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Analysis of Scattering by PEC Objects in Layered Medium with Calderón Preconditioner

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Abstract—Electromagnetic scattering by perfect electrically conducting (PEC) objects in a layered medium is studied in this paper. The layered medium Green’s function is adopted as the kernel of the electric field integral equation (EFIE) so that the effects from the multilayered background can be accounted for automatically. However, the spectrum of the EFIE with this kernel, is unfortunately undesirable. This leads to slow convergence of the iterative solution. To improve the convergence, the Calderón identities are derived and leveraged to precondition the EFIE. By utilizing Buffa-Christiansen (BC) basis function in discretizing the preconditioning operator, the preconditioner can be made completely multiplicative. Different numerical examples are designed to show the performance of the preconditioner. It is shown that the proposed preconditioner makes the EFIE system with layered kernel converge rapidly, independent of the discretization density.

I. INTRODUCTION

The electric field integral equation (EFIE) with layered medium Green’s function plays an important role in the analysis of electromagnetic radiation and scattering in layered medium [1]. Although the EFIE is more robust and accurate than the MFIE (magnetic field integral equation), the spectrum of the EFIE operator is undesirable since it contains two branches clustering at zero and infinity. This leads to an ill-conditioned matrix and decreases the iterative convergence. Preconditioners are therefore indispensable to improve the condition number.

Recently, a Calderón multiplicative preconditioner was developed in free space applications [2]. This preconditioner is constructed by leveraging the self-regularizing property of the EFIE operator, which is indicated by the Calderón identities [3]. The identities reveal that the square of the EFIE operator equals to $-1/4$ perturbed by a compact MFIE operator. Therefore, the EFIE operator is an excellent preconditioner to itself. In [2], the Buffa-Christiansen (BC) basis function [4], a subset of the Chen-Wilton (CW) dual basis function [5], is adopted to discretize the preconditioning EFIE operator. For the original EFIE operator, the traditional Rao-Wilton-Glisson (RWG) basis function [6] is used. Hence, the preconditioner is purely multiplicative and compatible with the existing EFIE code.

However, the work so far only considers the radiation or scattering in free space. In this paper, we will study the extension of the Calderón preconditioner for a layered medium kernel, where the Calderón identities need to be re-investigated.

II. FORMULATION

A. EFIE in Layered Medium

Consider a PEC object located in the background of a layered medium. It is illuminated by an incident field $E'$. The induced surface current $J$ can be determined via the EFIE as:

$$0 = \hat{n} \times \mathcal{L}_E(J) + \hat{n} \times E'. \quad (1)$$

The integration operator $\mathcal{L}_E$ maps the electric current $J$ at source position $r'$ to the electric field $E$ at observation point $r$ via the Green’s function [7]:

$$\mathcal{L}_E(J) = i \omega \int dr' \mathcal{G}_e(r, r') \mu(r') \cdot J(r') \quad (2)$$

where $\mathcal{G}_e(r, r')$ is the $e$-type dyadic Green’s function in layered medium, which can further be expressed as:

$$\mathcal{G}_e(r, r') = \left( \nabla \times \hat{z} \right) \left( \nabla' \times \hat{z} \right) g_{\text{TE}}(r, r') + \frac{1}{k_0^2} \left( \nabla \times \nabla \times \hat{z} \right) \left( \nabla' \times \nabla' \times \hat{z} \right) g_{\text{TM}}(r, r'). \quad (3)$$

A direct Galerkin discretization to the EFIE in (1) usually leads to an ill-conditioned system. In the next section, a Calderón preconditioner will be constructed to improve the EFIE in layered medium.

B. Calderón Preconditioner

The Calderón identities in an inhomogeneous medium can be derived as [8]:

$$\left( \hat{n} \times \mathcal{L}_H \right) \left( \hat{n} \times \mathcal{L}_E \right) = \left( \hat{n} \times \mathcal{K}_H \right)^2 - \frac{1}{4} \quad (4)$$

$$\left( \hat{n} \times \mathcal{K}_H \right) \left( \hat{n} \times \mathcal{L}_H \right) - \left( \hat{n} \times \mathcal{L}_H \right) \left( \hat{n} \times \mathcal{K}_E \right) = 0 \quad (5)$$

$$- \left( \hat{n} \times \mathcal{L}_E \right) \left( \hat{n} \times \mathcal{K}_H \right) + \left( \hat{n} \times \mathcal{K}_E \right) \left( \hat{n} \times \mathcal{L}_E \right) = 0 \quad (6)$$

$$\left( \hat{n} \times \mathcal{L}_E \right) \left( \hat{n} \times \mathcal{L}_H \right) = \left( \hat{n} \times \mathcal{K}_E \right)^2 - \frac{1}{4} \quad (7)$$

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It is shown in (4) that a “dual-squared” system of 
\((\hat{n} \times \mathcal{L}_H)(\hat{n} \times \mathcal{L}_E)\) leads to a system with \(-1=4\) perturbed by 
a compact operator. Such a system is well-behaved. Therefore, 
\(\hat{n} \times \mathcal{L}_H\) can be utilized to precondition the EFIE:

\[
\hat{n} \times \mathcal{L}_H [\hat{n} \times \mathcal{L}_E (J)] = -\hat{n} \times \mathcal{L}_H (\hat{n} \times \mathcal{E}_i)
\]  

(8)

Similarly as in free space [2], the inner \(\hat{n} \times \mathcal{L}_E\) is discretized by 
using the RWG basis functions \(\mathbf{f}_{\text{RWG}}\) and the outer \(\hat{n} \times \mathcal{L}_H\) is 
discretized by the BC basis functions \(\mathbf{f}_{\text{BC}}\). After discretization, 
the matrix system is

\[
[(\hat{n} \times \mathcal{L}_H) (\hat{n} \times \mathcal{L}_E)]_{\text{dis}} = \mathbf{Z}_{\text{BC}}^H \mathbf{G}_m^{-1} \mathbf{Z}_{\text{RWG}}^E
\]  

(9)

where the Gram matrix is

\[
[\mathbf{G}_m]_{ji} = \langle n \times \mathbf{f}_{\text{RWG}j}, \mathbf{f}_{\text{BC}i} \rangle
\]  

(10)

and the impedance matrices are

\[
[\mathbf{Z}_{\text{RWG}}^E]_{ji} = \langle n \times \mathbf{f}_{\text{RWG}j}, \mathbf{L}_E, \mathbf{f}_{\text{RWG}i} \rangle
\]  

(11)

\[
[\mathbf{Z}_{\text{BC}}^H]_{ji} = \langle n \times \mathbf{f}_{\text{BC}j}, \mathbf{L}_H, \mathbf{f}_{\text{BC}i} \rangle.
\]  

(12)

III. NUMERICAL RESULTS

The scattering of a PEC sphere with \(r = 1\) m situated at the 
top layer of a 3-layer medium is analyzed. The configuration 
as well as the material parameters of the layered medium are 
shown in Fig. 1. The sphere is illuminated by a \(y\)-polarized 
plane wave of \(f = 150\) MHz with normal incidence. Fig. 2 
shows the number of iterations required in GMRES to achieve 
a targeted relative residual error of \(10^{-6}\), with respect to the 
discretization density (wavelength \(\lambda \) / mesh size \(\delta\)). It is shown 
that the number of iterations in EFIE increases rapidly as the 
mesh becomes denser. However, the Calderón preconditioned 
EFIE has very low and stable numbers of iterations.

**Fig. 1.** A PEC sphere located above a 3-layer medium is excited by a plane wave of \(f = 150\) MHz, where \(r = 1\), \(h_2 = 1\) (unit: m). The material 
parameters of each layer are shown in the figure.

**Fig. 2.** Number of iterations required in GMRES to achieve a relative residual 
error of \(10^{-6}\) versus discretization density (\(\lambda / \delta\)).

IV. CONCLUSION

Analysis of EM scattering by PEC objects in layered 
medium is conducted in this paper. The Calderón identities 
for inhomogeneous medium are derived, which is leveraged to 
precondition the EFIE with layered kernel. Numerical results 
show that excellent convergence can be achieved by using this 
novel preconditioner. It is also shown that the convergence of 
the iteration is independent of the discretization density.

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