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Accelerated A-EFIE with Perturbation Method
Using Fast Fourier Transform

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Abstract—In this paper, we apply the fast Fourier transform on the perturbation-based augmented electric field integral equation (A-EFIE). The main idea lies on the Lagrangian interpolation of a series $R^n$, $n = -1, 0, 1, 2, \ldots$, obtained by expanding Green's function using Taylor series. By utilizing the Toeplitz property of the $R^n$ on the uniform cartesian grids, the multiplication of expanded kernels in the vector and scalar potentials can be accelerated effectively. The oscillation of these expanded kernels has less variation compared to the original kernel with $e^{ikR}/R$. In addition, we do not need to do any near field amendment when $n \geq 0$. Numerical example validates the feasibility and validity of this method.

I. INTRODUCTION

It is well-known that there is a low-frequency breakdown problem in electric field integral equation (EFIE) when Rao-Wilton-Glisson (RWG) basis functions are employed in moments method. In the past years, different solutions to this low-frequency problem have been proposed, such as Helmholtz decomposition of basis functions [1], [2] and Calderón preconditioner [3]. Among them, the augmented electric field integral equation (A-EFIE) becomes one of the most effective methods to remedy the low-frequency breakdown [4], [5], which is the only method independent of the basis functions. Later on, a perturbation method was introduced to enhance the accuracy of A-EFIE [6]. However, the perturbation method generates more matrix vector products at different frequency orders, which have to be accelerated before solving the real-world problems.

As the number of unknowns increases, a fast algorithm has to be incorporated into the iterative solver. The most popular multilevel implementation of fast multipole algorithm (MLPMA) [7], the mixed-form FMA [8], LF-FMA algorithm [9] and accelerated cartesian expansion (ACE) instead of $e^{ikR}/R$. In this work, we expand these series using Lagrangian interpolation. The oscillation of the kernels in this method has less variation compared to the original kernel with $e^{ikR}/R$. It is important to notice that near field amendment is not needed when $n \geq 0$.

II. BACKGROUND

For a perfect electric conductor (PEC) surface $S'$, the perturbation based A-EFIE in its mixed potential form can be written as

$$
\begin{align*}
\vec{V} &= \vec{V}^{(0)} + \delta \vec{V}^{(1)} + \delta^2 \vec{V}^{(2)} + \delta^3 \vec{V}^{(3)} + \ldots + \delta^n \vec{V}^{(n)} + \ldots (3)
\end{align*}
$$

$$
\begin{align*}
\vec{P} &= \vec{P}^{(0)} + \delta \vec{P}^{(1)} + \delta^2 \vec{P}^{(2)} + \delta^3 \vec{P}^{(3)} + \ldots + \delta^n \vec{P}^{(n)} + \ldots (4)
\end{align*}
$$

where

$$
\begin{align*}
\vec{V}^{(N)}_{mn} &= \mu_r \int_{S_m} \int_{S_n} \vec{A}_m(r) \cdot \vec{g}^{[N]}(r, r') \vec{A}_n(r') dS' dS \\
\vec{P}^{(N)}_{mn} &= \frac{1}{\varepsilon_r} \int_{S_m} \int_{S_n} \vec{h}_m(r) \cdot \vec{g}^{[N]}(r, r') \vec{h}_n(r') dS' dS (5)
\end{align*}
$$

For the perturbation method, we can expand Green’s function at different orders into Cartesian Lagrange polynomial as,

$$
\begin{align*}
g^{[N]}(r, r') &= \sum_{n=0}^{N_g-1} \sum_{m=0}^{N_g-1} \beta^p_{r_m} \beta^p_{r_n} (7)
\end{align*}
$$

where $p$ is the order of Lagrange polynomial interpolation, $N_g$ is the number of grid points [12], and

$$
\begin{align*}
g^{[N]}(r, r') &= \frac{1}{4\pi} \frac{R^{n-1}}{n!} (8)
\end{align*}
$$

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TABLE I

<table>
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<tr>
<th>methods</th>
<th>solving matrix equation times(s)</th>
<th>total memory (MB)</th>
<th>iteration numbers</th>
</tr>
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<tbody>
<tr>
<td>AEFIE</td>
<td>—</td>
<td>29618.90</td>
<td>262</td>
</tr>
<tr>
<td>AEFIE+FFT</td>
<td>3511.61</td>
<td>241.27</td>
<td>262</td>
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IV. CONCLUSION

The FFT technique has been adopted to accelerate the expanded integral equation kernels in the A-EFIE with perturbation method. It makes the perturbation method possible to efficiently solve the complicated real world problems with over one million unknowns without using a parallel computer. The effectiveness and accuracy of the proposed method has been verified by the given numerical example.

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REFERENCES