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Calderón Preconditioned PMCHWT Equation for Layered Medium Problems

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Abstract—Electromagnetic scattering by dielectric objects in the presence of a layered medium is investigated by applying the Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) equation combined with the layered medium Green’s function. Due to the electric field integral equation (EFIE) operator involved, the spectrum of the PMCHWT equation is also undesired. When the surface is densely discretized, the condition of the resulting matrix is extremely bad. An effective Calderón preconditioner is developed in this paper to improve the convergence. Different from its free space counterpart, the Calderón identities for inhomogeneous medium need to be re-derived. It is shown from numerical examples that the convergence of the PMCHWT system in layered medium can be significantly improved by using the proposed Calderón preconditioner.

I. INTRODUCTION

Electromagnetic scattering by dielectric objects located in layered medium is of great importance in various applications. At low frequency or microwave regime, it is commonly encountered in sub-surface detection and geophysical exploration. While at the infrared or visible region, it is also a fundamental problem in the design of photovoltaic devices, where metallic nano-particles (usually embedded in layered structure) become partially penetrable. To analyze these problems, surface integral equation method [1] is preferred since it has relatively small number of unknowns. However, the popular Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) equation, though more accurate than other formulations, suffers from poor spectrum due to the electric field integral equation (EFIE) operator involved.

Recently, a highly effective preconditioner based on Calderón identities [2] was developed firstly in EFIE [3] and then extended in PMCHWT formulation [4], [5]. However, most existing work only considers the radiation or scattering in free space. It is clear that the commonly used Calderón identities are derived based on the free space Green’s function. In layered media, such identities need to be re-investigated. Currently, we have reported a work on the Calderón preconditioner for the EFIE analysis for PEC scattering problems in a layered medium [6]. In this paper, we will further extend our previous work to the PMCHWT analysis for scattering by dielectric objects in the same inhomogeneous medium.

II. FORMULATION

A. PMCHWT Equation in Layered Medium

Consider that a dielectric object is located in the background of a layered medium. It is illuminated by an incident field \( \mathbf{E}^o \). The induced electric current \( \mathbf{J} \) and magnetic current \( \mathbf{M} \) can be determined via the PMCHWT equation:

\[
\begin{bmatrix}
\hat{n} \times (L_E^o + L_E^H) \\
\hat{n} \times (K_H^o + K_H^E)
\end{bmatrix}
\begin{bmatrix}
\hat{n} \times (K_E^o + K_E^H) \\
\hat{n} \times (L_H^o + L_H^E)
\end{bmatrix}
= \begin{bmatrix}
-\hat{n} \times \mathbf{E}^o \\
-\hat{n} \times \mathbf{H}^o
\end{bmatrix}
\]

(1)

where superscript \( o \) means outside (the scatterer) region and \( i \) means inside region. The detailed information of the linear integral operators in layered medium can be found in [7] and will not be repeated here. The equation in (1) is shown to be imbalanced; hence, the normalization in [4] is adopted:

\[
\mathcal{P} \begin{bmatrix}
\eta_m \mathbf{J} \\
\mathbf{M}
\end{bmatrix} = \begin{bmatrix}
-\hat{n} \times \mathbf{E}^o \\
-\hat{n} \times \mathbf{H}^o
\end{bmatrix}
\]

(2)

where \( \eta_m \) is the wave impedance in the source region and

\[
\mathcal{P} = \begin{bmatrix}
\hat{n} \times (L_E^o + L_E^H)/\eta_m \\
\hat{n} \times (K_H^o + K_H^E)
\end{bmatrix}
\begin{bmatrix}
\hat{n} \times (K_E^o + K_E^H) \\
\hat{n} \times (L_H^o + L_H^E) \cdot \eta_m
\end{bmatrix}
\]

(3)

A direct Galerkin discretization by the Rao-Wilton-Glisson (RWG) basis function [8] to the PMCHWT equation usually leads to an ill-conditioned system. In the next section, the Calderón preconditioner developed in [6] will be extended to the PMCHWT equation in layered medium.

B. Calderón Preconditioner

The Calderón identities in an inhomogeneous medium can be derived as [6]:

\[
(\hat{n} \times L_H) (\hat{n} \times L_E) = (\hat{n} \times K_H)^2 - \frac{1}{4}
\]

(4)

\[
(\hat{n} \times K_H) (\hat{n} \times L_H) - (\hat{n} \times L_H) (\hat{n} \times K_E) = 0
\]

(5)

\[
- (\hat{n} \times L_E) (\hat{n} \times K_H) + (\hat{n} \times K_E) (\hat{n} \times L_E) = 0
\]

(6)

\[
(\hat{n} \times L_E) (\hat{n} \times L_H) = (\hat{n} \times K_E)^2 - \frac{1}{4}
\]

(7)
Motivated by the identities, we can construct the preconditioning operator as:
\[
Q = \left[ \begin{array}{cc}
\eta_m \times (\mathcal{L}_H^o + \mathcal{L}_H^i) \\
\eta_m \times (\mathcal{K}_H^o + \mathcal{K}_H^i)
\end{array} \right]
\left[ \begin{array}{c}
\eta_m \\
\eta_m \\
\eta_m \times (\mathcal{L}_E^o + \mathcal{L}_E^i) \end{array} \right]^{-1} \left[ \begin{array}{c}
\eta_m \\
\eta_m \\
\eta_m \\
\eta_m \times (\mathcal{L}_E^o + \mathcal{L}_E^i) \end{array} \right]^{-1}
\]
(8)
so that the preconditioned PMCHWT equation is well-behaved:
\[
\mathcal{M} \left[ \begin{array}{c}
\eta_m \times \mathbf{J} \\
\mathbf{M}
\end{array} \right] = Q \left[ \begin{array}{c}
-\hat{n} \times \mathbf{E}_{\text{inc}}^o \\
-\hat{n} \times \mathbf{H}_{\text{inc}}^o
\end{array} \right]
\]
where
\[
\mathcal{M} = Q \mathcal{P}
\]
(10)

Similarly as in EFIE [3], [6], the inner matrix operator \( \mathcal{P} \) is discretized by using the RWG basis functions; the outer matrix operator \( Q \) is discretized by the Buffa-Christiansen (BC) basis functions [9], which is a subset of the Chen-Wilton (CW) dual basis function [10]. After discretization, the matrix system becomes
\[
\mathcal{M}_{\text{dis}} = Q_{\text{BC}} \cdot \left[ \begin{array}{c}
\mathcal{G}_m^{-1} \\
\mathcal{G}_m^{-1}
\end{array} \right] \cdot \mathcal{P}_{\text{RWG}}
\]
(11)
where \( \mathcal{G}_m \) is the Gram matrix.

III. NUMERICAL RESULTS

The scattering of a dielectric sphere with \( \epsilon = 4, \mu = 1 \) and \( r = 1 \) m is analyzed. It is situated at the top layer of a 3-layer medium, shown in Fig. 1. A \( y \)-polarized plane wave of \( f = 100 \) MHz is illuminating the object along the \( -z \) direction. Fig. 2 shows the number of iterations versus the mesh density, where a targeted relative residual error of \( 10^{-6} \) is set in the GMRES solver. It is shown that the preconditioned system performs well and is stable with respect to the mesh configuration. However, the number of iterations in the PMCHWT equation is much bigger and increases rapidly as the mesh becomes denser.

![Fig. 1. A dielectric sphere with unit radius and \( \epsilon = 4, \mu = 1 \) is under the illumination of a \( f = 100 \) MHz plane wave. (a) 2D profile of the configuration; (b) 3D model of the object.](image)

![Fig. 2. Performance comparison of the PMCHWT formulation and the preconditioned system. Number of iterations required in GMRES to achieve a relative residual error of \( 10^{-6} \) versus discretization density (\( \lambda/\delta \)).](image)

IV. CONCLUSION

Analysis of EM scattering by dielectric objects in layered medium is conducted in this paper. The Calderón identities for inhomogeneous medium are leveraged to precondition the PMCHWT equation with layered kernel. Numerical results show the effectiveness of this preconditioner.

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