as A-EFIE [7], CMP-EFIE [8]. Fig. 4 shows the convergent histories of the simulation by using the EFIE and A-EFIE. It can be noticed that the simulation using the EFIE cannot be converged, while the A-EFIE method is sufficiently fast in convergence over all the three selected frequencies.

As shown in Fig. 5, the convergent histories of the simulation with different mesh densities are plotted. It can be noticed that the convergence at 60 MHz is slightly slower than those at 30 MHz and 120 MHz. This is because the 60 MHz is very close to the second resonance of the system. As the number of unknowns increases from 2000 to 8000, the minimum mesh size becomes much smaller and the convergence becomes slower due to the increased condition number. This situation is more and more obvious when the frequency is close to the resonant frequency. However, the system can still get converged at both resonant and off-resonant frequencies.

IV. CONCLUSION

We have successfully applied the low-frequency integral equation solver on the modeling of wireless power transfer system. The presented numerical solution has a good capability in modeling and optimizing the transfer efficiency, especially for multiple magnetic-coupled resonators with large number of unknowns. To improve the spectrum of the system matrix around resonance, the recent proposed Calderón based preconditioner can be further employed [8], [9].
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Fig. 5. The convergent history at different frequencies. (a) 30 MHz; (b) 60 MHz; (c) 80 MHz.