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<th>Study of late time stability for marching on-in-time solution of TDIE</th>
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Abstract—In this paper, the late time stability is studied for marching on-in-time scheme in the time domain integral equation. First, the numerical evaluation of the convolution between retarded potential and time basis function is employed to ensure the accuracy. Then, we solve the first few time steps’ current coefficients as eigenvectors to derive the remaining coefficients. By projecting the system onto a modal order reduced system, the late time stability can be still ensured. Numerical results show that the accuracy and stability of the solutions obtained with the proposed methods in MOT-TD-CFIE can be guaranteed.

I. INTRODUCTION

For the time domain integral equation (TDIE), the unknown current density is often solved by using the marching on-in-time (MOT) scheme. The computational efficiency, accuracy and late time stability are the three major issues of the TDIE system [1]-[4]. Many research works have been carried out in remedying the late time stability. Perhaps the most promising scheme for solving instability is to obtain a more accurate impedance matrix elements [5], [6]. By using the closed-form expressions in the MOT solution, the accuracy of calculation for the MOT matrix elements can be largely improved. However, it is heavily dependent on the choice of the temporal basis functions (only for piece-wise polynomials). If the order of the time basis function is higher or it is not the piece-wise polynomial, the analytical result will become complicated or even cannot be obtained. Hence, the smoothing integrand based on variable transformation can be employed to guarantee the smoothness of the integral kernel, which results a much fast convergence [7], [8]. In addition, the integration area can also be reduced for different situations to avoid the unnecessary calculation.

In this work, the numerical evaluation of the convolution between retarded potential and time basis function is first employed to ensure the accuracy of impedance matrix [9]. Then, the eigenvectors associated with stable modes are first obtained by solving the first few time steps in the MOT scheme. By removing the unstable modes, the original matrix system can be projected onto a smaller system with only stable modes. As a result, the computational efficiency can be improved without influencing the late time stability. The similar idea was also successfully implemented in explicit time-domain finite-element method [10]. This is the first implementation based on the MOT-TDIE formulation.

II. BACKGROUND

For a perfect electric conductor (PEC) surface, the TD-EFIE with time derivation can be written as

\[ \mathbf{n} \times \partial_t E^{inc}(r, t) = \mathbf{n} \times \left[ \frac{\partial^2}{\partial t^2} \mathbf{A}(r, t) + \nabla \partial_t \Psi(r, t) \right] \] (1)

where \( E^{inc}(r, t) \) is the incident electric field, \( \mathbf{A}(r, t) \) and \( \Psi(r, t) \) denote the vector and scalar potentials, respectively. Using the point match at \( t = t_k = k \Delta t \) in time domain and testing the equation (1) with Galerkin methods in spatial domain, the TDIEs can be written as,

\[ \mathbf{M}_o^k \cdot \mathbf{I}^k = \mathbf{V}^{inc,c} - \sum_{l=1}^{k-1} \mathbf{M}_l^k \cdot \mathbf{I}^{k-l} \] (2)

where

\[ I^k(m) = f^k_m \] (3)

\[ V^{inc,e}(m) = 4\pi \int_{S_m} f_m(r) \cdot \partial_t E^{inc}(r, t_k) dS \] (4)

\[ M_l^k(m, n) = \mu \int_{S_m} f_m(r) \cdot \left[ E_n(r, t) \ast \frac{\partial^2 T(t)}{\partial t^2} \right]_{t=\Delta t} dS \]

\[ -\frac{1}{\varepsilon} \int_{S_m} \nabla \cdot f_m(r) \left[ \Phi_n(r, t) \ast T(t) \right]_{t=\Delta t} dS \] (5)

Meanwhile, the magnetic field integral equations (MFIE) can also be derived using the similar formulation. Here, we adopts the combined field integral equations (CFIE) with the weighting coefficient of 0.2. Here, we employ the numerical evaluation of the convolution between retarded potential and time basis function to improve the accuracy of impedance matrix in (5). However, we need to solve the similar equation of \( \mathbf{M}_o \mathbf{x} = \mathbf{b} \) in every time step. As the dimension \( N \) of matrix \( \mathbf{M}_o \) increases, more iterations have to be solved, which downgrading the computational efficiency dramatically.

First, we solve the first \( M \) step currents in the enhanced MOT method to determine the stable mode as normalized eigenvector \( E \). Then, for the other \( k-M \) time steps, we can project the original system onto a reduced system as

\[ \mathbf{E}^T \cdot \mathbf{M}_o^e \cdot \mathbf{E} = \mathbf{E}^T \cdot \mathbf{V}^{inc,e} - \mathbf{E}^T \cdot \sum_{l=1}^{k-1} \mathbf{M}_l^e \cdot \mathbf{E}^{k-l} \] (6)

By defining \( \mathbf{A} = \mathbf{E}^T \mathbf{M}_o^e \mathbf{E} \), we have...
Again, we need to solve $A_0x = b$ in every time step. Fortunately, the dimension of projected matrix $A_0$ becomes only $M$.

### III. RESULTS

In this example, a unit PEC sphere is discretized into 280 triangle patches with 420 unknowns. Fig. 1 shows excellent agreement between the currents obtained by the MOT-TD-CFIE solver with the numerical evaluation and reduced system. Note that the first few modes are enough in describing the system at remaining time steps. Fig. 2 compares the RCS at two frequencies 30 MHz and 300 MHz. As expected, they agree well demonstrating the accuracy of the proposed methods.

### IV. CONCLUSION

By keeping only the first several modes in the MOT-TD-CFIE solver, we have found that the proposed method can effectively ensure the late time stability and reduce the computational complexity. For the given example, the first 20 modes are enough to describe the system without influencing the accuracy of the solutions.

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### REFERENCES


